Diploma Programme
Programme du diplôme
Programa del Diploma

# Mathematics: applications and interpretation
# Higher level
# Paper 3

Tuesday 11 May 2021 (morning)

1 hour

---

**Instructions to candidates**

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

2221 – 7208

6 pages

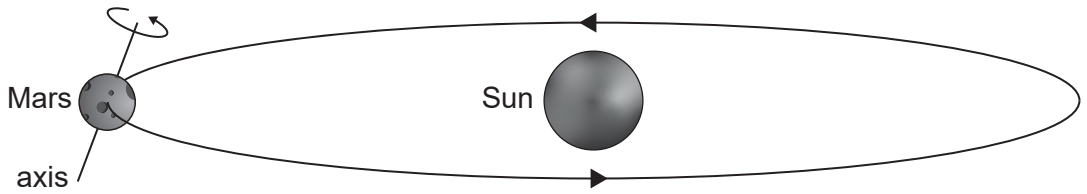
Answer **both** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**1.** [Maximum mark: 27]

**A suitable site for the landing of a spacecraft on the planet Mars is identified at a point, A. The shortest time from sunrise to sunset at point A must be found.**

Radians should be used throughout this question. All values given in the question should be treated as exact.

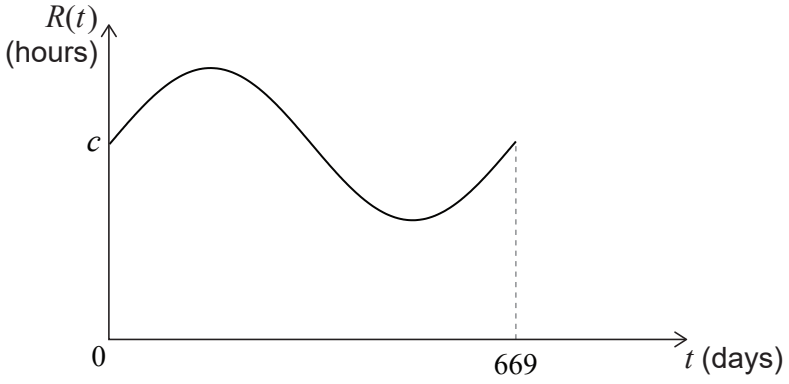
Mars completes a full orbit of the Sun in $669$ Martian days, which is one Martian year.




On day $t$, where $t \in \mathbb{Z}$, the length of time, in hours, from the start of the Martian day until sunrise at point A can be modelled by a function, $R(t)$, where

$$R(t) = a\sin(bt) + c,\ t \in \mathbb{R}.$$

The graph of $R$ is shown for one Martian year.




(a) Show that $b \approx 0.00939$. [2]

Mars completes a full rotation on its axis in $24$ hours and $40$ minutes.

(b) Find the angle through which Mars rotates on its axis each hour. [3]

**(This question continues on the following page)**

**(Question 1 continued)**

The time of sunrise on Mars depends on the angle, $\delta$, at which it tilts towards the Sun. During a Martian year, $\delta$ varies from $-0.440$ to $0.440$ radians.

The angle, $\omega$, through which Mars rotates on its axis from the start of a Martian day to the moment of sunrise, at point A, is given by $\cos\omega = 0.839\tan\delta,\ 0 \le \omega \le \pi$.

(c) (i) Show that the maximum value of $\omega = 1.98$, correct to three significant figures. [3]

(ii) Find the minimum value of $\omega$. [1]

(d) Use your answers to parts (b) and (c) to find

(i) the maximum value of $R(t)$; [2]

(ii) the minimum value of $R(t)$. [1]

(e) Hence show that $a = 1.6$, correct to two significant figures. [2]

(f) Find the value of $c$. [2]

Let $S(t)$ be the length of time, in hours, from the start of the Martian day until **sunset** at point A on day $t$. $S(t)$ can be modelled by the function

$$S(t) = 1.5\sin(0.00939t + 2.83) + 18.65.$$

The length of time between sunrise and sunset at point A, $L(t)$, can be modelled by the function

$$L(t) = 1.5\sin(0.00939t + 2.83) - 1.6\sin(0.00939t) + d.$$

(g) Find the value of $d$. [2]

Let $f(t) = 1.5\sin(0.00939t + 2.83) - 1.6\sin(0.00939t)$ and hence $L(t) = f(t) + d$.

$f(t)$ can be written in the form $\mathrm{Im}(z_1 - z_2)$, where $z_1$ and $z_2$ are complex functions of $t$.

(h) (i) Write down $z_1$ and $z_2$ in exponential form, with a constant modulus. [3]

(ii) Hence or otherwise find an equation for $L$ in the form $L(t) = p\sin(qt + r) + d$, where $p,\ q,\ r,\ d \in \mathbb{R}$. [4]

(iii) Find, in hours, the shortest time from sunrise to sunset at point A that is predicted by this model. [2]

**2.** [Maximum mark: 28]

**A firm wishes to review its recruitment processes. This question considers the validity and reliability of the methods used.**

Every year an accountancy firm recruits new employees for a trial period of one year from a large group of applicants.

At the start, all applicants are interviewed and given a rating. Those with a rating of either *Excellent*, *Very good* or *Good* are recruited for the trial period. At the end of this period, some of the new employees will stay with the firm.

It is decided to test how valid the interview rating is as a way of predicting which of the new employees will stay with the firm.

