Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 1

Thursday 6 May 2021 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2221 – 7109

9 pages

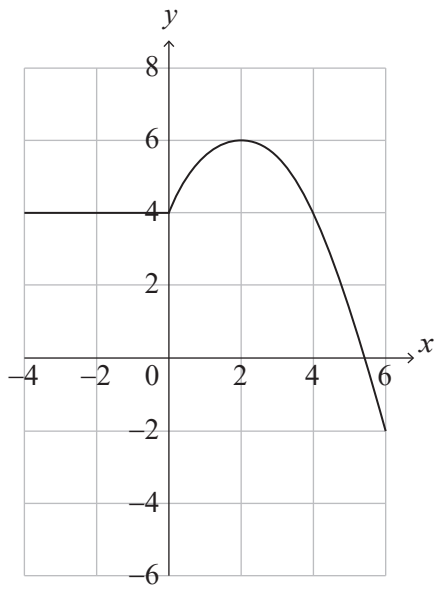
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The graph of $y = f(x)$ for $-4 \le x \le 6$ is shown in the following diagram.



(a) Write down the value of

(i) $f(2)$;

(ii) $(f \circ f)(2)$. [2]

(b) Let $g(x) = \frac{1}{2} f(x) + 1$ for $-4 \le x \le 6$. On the axes above, sketch the graph of g. [3]

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2. [Maximum mark: 4]

The diameter of a spherical planet is $6 \times 10^4 \text{km}$.

(a) Write down the radius of the planet. [1]

The volume of the planet can be expressed in the form $\pi(a \times 10^k)\text{km}^3$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

(b) Find the value of a and the value of k. [3]

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3. [Maximum mark: 5]

Consider an arithmetic sequence where $u_8 = S_8 = 8$. Find the value of the first term, u_1, and the value of the common difference, d.

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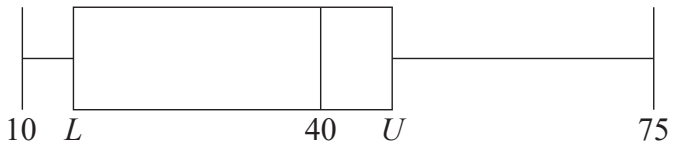
4. [Maximum mark: 5]

A research student weighed lizard eggs in grams and recorded the results. The following box and whisker diagram shows a summary of the results where L and U are the lower and upper quartiles respectively.

diagram not to scale



The interquartile range is 20 grams and there are no outliers in the results.

(a) Find the minimum possible value of U. [3]

(b) Hence, find the minimum possible value of L. [2]

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5. [Maximum mark: 7]

Consider the functions $f(x) = -(x - h)^2 + 2k$ and $g(x) = e^{x-2} + k$ where $h, k \in \mathbb{R}$.

(a) Find $f'(x)$. [1]

The graphs of f and g have a common tangent at $x = 3$.

(b) Show that $h = \frac{e+6}{2}$. [3]

(c) Hence, show that $k = e + \frac{e^2}{4}$. [3]

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6. [Maximum mark: 8]

(a) Show that $\sin 2x + \cos 2x - 1 = 2\sin x(\cos x - \sin x)$. [2]

(b) Hence or otherwise, solve $\sin 2x + \cos 2x - 1 + \cos x - \sin x = 0$ for $0 < x < 2\pi$. [6]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

Let $f(x) = mx^2 - 2mx$, where $x \in \mathbb{R}$ and $m \in \mathbb{R}$. The line $y = mx - 9$ meets the graph of f at exactly one point.

(a) Show that $m = 4$. [6]

The function f can be expressed in the form $f(x) = 4(x - p)(x - q)$, where $p, q \in \mathbb{R}$.

(b) Find the value of p and the value of q. [2]

The function f can also be expressed in the form $f(x) = 4(x - h)^2 + k$, where $h, k \in \mathbb{R}$.

(c) Find the value of h and the value of k. [3]

(d) Hence find the values of x where the graph of f is both negative and increasing. [3]

8. [Maximum mark: 16]

Let $y = \frac{\ln x}{x^4}$ for $x > 0$.

(a) Show that $\frac{dy}{dx} = \frac{1 - 4\ln x}{x^5}$. [3]

Consider the function defined by $f(x) = \frac{\ln x}{x^4}$ for $x > 0$ and its graph $y = f(x)$.

(b) The graph of f has a horizontal tangent at point P. Find the coordinates of P. [5]

(c) Given that $f''(x) = \frac{20\ln x - 9}{x^6}$, show that P is a local maximum point. [3]

(d) Solve $f(x) > 0$ for $x > 0$. [2]

(e) Sketch the graph of f, showing clearly the value of the x-intercept and the approximate position of point P. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

A biased four-sided die, A, is rolled. Let X be the score obtained when die A is rolled. The probability distribution for X is given in the following table.

x	1	2	3	4
$\mathrm{P}(X = x)$	p	p	p	$\frac{1}{2}p$

(a) Find the value of p. [2]

(b) Hence, find the value of $\mathrm{E}(X)$. [2]

A second biased four-sided die, B, is rolled. Let Y be the score obtained when die B is rolled. The probability distribution for Y is given in the following table.

y	1	2	3	4
$\mathrm{P}(Y = y)$	q	q	q	r

(c) (i) State the range of possible values of r.

(ii) Hence, find the range of possible values of q. [3]

(d) Hence, find the range of possible values for $\mathrm{E}(Y)$. [3]

Agnes and Barbara play a game using these dice. Agnes rolls die A once and Barbara rolls die B once. The probability that Agnes' score is less than Barbara's score is $\frac{1}{2}$.

(e) Find the value of $\mathrm{E}(Y)$. [6]

References:

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