

© International Baccalaureate Organization 2021

All rights reserved. No part of this product may be reproduced in any form or by any electronic or mechanical means, including information storage and retrieval systems, without the prior written permission from the IB. Additionally, the license tied with this product prohibits use of any selected files or extracts from this product. Use by third parties, including but not limited to publishers, private teachers, tutoring or study services, preparatory schools, vendors operating curriculum mapping services or teacher resource digital platforms and app developers, whether fee-covered or not, is prohibited and is a criminal offense.

More information on how to request written permission in the form of a license can be obtained from <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.

© Organisation du Baccalauréat International 2021

Tous droits réservés. Aucune partie de ce produit ne peut être reproduite sous quelque forme ni par quelque moyen que ce soit, électronique ou mécanique, y compris des systèmes de stockage et de récupération d'informations, sans l'autorisation écrite préalable de l'IB. De plus, la licence associée à ce produit interdit toute utilisation de tout fichier ou extrait sélectionné dans ce produit. L'utilisation par des tiers, y compris, sans toutefois s'y limiter, des éditeurs, des professeurs particuliers, des services de tutorat ou d'aide aux études, des établissements de préparation à l'enseignement supérieur, des fournisseurs de services de planification des programmes d'études, des gestionnaires de plateformes pédagogiques en ligne, et des développeurs d'applications, moyennant paiement ou non, est interdite et constitue une infraction pénale.

Pour plus d'informations sur la procédure à suivre pour obtenir une autorisation écrite sous la forme d'une licence, rendez-vous à l'adresse <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.

© Organización del Bachillerato Internacional, 2021

Todos los derechos reservados. No se podrá reproducir ninguna parte de este producto de ninguna forma ni por ningún medio electrónico o mecánico, incluidos los sistemas de almacenamiento y recuperación de información, sin la previa autorización por escrito del IB. Además, la licencia vinculada a este producto prohíbe el uso de todo archivo o fragmento seleccionado de este producto. El uso por parte de terceros —lo que incluye, a título enunciativo, editoriales, profesores particulares, servicios de apoyo académico o ayuda para el estudio, colegios preparatorios, desarrolladores de aplicaciones y entidades que presten servicios de planificación curricular u ofrezcan recursos para docentes mediante plataformas digitales—, ya sea incluido en tasas o no, está prohibido y constituye un delito.

En este enlace encontrará más información sobre cómo solicitar una autorización por escrito en forma de licencia: <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.

# Mathematics: analysis and approaches

## Standard level

### Paper 1

Thursday 6 May 2021 (afternoon)

Candidate session number

--	--	--	--	--	--	--	--	--	--

1 hour 30 minutes

#### Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



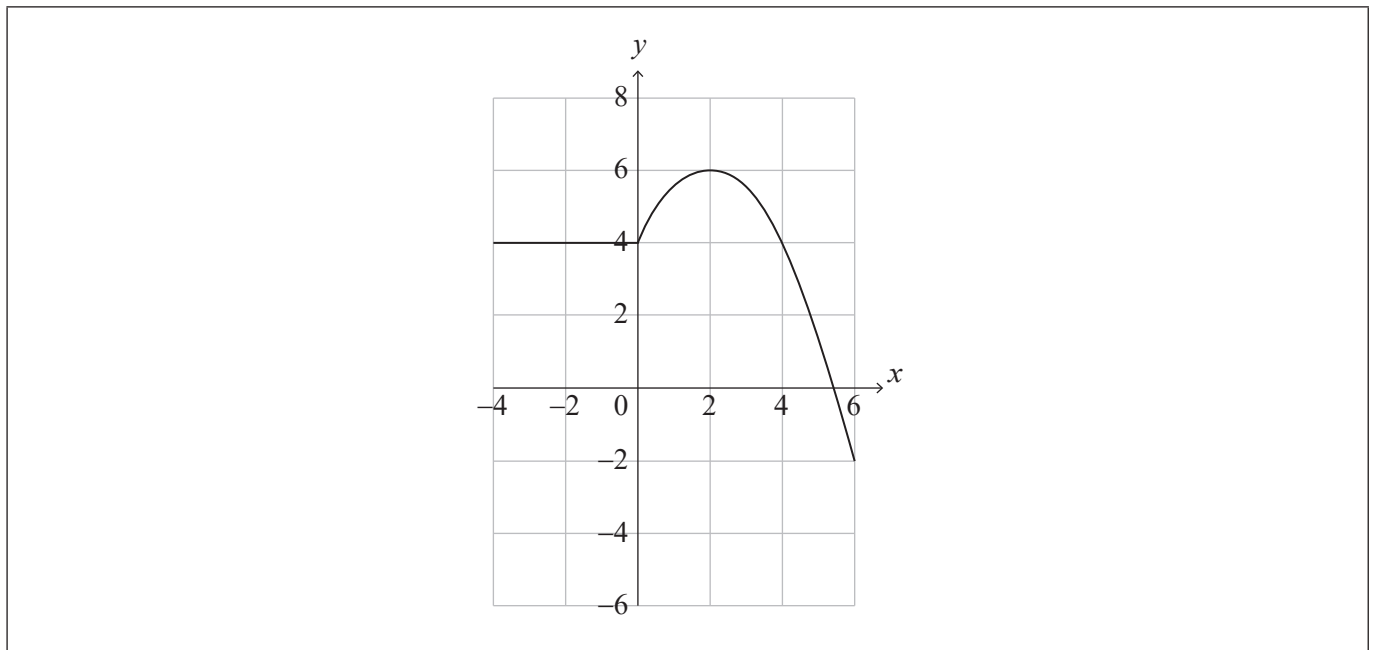
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

### Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The graph of  $y = f(x)$  for  $-4 \leq x \leq 6$  is shown in the following diagram.



