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# Mathematics: applications and interpretation
# Standard level
# Paper 2

Friday 7 May 2021 (morning)

1 hour 30 minutes

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**Instructions to candidates**

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

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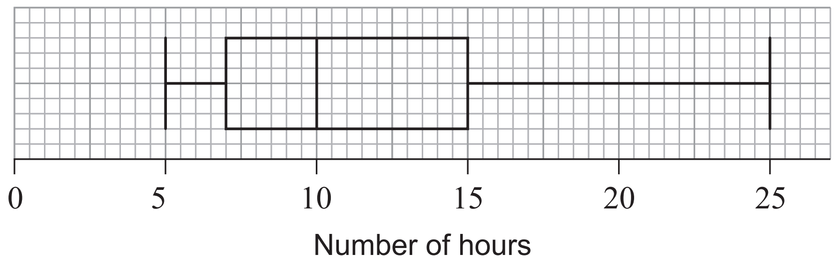
Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**1.** [Maximum mark: 18]

As part of his mathematics exploration about classic books, Jason investigated the time taken by students in his school to read the book *The Old Man and the Sea*. He collected his data by stopping and asking students in the school corridor, until he reached his target of $10$ students from **each** of the literature classes in his school.

(a) State which of the two sampling methods, systematic or quota, Jason has used. [1]

Jason constructed the following box and whisker diagram to show the number of hours students in the sample took to read this book.



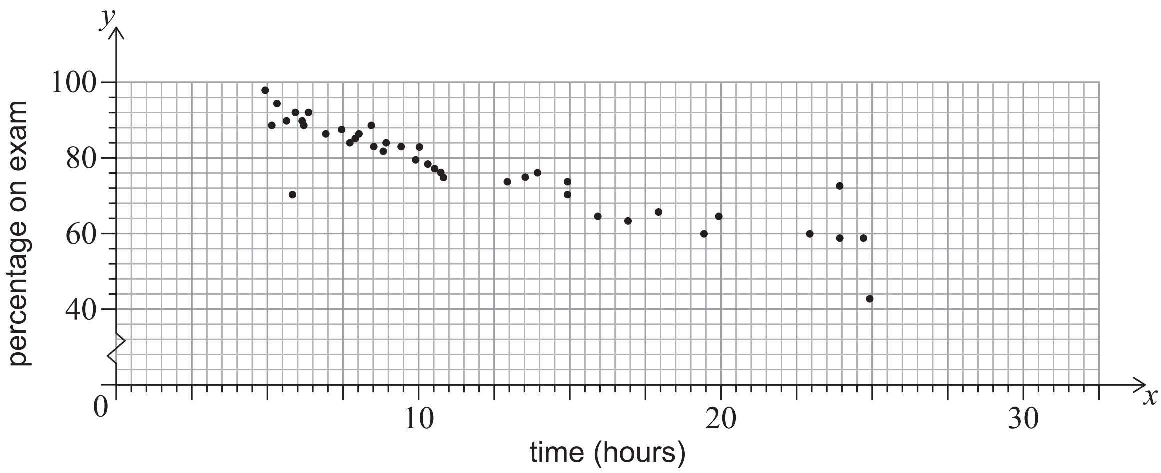

(b) Write down the median time to read the book. [1]

(c) Calculate the interquartile range. [2]

Mackenzie, a member of the sample, took $25$ hours to read the novel. Jason believes Mackenzie's time is not an outlier.

(d) Determine whether Jason is correct. Support your reasoning. [4]

For each student interviewed, Jason recorded the time taken to read *The Old Man and the Sea* ($x$), measured in hours, and paired this with their percentage score on the final exam ($y$). These data are represented on the scatter diagram.




(e) Describe the correlation. [1]

**(This question continues on the following page)**

**(Question 1 continued)**

Jason correctly calculates the equation of the regression line $y$ on $x$ for these students to be

$$y = -1.54x + 98.8\,.$$

He uses the equation to estimate the percentage score on the final exam for a student who read the book in $1.5$ hours.

(f) Find the percentage score calculated by Jason. [2]

(g) State whether it is valid to use the regression line $y$ on $x$ for Jason's estimate. Give a reason for your answer. [2]

Jason found a website that rated the 'top $50$' classic books. He randomly chose eight of these classic books and recorded the number of pages. For example, Book H is rated $44$th and has $281$ pages. These data are shown in the table.

| **Book** | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| **Number of pages ($n$)** | 4215 | 863 | 585 | 1225 | 366 | 209 | 624 | 281 |
| **Top 50 rating ($t$)** | 1 | 2 | 5 | 7 | 13 | 22 | 40 | 44 |

Jason intends to analyse the data using Spearman's rank correlation coefficient, $r_s$.