Data is collected and recorded in a contingency table.

| | **Interview rating** | | |
|---|---|---|---|
| | **Excellent** | **Very good** | **Good** |
| **Stay** | 12 | 20 | 19 |
| **Leave** | 10 | 15 | 24 |

(a) Use an appropriate test, at the $5\%$ significance level, to determine whether a new employee staying with the firm is independent of their interview rating. State the null and alternative hypotheses, the $p$-value and the conclusion of the test. [6]

The next year's group of applicants are asked to complete a written assessment which is then analysed. From those recruited as new employees, a random sample of size $18$ is selected.

The sample is stratified by department. Of the $91$ new employees recruited that year, $55$ were placed in the national department and $36$ in the international department.

(b) Show that $11$ employees are selected for the sample from the national department. [2]

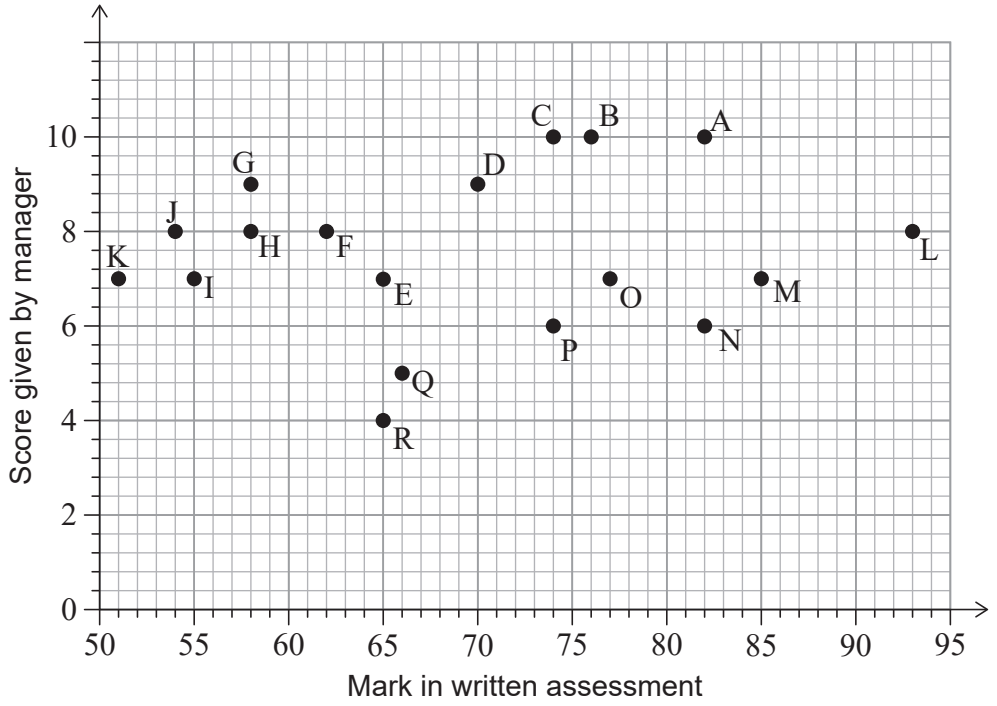
**(This question continues on the following page)**

**(Question 2 continued)**

At the end of their first year, the level of performance of each of the 18 employees in the sample is assessed by their department manager. They are awarded a score between 1 (low performance) and 10 (high performance).

The marks in the written assessment and the scores given by the managers are shown in both the table and the scatter diagram.

| | National department | | | | | | | | | | | International department | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Employee** | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R |
| **Mark in written assessment** | 82 | 76 | 74 | 70 | 65 | 62 | 58 | 58 | 55 | 54 | 51 | 93 | 85 | 82 | 77 | 74 | 66 | 65 |
| **Score given by manager** | 10 | 10 | 10 | 9 | 7 | 8 | 9 | 8 | 7 | 8 | 7 | 8 | 7 | 6 | 7 | 6 | 5 | 4 |




The firm decides to find a Spearman's rank correlation coefficient, $r_s$, for this data.

(c) (i) Without calculation, explain why it might not be appropriate to calculate a correlation coefficient for the whole sample of 18 employees. [2]

(ii) Find $r_s$ for the seven employees working in the **international** department. [4]

(iii) Hence comment on the validity of the written assessment as a measure of the level of performance of employees in this department. Justify your answer. [2]

**(This question continues on the following page)**

**(Question 2 continued)**

The same seven employees are given the written assessment a second time, at the end of the first year, to measure its reliability. Their marks are shown in the table below.

| | **International department** | | | | | | |
|---|---|---|---|---|---|---|---|
| | L | M | N | O | P | Q | R |
| **First mark** | 93 | 85 | 82 | 77 | 74 | 66 | 65 |
| **Second mark** | 90 | 92 | 85 | 73 | 79 | 71 | 65 |

(d) (i) State the name of this type of test for reliability. [1]

(ii) For the data in this table, test the null hypothesis, $H_0 : \rho = 0$, against the alternative hypothesis, $H_1 : \rho > 0$, at the $5\%$ significance level. You may assume that all the requirements for carrying out the test have been met. [4]

(iii) Hence comment on the reliability of the written assessment. [1]

(e) The written assessment is in five sections, numbered $1$ to $5$. At the end of the year, the employees are also given a score for each of five professional attributes: V, W, X, Y and Z.

The firm decides to test the hypothesis that there is a correlation between the mark in a section and the score for an attribute.

They compare marks in **each** of the sections with scores for **each** of the attributes.

(i) Write down the number of tests they carry out. [1]

(ii) The tests are performed at the $5\%$ significance level.

Assuming that:
- there is no correlation between the marks in any of the sections and scores in any of the attributes,
- the outcome of each hypothesis test is independent of the outcome of the other hypothesis tests,

find the probability that at least one of the tests will be significant. [4]

(iii) The firm obtains a significant result when comparing section $2$ of the written assessment and attribute X. Interpret this result. [1]

**References:**