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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 2

Friday 7 May 2021 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2221 – 7215

7 pages

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 19]

A medical centre is testing patients for a certain disease. This disease occurs in $5\,\%$ of the population.

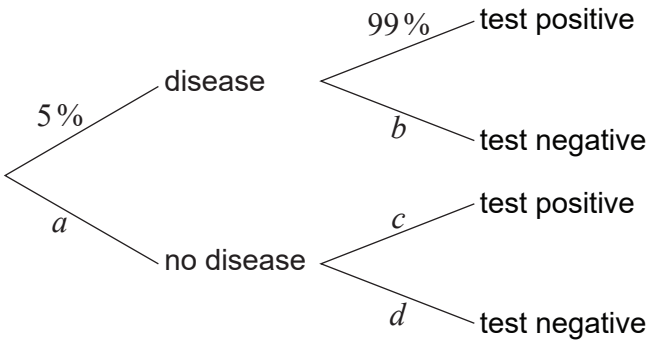
They test every patient who comes to the centre on a particular day.

(a) State the sampling method being used. [1]

It is intended that if a patient has the disease, they test "positive", and if a patient does not have the disease, they test "negative".

However, the tests are not perfect, and only $99\,\%$ of people who have the disease test positive. Also, $2\,\%$ of people who **do not** have the disease test positive.

The tree diagram shows some of this information.



(b) Write down the value of

(i) a.

(ii) b.

(iii) c.

(iv) d. [4]

(This question continues on the following page)

(Question 1 continued)

(c) Use the tree diagram to find the probability that a patient selected at random

(i) will not have the disease and will test positive.

(ii) will test negative.

(iii) has the disease given that they tested negative. [8]

(d) The medical centre finds the actual number of positive results in their sample is different than predicted by the tree diagram. Explain why this might be the case. [1]

The staff at the medical centre looked at the care received by all visiting patients on a randomly chosen day. All the patients received at least one of these services: they had medical tests (M), were seen by a nurse (N), or were seen by a doctor (D). It was found that:

- 78 had medical tests,
- 45 were seen by a nurse;
- 30 were seen by a doctor;
- 9 had medical tests and were seen by a doctor and a nurse;
- 18 had medical tests and were seen by a doctor but were not seen by a nurse;
- 11 patients were seen by a nurse and had medical tests but were not seen by a doctor;
- 2 patients were seen by a doctor without being seen by nurse and without having medical tests.

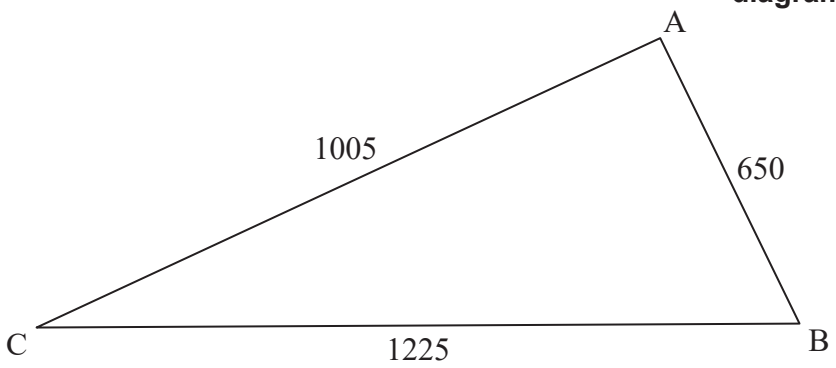
(e) Draw a Venn diagram to illustrate this information, placing all relevant information on the diagram. [3]

(f) Find the total number of patients who visited the centre during this day. [2]

2. [Maximum mark: 15]

A farmer owns a field in the shape of a triangle ABC such that $AB = 650\,\text{m}$, $AC = 1005\,\text{m}$ and $BC = 1225\,\text{m}$.

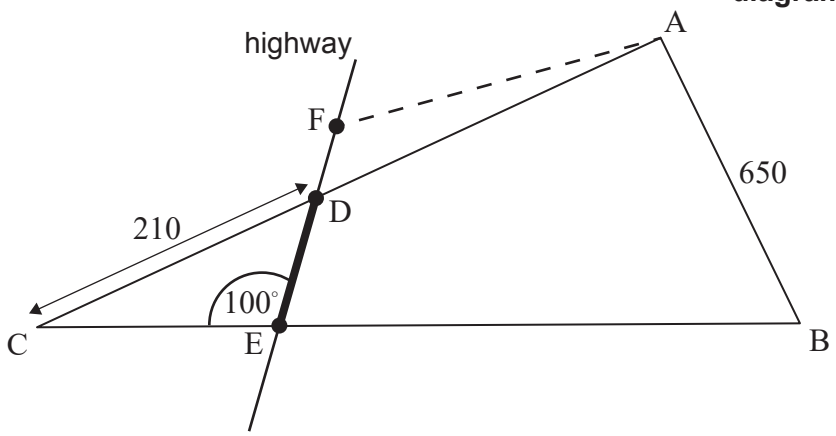
diagram not to scale



(a) Find the size of $A\hat{C}B$. [3]

The local town is planning to build a highway that will intersect the borders of the field at points D and E, where $DC = 210\,\text{m}$ and $C\hat{E}D = 100^\circ$, as shown in the diagram below.

diagram not to scale



(b) Find DE. [3]

The town wishes to build a carpark here. They ask the farmer to exchange the part of the field represented by triangle DCE. In return the farmer will get a triangle of equal area ADF, where F lies on the same line as D and E, as shown in the diagram above.