(h) Copy and complete the information in the following table. [2]

| **Book** | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| **Rank – Number of pages** | 1 | | | | | | | |
| **Rank – Top 50 Rating** | 1 | | | | | | | |

(i) (i) Calculate the value of $r_s$.

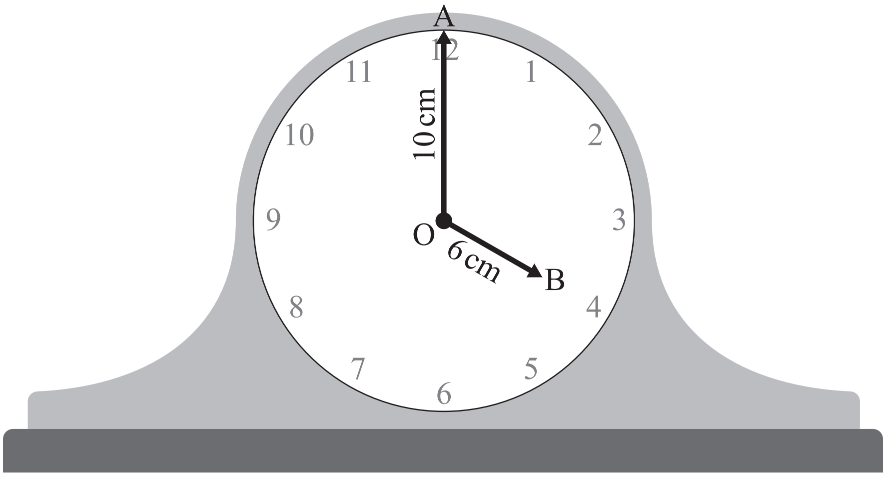
(ii) Interpret your result. [3]

**2.** [Maximum mark: 17]

The diagram below shows a circular clockface with centre O. The clock's minute hand has a length of $10\,\text{cm}$. The clock's hour hand has a length of $6\,\text{cm}$.

At 4:00 pm the endpoint of the minute hand is at point A and the endpoint of the hour hand is at point B.

**diagram not to scale**

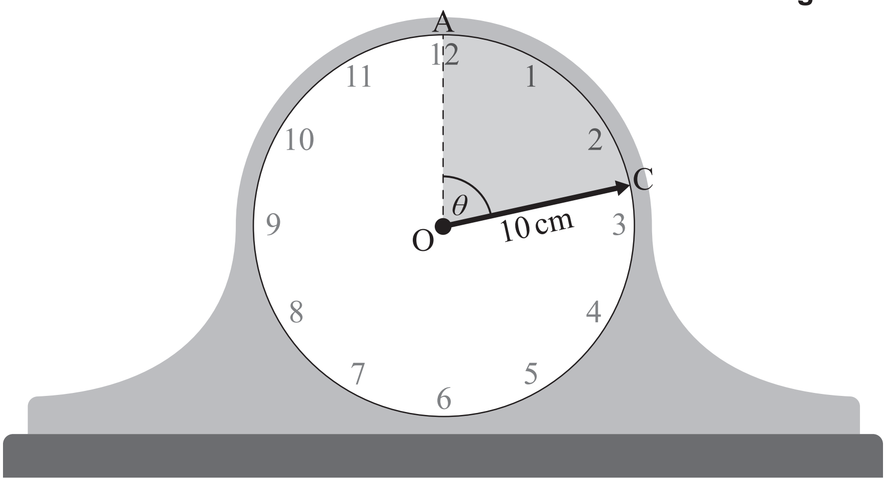



(a) Find the size of angle $\text{A}\hat{\text{O}}\text{B}$ in degrees. [2]

(b) Find the distance between points A and B. [3]

Between 4:00 pm and 4:13 pm, the endpoint of the **minute hand** rotates through an angle, $\theta$, from point A to point C. This is illustrated in the diagram.