(a) Write down the value of

(i)  $f(2)$ ;

(ii)  $(f \circ f)(2)$ .

[2]

(b) Let  $g(x) = \frac{1}{2}f(x) + 1$  for  $-4 \leq x \leq 6$ . On the axes above, sketch the graph of  $g$ .

[3]

.....

.....

.....

.....

.....

.....



2. [Maximum mark: 4]

The diameter of a spherical planet is  $6 \times 10^4$  km.

(a) Write down the radius of the planet. [1]

The volume of the planet can be expressed in the form  $\pi(a \times 10^k)\text{km}^3$  where  $1 \leq a < 10$  and  $k \in \mathbb{Z}$ .

(b) Find the value of  $a$  and the value of  $k$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



3. [Maximum mark: 5]

Consider an arithmetic sequence where  $u_8 = S_8 = 8$ . Find the value of the first term,  $u_1$ , and the value of the common difference,  $d$ .

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





5. [Maximum mark: 7]

Consider the functions  $f(x) = -(x - h)^2 + 2k$  and  $g(x) = e^{x-2} + k$  where  $h, k \in \mathbb{R}$ .

(a) Find  $f'(x)$ . [1]

The graphs of  $f$  and  $g$  have a common tangent at  $x = 3$ .

(b) Show that  $h = \frac{e+6}{2}$ . [3]

(c) Hence, show that  $k = e + \frac{e^2}{4}$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



6. [Maximum mark: 8]

(a) Show that  $\sin 2x + \cos 2x - 1 = 2 \sin x (\cos x - \sin x)$ . [2]

(b) Hence or otherwise, solve  $\sin 2x + \cos 2x - 1 + \cos x - \sin x = 0$  for  $0 < x < 2\pi$ . [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



Turn over



Do **not** write solutions on this page.

### Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

Let  $f(x) = mx^2 - 2mx$ , where  $x \in \mathbb{R}$  and  $m \in \mathbb{R}$ . The line  $y = mx - 9$  meets the graph of  $f$  at exactly one point.

(a) Show that  $m = 4$ . [6]

The function  $f$  can be expressed in the form  $f(x) = 4(x - p)(x - q)$ , where  $p, q \in \mathbb{R}$ .

(b) Find the value of  $p$  and the value of  $q$ . [2]

The function  $f$  can also be expressed in the form  $f(x) = 4(x - h)^2 + k$ , where  $h, k \in \mathbb{R}$ .

(c) Find the value of  $h$  and the value of  $k$ . [3]

(d) Hence find the values of  $x$  where the graph of  $f$  is both negative and increasing. [3]

8. [Maximum mark: 16]

Let  $y = \frac{\ln x}{x^4}$  for  $x > 0$ .

(a) Show that  $\frac{dy}{dx} = \frac{1 - 4 \ln x}{x^5}$ . [3]

Consider the function defined by  $f(x) = \frac{\ln x}{x^4}$  for  $x > 0$  and its graph  $y = f(x)$ .

(b) The graph of  $f$  has a horizontal tangent at point P. Find the coordinates of P. [5]

(c) Given that  $f''(x) = \frac{20 \ln x - 9}{x^6}$ , show that P is a local maximum point. [3]

(d) Solve  $f(x) > 0$  for  $x > 0$ . [2]

(e) Sketch the graph of  $f$ , showing clearly the value of the  $x$ -intercept and the approximate position of point P. [3]



Do **not** write solutions on this page.

9. [Maximum mark: 16]

A biased four-sided die, A, is rolled. Let  $X$  be the score obtained when die A is rolled. The probability distribution for  $X$  is given in the following table.

$x$	1	2	3	4
$P(X=x)$	$p$	$p$	$p$	$\frac{1}{2}p$

- (a) Find the value of  $p$ . [2]
- (b) Hence, find the value of  $E(X)$ . [2]

A second biased four-sided die, B, is rolled. Let  $Y$  be the score obtained when die B is rolled. The probability distribution for  $Y$  is given in the following table.

$y$	1	2	3	4
$P(Y=y)$	$q$	$q$	$q$	$r$

- (c) (i) State the range of possible values of  $r$ .
- (ii) Hence, find the range of possible values of  $q$ . [3]
- (d) Hence, find the range of possible values for  $E(Y)$ . [3]

Agnes and Barbara play a game using these dice. Agnes rolls die A once and Barbara rolls die B once. The probability that Agnes' score is less than Barbara's score is  $\frac{1}{2}$ .

- (e) Find the value of  $E(Y)$ . [6]

References:



Please **do not** write on this page.

Answers written on this page  
will not be marked.



12EP10

Please **do not** write on this page.

Answers written on this page  
will not be marked.



12EP11

Please **do not** write on this page.

Answers written on this page  
will not be marked.



12EP12