Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Tuesday 14 November 2017 (morning)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

16EP01

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.
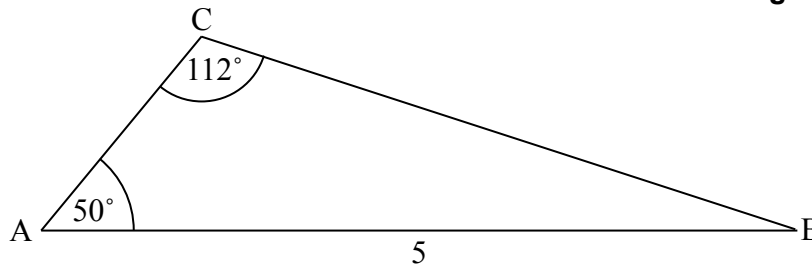
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1.   [Maximum mark: 6]

The following diagram shows a triangle ABC.

diagram not to scale

$$AB = 5\,cm, \ C\hat{A}B = 50° \ and \ A\hat{C}B = 112°$$

(a)   Find BC.   [3]

(b)   Find the area of triangle ABC.   [3]

....
....
....
....
....
....
....
....
....
....
....
....
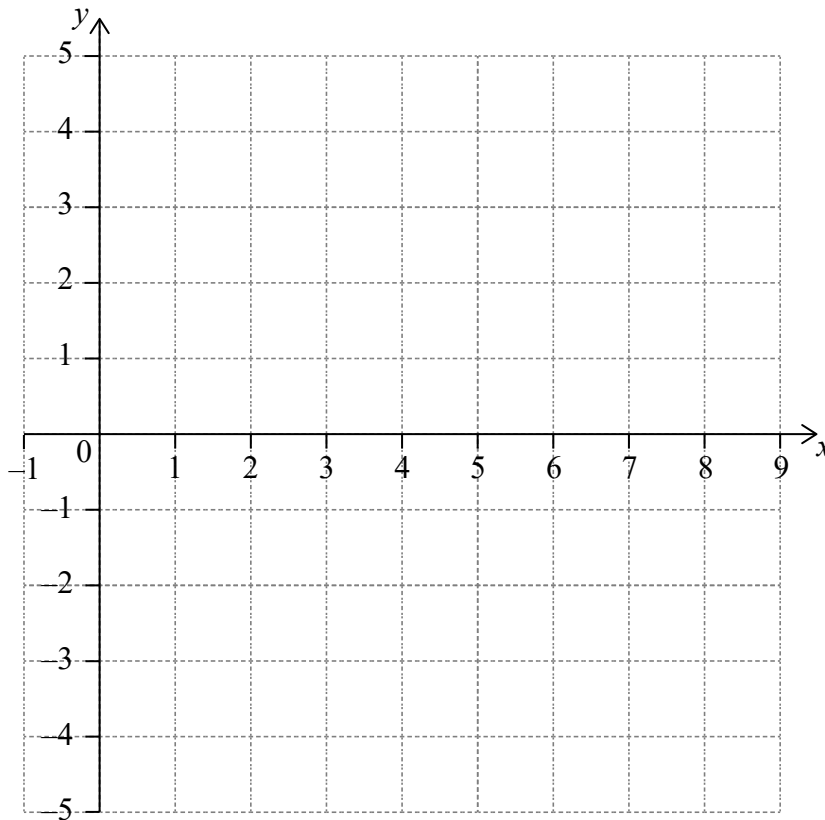
2. [Maximum mark: 7]

Let $f(x) = \dfrac{6x^2 - 4}{e^x}$, for $0 \leq x \leq 7$.

(a) Find the x-intercept of the graph of f. [2]

(b) The graph of f has a maximum at the point A. Write down the coordinates of A. [2]

(c) On the following grid, sketch the graph of f. [3]

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

3. [Maximum mark: 6]

Let $\vec{AB} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$.

(a)   Find $\left| \vec{AB} \right|$. [2]

(b)   Let $\vec{AC} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$. Find $B\hat{A}C$. [4]

4. [Maximum mark: 8]

A discrete random variable X has the following probability distribution.

X	0	1	2	3
$P(X=x)$	0.475	$2k^2$	$\dfrac{k}{10}$	$6k^2$

(a) Find the value of k. [4]

(b) Write down $P(X=2)$. [1]

(c) Find $P(X=2 \mid X>0)$. [3]

. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .

5.   [Maximum mark: 5]

Let $f(x) = 6 - \ln(x^2 + 2)$, for $x \in \mathbb{R}$. The graph of f passes through the point $(p, 4)$, where $p > 0$.

(a)   Find the value of p.   [2]

(b)   The following diagram shows part of the graph of f.

The region enclosed by the graph of f, the x-axis and the lines $x = -p$ and $x = p$ is rotated $360°$ about the x-axis. Find the volume of the solid formed.   [3]

Turn over

6.   [Maximum mark: 6]

In the expansion of $ax^3(2 + ax)^{11}$, the coefficient of the term in x^5 is 11880.
Find the value of a.

7.   [Maximum mark: 7]

The heights of adult males in a country are normally distributed with a mean of $180\,\text{cm}$ and a standard deviation of $\sigma\,\text{cm}$. 17% of these men are shorter than $168\,\text{cm}$. 80% of them have heights between $(192 - h)\,\text{cm}$ and $192\,\text{cm}$.

Find the value of h.

Turn over

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8.   [Maximum mark: 14]

Adam is a beekeeper who collected data about monthly honey production in his bee hives.
The data for six of his hives is shown in the following table.

Number of bees (N)	190	220	250	285	305	320
Monthly honey production in grams (P)	900	1100	1200	1500	1700	1800

The relationship between the variables is modelled by the regression line with
equation $P = aN + b$.

(a)   Write down the value of a and of b. [3]

(b)   Use this regression line to estimate the monthly honey production from a hive that has
270 bees. [2]

Adam has 200 hives in total. He collects data on the monthly honey production of all the
hives. This data is shown in the following cumulative frequency graph.
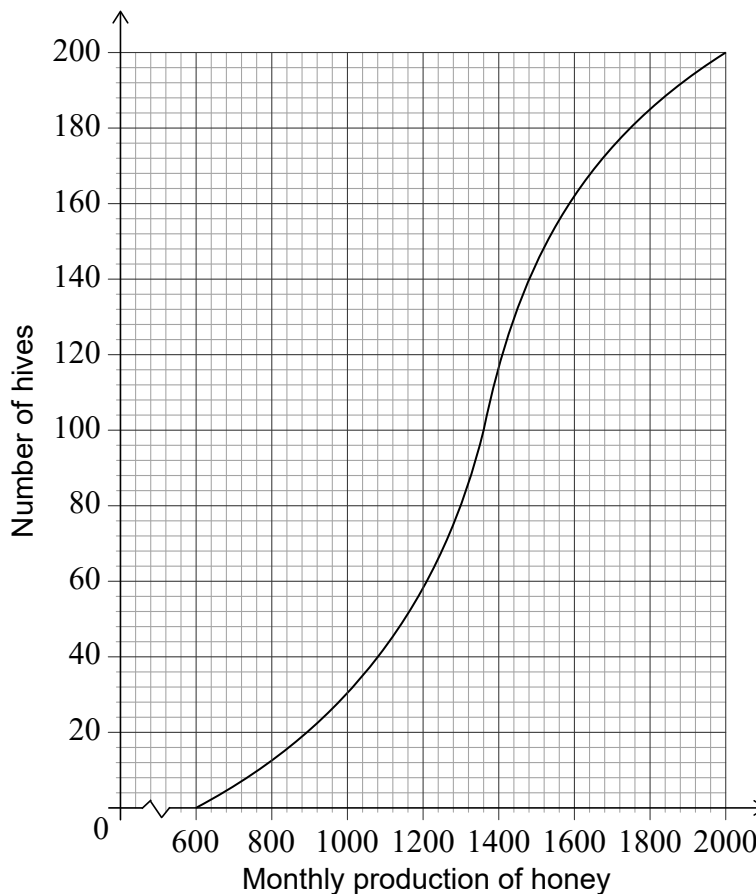
(This question continues on the following page)
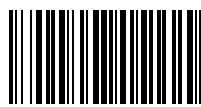
Do **not** write solutions on this page.

(Question 8 continued)

Adam's hives are labelled as low, regular or high production, as defined in the following table.

