FURTHER MATHEMATICS

Overall grade boundaries

Higher level

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 11	12 - 22	23 - 33	34 - 45	46 - 57	58 - 69	70 - 100

Higher level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 35	36 - 52	53 - 70	71 - 88	89 - 106	107 - 150

The areas of the programme and examination which appeared difficult for the candidates

Many candidates seemed unfamiliar with probability generating functions.

Some candidates failed to attempt any of the geometry questions, suggesting that this topic was not covered in class.

Some candidates failed to appreciate the level of rigour required in questions on bijective functions.

Many candidates showed very little knowledge and understanding of the notion of cosets.

The areas of the programme and examination in which candidates appeared well prepared

Number bases are well understood in general.

It was encouraging to see that many candidates showed a good understanding of matrices, eigenvalues and eigenvectors.

Convergence of infinite series is well understood in general.

Candidates have a good understanding of linear congruences.

The strengths and weaknesses of the candidates in the treatment of individual questions

The candidature was disappointing with a number of excellent candidates but also a significant minority who should never have been entered for this examination. There was evidence in some responses that the whole syllabus had not been covered.

Question 1

This question was well answered in general.

Question 2

This question was well answered in general. Candidates should perhaps be advised to give their solution in the form of a table.

Question 3

Although most candidates were able to derive the probability generating function of X , many were then unable to deduce the probability generating function of $\Sigma\ X$ with some candidates multiplying by 4 instead of raising to the fourth power.

Question 4

In (a), Most candidates were able to derive the characteristic equation but it was extremely disappointing to see that many candidates, instead of using Vieta's formula, found the roots and used those to find the required sum and product. Solutions to (b) were often disappointing with a page of algebra going nowhere. It was pleasing to see that many candidates solved (c) correctly.

Question 5

Part (a) was well done in general but some candidates seemed to find it difficult to interpret 1(mod 2). Candidates should be aware that some questions on the Further Mathematics examination papers may involve topics from several different sections of the syllabus.

Question 6

Part (a) was well answered in general with most candidates using some sort of perpendicularity condition. It was surprising to see that some candidates used Pythagoras' Theorem which, although valid, was a much longer method. The remaining parts of the question were well answered in general although it was noted that a significant minority of candidates did not attempt this question, suggesting perhaps that some centres did not cover the geometry content.

Question 7

Part (a) was well answered in general although some candidates used their calculator to give them values of Σx and Σx^2 and then found the required quantities manually instead of using a different part of the statistics menu which would give all the answers automatically. Part (b) caused problems for some candidates who were not quite sure how to proceed.

Question 8

Most candidates knew what had to be done here although solutions were sometimes incomplete, eg in proving symmetry, some candidates simply stated that

$x^{-1}y \in H \Rightarrow y^{-1}x \in H$ without the appropriate reason. Many candidates failed to give a solution to (b).

Question 9

This was reasonably well answered in general although some candidates assumed, incorrectly, that the hexagon was regular. Some candidates thought it necessary to prove that the two tangents from a point to a circle are equal in length although this was considered to be a basic result which may be assumed.

Question 10

Part (a) was very well done by many candidates. In (b), however, many candidates ignored the word 'hence' and simply found the inverse matrix using their calculator. This, of course, was given no credit.

Question 11

Solutions to this question were generally good although, in (b), many candidates calculated the *p*-value via r and t instead of using appropriate calculator software to give the *p*-value immediately. Although the markscheme gives this method, that is so that marks can be allocated appropriately. The expectation was that candidates would write down the answer using appropriate software. It was pleasing to note that almost every candidate stated in (c) that the regression line of y on x was not suitable for predicting the value of y for a given value of x since the *p*-value suggested that the variables were independent.

Question 12

This question was well answered in general.

Question 13

Solutions to this question were often disappointing. Some candidates stated that *f* would be injective if $f\left(x,y\right)$ were single-valued. Others stated that *f* was surjective because $f\left(x,y\right) \in \mathbb{R}^{+} \times \mathbb{R}^{+}$ without mentioning that the whole space had to be covered. In many solutions, there was a marked lack of rigour.

Question 14

Part (a)(i) was well answered by many candidates although (a)(ii) proved difficult for some candidates especially those who failed to draw a diagram. In (b), most candidates solved (i) correctly but (ii) caused problems for many. Most candidates obtained the required quadratic equation but, as in Ques 4(a), many solved the equation using the quadratic formula instead of using Vieta's formula. Some candidates ignored the word 'hence' in (b)(ii) and used the tangent-secant formula for which they were given no credit. It was noted, however, that some candidates failed to attempt this question again suggesting that some centres did not cover the geometry syllabus.