**diagram not to scale**




(c) Find the size of angle $\theta$ in degrees. [2]

(d) Calculate the length of arc AC. [2]

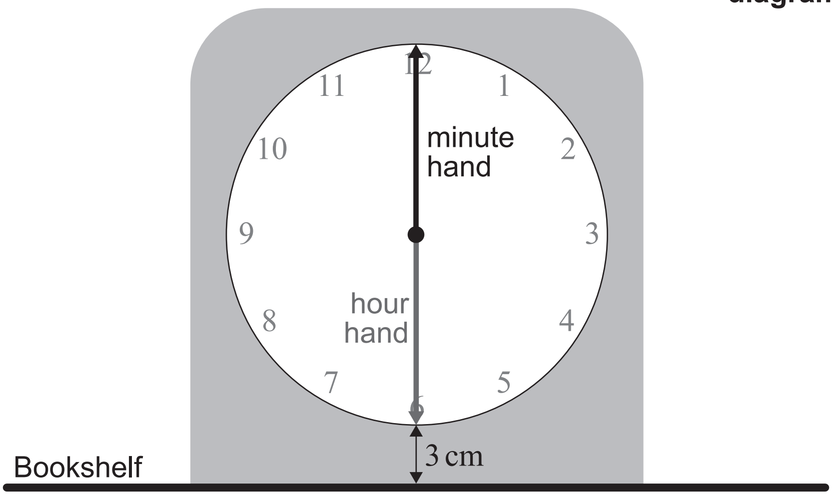
(e) Calculate the area of the shaded sector, AOC. [2]

**(This question continues on the following page)**

**(Question 2 continued)**

A **second** clock is illustrated in the diagram below. The clock face has radius $10\,\text{cm}$ with minute and hour hands both of length $10\,\text{cm}$. The time shown is $6{:}00$ am. The bottom of the clock face is located $3\,\text{cm}$ above a horizontal bookshelf.

**diagram not to scale**




(f) Write down the height of the endpoint of the minute hand above the bookshelf at $6{:}00$ am. [1]

The height, $h$ centimetres, of the endpoint of the minute hand above the bookshelf is modelled by the function

$$h(\theta) = 10\cos\theta + 13\,,\ \theta \geq 0\,,$$

where $\theta$ is the angle rotated by the minute hand from $6{:}00$ am.

(g) Find the value of $h$ when $\theta = 160^{\circ}$. [2]

The height, $g$ centimetres, of the endpoint of the **hour hand** above the bookshelf is modelled by the function

$$g(\theta) = -10\cos\left(\frac{\theta}{12}\right) + 13\,,\ \theta \geq 0\,,$$

where $\theta$ is the angle in degrees rotated by the minute hand from $6{:}00$ am.

(h) Write down the amplitude of $g(\theta)$. [1]

The endpoints of the minute hand and hour hand meet when $\theta = k$.

(i) Find the smallest possible value of $k$. [2]

**3.** [Maximum mark: 19]

**Give your answers in parts (a), (d)(i), (e) and (f) to the nearest dollar.**

Daisy invested $37\,000$ Australian dollars $(\text{AUD})$ in a fixed deposit account with an annual interest rate of $6.4\%$ compounded **quarterly**.

(a) Calculate the value of Daisy's investment after $2$ years. [3]

After $m$ months, the amount of money in the fixed deposit account has appreciated to more than $50\,000$ AUD.

(b) Find the minimum value of $m$, where $m \in \mathbb{N}$. [4]

Daisy is saving to purchase a new apartment. The price of the apartment is $200\,000$ AUD.

Daisy makes an initial payment of $25\%$ and takes out a loan to pay the rest.

(c) Write down the amount of the loan. [1]

The loan is for $10$ years, compounded monthly, with equal monthly payments of $1700$ AUD made by Daisy at the end of each month.

(d) For this loan, find

(i) the amount of interest paid by Daisy.

(ii) the annual interest rate of the loan. [5]

After $5$ years of paying off this loan, Daisy decides to pay the **remainder** in one final payment.

(e) Find the amount of Daisy's final payment. [3]

(f) Find how much money Daisy saved by making one final payment after $5$ years. [3]

**4.** [Maximum mark: 13]

The stopping distances for bicycles travelling at $20\,\text{km h}^{-1}$ are assumed to follow a normal distribution with mean $6.76\,\text{m}$ and standard deviation $0.12\,\text{m}$.

(a) Under this assumption, find, correct to four decimal places, the probability that a bicycle chosen at random travelling at $20\,\text{km h}^{-1}$ manages to stop

(i) in less than $6.5\,\text{m}$.