(c) Find the area of triangle DCE. [5]

(d) Estimate DF. You may assume the highway has a width of zero. [4]

3. [Maximum mark: 16]

A new concert hall was built with 14 seats in the first row. Each subsequent row of the hall has two more seats than the previous row. The hall has a total of 20 rows.

(a) Find:

(i) the number of seats in the last row.

(ii) the total number of seats in the concert hall. [5]

The concert hall opened in 2019. The average number of visitors per concert during that year was 584. In 2020, the average number of visitors per concert increased by 1.2%.

(b) Find the average number of visitors per concert in 2020. [2]

The concert organizers use this data to model future numbers of visitors. It is assumed that the average number of visitors per concert will continue to increase each year by 1.2%.

(c) Determine the first year in which this model predicts the average number of visitors per concert will exceed the total seating capacity of the concert hall. [5]

It is assumed that the concert hall will host 50 concerts each year.

(d) Use the average number of visitors per concert per year to predict the **total** number of people expected to attend the concert hall from when it opens until the end of 2025. [4]

4. [Maximum mark: 14]

It is known that the weights of male Persian cats are normally distributed with mean $6.1\,\text{kg}$ and variance $0.5^2\,\text{kg}^2$.

(a) Sketch a diagram showing the above information. [2]

(b) Find the proportion of male Persian cats weighing between $5.5\,\text{kg}$ and $6.5\,\text{kg}$. [2]

A group of 80 male Persian cats are drawn from this population.

(c) Determine the expected number of cats in this group that have a weight of less than $5.3\,\text{kg}$. [3]

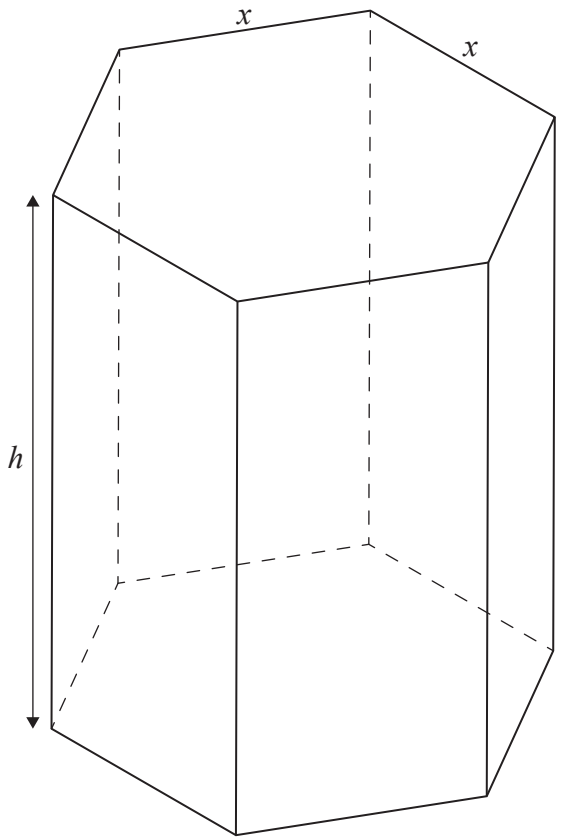
(d) It is found that 12 of the cats weigh more than $x\,\text{kg}$. Estimate the value of x. [3]

(e) Ten of the cats are chosen at random. Find the probability that exactly one of them weighs over $6.25\,\text{kg}$. [4]

5. [Maximum mark: 16]

A hollow chocolate box is manufactured in the form of a right prism with a regular hexagonal base. The height of the prism is $h\,\text{cm}$, and the top and base of the prism have sides of length $x\,\text{cm}$.

diagram not to scale



(a) Given that $\sin 60^\circ = \frac{\sqrt{3}}{2}$, show that the area of the base of the box is equal to $\frac{3\sqrt{3}x^2}{2}$. [2]

(b) Given that the total external surface area of the box is $1200\,\text{cm}^2$, show that the volume of the box may be expressed as $V = 300\sqrt{3}\,x - \frac{9}{4}x^3$. [5]

(c) Sketch the graph of $V = 300\sqrt{3}\,x - \frac{9}{4}x^3$, for $0 \le x \le 16$. [2]

(d) Find an expression for $\frac{\mathrm{d}V}{\mathrm{d}x}$. [2]

(e) Find the value of x which maximizes the volume of the box. [2]

(f) Hence, or otherwise, find the maximum possible volume of the box. [2]

The box will contain spherical chocolates. The production manager assumes that they can calculate the exact number of chocolates in each box by dividing the volume of the box by the volume of a single chocolate and then rounding down to the nearest integer.

(g) Explain why the production manager is incorrect. [1]

References:

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