Type of hive	low	regular	high
Monthly honey production in grams (P)	$P \leq 1080$	$1080 < P \leq k$	$P > k$

(c)   Write down the number of low production hives.   [1]

Adam knows that 128 of his hives have a regular production.

(d)   Find

     (i)   the value of k;

     (ii)   the number of hives that have a high production.   [5]

(e)   Adam decides to increase the number of bees in each low production hive. Research suggests that there is a probability of 0.75 that a low production hive becomes a regular production hive. Calculate the probability that 30 low production hives become regular production hives.   [3]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Do **not** write solutions on this page.

9.   [Maximum mark: 14]

   Note: In this question, distance is in metres and time is in seconds.

   A particle P moves in a straight line for five seconds. Its acceleration at time t is given
   by $a = 3t^2 - 14t + 8$, for $0 \le t \le 5$.

   (a)   Write down the values of t when $a = 0$.   [2]

   (b)   Hence or otherwise, find all possible values of t for which the velocity of P is
          decreasing.   [2]

   When $t = 0$, the velocity of P is $3\,\mathrm{m\,s^{-1}}$.

   (c)   Find an expression for the velocity of P at time t.   [6]

   (d)   Find the total distance travelled by P when its velocity is increasing.   [4]

Do **not** write solutions on this page.

10.   [Maximum mark: 17]

Note: In this question, distance is in millimetres.

Let $f(x) = x + a\sin\left(x - \dfrac{\pi}{2}\right) + a$, for $x \geq 0$.

(a)   Show that $f(2\pi) = 2\pi$.   [3]

The graph of f passes through the origin. Let P_k be any point on the graph of f
with x-coordinate $2k\pi$, where $k \in \mathbb{N}$. A straight line L passes through all the points P_k.

(b)   (i)   Find the coordinates of P_0 and of P_1.

(ii)   Find the equation of L.   [6]

(c)   Show that the distance between the x-coordinates of P_k and P_{k+1} is 2π.   [2]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 10 continued)

Diagram 1 shows a saw. The length of the toothed edge is the distance AB.
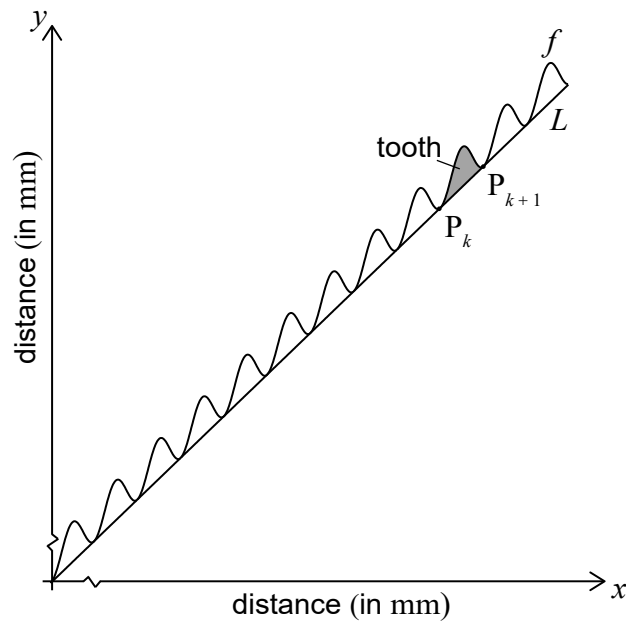
Diagram 1

diagram not to scale
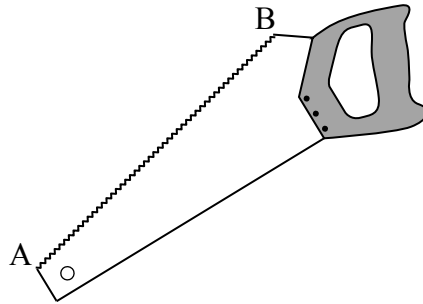
The toothed edge of the saw can be modelled using the graph of f and the line L.
Diagram 2 represents this model.

Diagram 2

diagram not to scale

The shaded part on the graph is called a tooth. A tooth is represented by the region enclosed by the graph of f and the line L, between P_k and P_{k+1}.

(d)　A saw has a toothed edge which is $300\,\text{mm}$ long. Find the number of complete teeth on this saw. [6]

Please **do not** write on this page.

Answers written on this page
will not be marked.