(ii) in more than $7\,\text{m}$. [3]

$1000$ randomly selected bicycles are tested and their stopping distances when travelling at $20\,\text{km h}^{-1}$ are measured.

(b) Find, correct to four significant figures, the expected number of bicycles tested that stop between

(i) $6.5\,\text{m}$ and $6.75\,\text{m}$.

(ii) $6.75\,\text{m}$ and $7\,\text{m}$. [3]

The measured stopping distances of the $1000$ bicycles are given in the table.

| **Measured stopping distance** | **Number of bicycles** |
|---|---|
| Less than $6.5\,\text{m}$ | 12 |
| Between $6.5\,\text{m}$ and $6.75\,\text{m}$ | 428 |
| Between $6.75\,\text{m}$ and $7\,\text{m}$ | 527 |
| More than $7\,\text{m}$ | 33 |

It is decided to perform a $\chi^2$ goodness of fit test at the $5\%$ level of significance to decide whether the stopping distances of bicycles travelling at $20\,\text{km h}^{-1}$ can be modelled by a normal distribution with mean $6.76\,\text{m}$ and standard deviation $0.12\,\text{m}$.

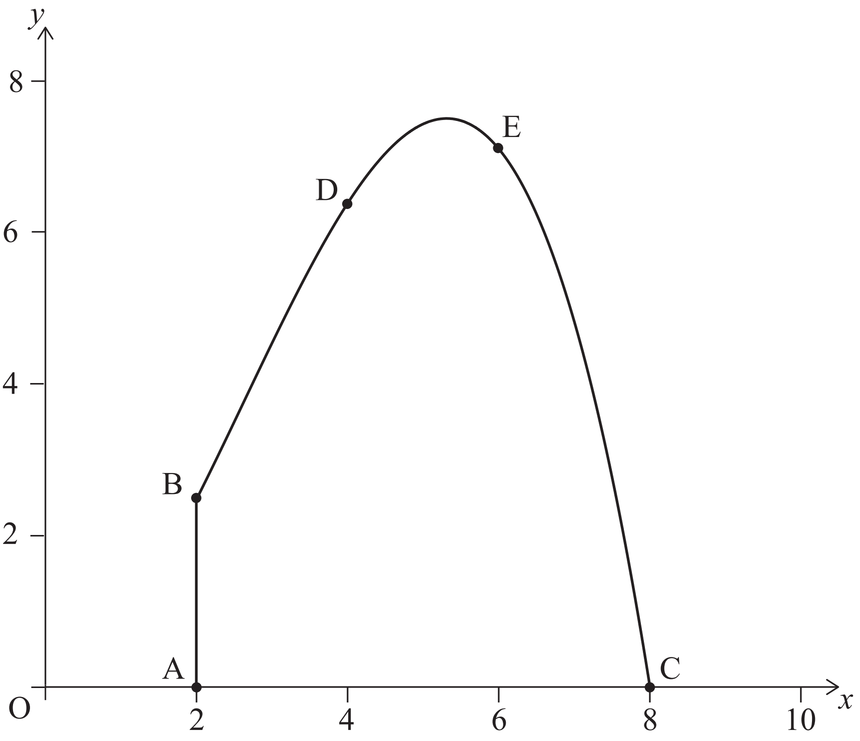
(c) State the null and alternative hypotheses. [2]

(d) Find the $p$-value for the test. [3]

(e) State the conclusion of the test. Give a reason for your answer. [2]

**5.** [Maximum mark: 13]

The cross-sectional view of a tunnel is shown on the axes below. The line [AB] represents a vertical wall located at the left side of the tunnel. The height, in metres, of the tunnel above the horizontal ground is modelled by $y = -0.1x^3 + 0.8x^2$, $2 \le x \le 8$, relative to an origin O.




Point A has coordinates (2 , 0), point B has coordinates (2 , 2.4), and point C has coordinates (8 , 0).

(a) (i) Find $\frac{dy}{dx}$.

(ii) Hence find the maximum height of the tunnel. [6]

When $x = 4$ the height of the tunnel is 6.4 m and when $x = 6$ the height of the tunnel is 7.2 m. These points are shown as D and E on the diagram, respectively.

(b) Use the trapezoidal rule, with three intervals, to estimate the cross-sectional area of the tunnel. [3]

(c) (i) Write down the integral which can be used to find the cross-sectional area of the tunnel.

(ii) Hence find the cross-sectional area of the tunnel. [4]

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**References:**