Question 15

Some solutions to (a) were disappointing with inappropriate expressions being derived, eg $a\,x \equiv b(\text{mod } p) \Rightarrow x \equiv \frac{b}{a}(\text{mod } p)$. Part (b) was reasonably well done in general with a variety of methods seen to simplify $7^{17} \times 13(\text{mod}19)$ and to solve the simultaneous congruences.

Question 16

This was the worst answered question on the paper with many candidates making no attempt at a solution and many other attempts lacking any rigour. It would appear that many centres failed to cover this topic adequately.

Recommendations and guidance for the teaching of future candidates

Centres should make every attempt to cover the whole syllabus.

Some candidates need to be more aware of the full capability of the statistics menu on their calculator.

Candidates need to cover the whole of the syllabus on Sets, Relations and Groups.

Higher level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 31	32 - 48	49 - 65	66 - 83	84 - 100	101 - 150

The areas of the programme and examination which appeared difficult for the candidates

On this paper candidates found difficulty with questions on unbiased estimators, using the nearest neighbour algorithm and the travelling salesman problem, working with the equation and properties of an ellipse, using probability density functions, Poisson distributions and the accurate use of the theorems of Ceva and Menelaus. For some candidates, timing was clearly a problem, with them running out of time on the later questions. There were also indications that some candidates had not been prepared for questions on the whole syllabus.

The areas of the programme and examination in which candidates appeared well prepared

On the whole candidates appeared to have been reasonably well prepared for questions on the properties of groups, basic graph theory, Maclaurin series, simple recurrence relations and L'Hôpital's rule. The vast majority of candidates were able to make some progress with the paper, and it was pleasing to see that there were a few exceptional scripts.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question1

There were a number of completely correct answers seen to this question and it was clear that this question was a very accessible beginning to the paper for many candidates. However, a number of candidates either made no attempt at the question at all or produced working that showed little or no understanding of the topic. For example a number of candidates assumed that $E\left(X^2\right)=\left[E\left(X\right)\right]^2$. For some candidates there is evidence to suggest that this style of question on this topic was entirely unfamiliar.

Question 2

For the vast majority of candidates this was the most accessible question on the paper. Most candidates showed an understanding of how to tackle part (a) although a minority decided to draw a Cayley table and work out the 64 elements of the table to show closure. This must have used a significant amount of time, which might have been used more effectively elsewhere. Also a small minority of candidates seemed to be unfamiliar with the correct terms for the axioms of a group. Part (b) was well done with many candidates gaining full marks.

Question 3

The vast majority of candidates found this question had an easy start and were successful with part (a). In part (b) candidates were able to find the answer to part (i) but were unable to use the nearest neighbour algorithm and in a number of cases did not recognise that a cycle was required. In part (c) the vast majority of candidates were unable to successfully solve the modified problem in (c) (i).

Question 4

There were very few totally correct solutions to this question, but most candidates were able to make a reasonable start. In (a) (ii) many candidates were either unable or forgot to solve the equations. It was only the better candidates who were able to fully complete part (b), but it was pleasing to see that there was a reasonable understanding of basis.

Question 5

The majority of candidates recognised the use of integrating factors in solving the differential equation, but a failure to simplify the integrating factor or simplify it incorrectly led to problems later on. The majority of candidates who attempted the Maclaurin series question gained some marks on it, but a number of candidates made either algebraic or arithmetic errors.

Question 6

Part (a) caused problems for many candidates with quite a number not starting the question and a number making no meaningful progress with the question. In comparison part (b) was well done with a good number of wholly correct answers seen.

Question 7

Most candidates found this question difficult with very few candidates gaining full marks. A number of candidates clearly knew the matrix for an anticlockwise rotation of θ about the origin but struggled to show how it could be deduced. A number of different methods were correctly used for part (b) (i) but only by a limited number of candidates. Very few candidates successfully found the coordinates of the foci.

Question 8

This started well for most candidates with a majority of students able to make substantial progress in using l'Hôpital's rule. After question 2, part (a) (i) of this question was the most successful on the paper. Part (a) (ii) was rather less successful with many students assuming the base case to be $n=1$ and with a lot of poor working in terms of the rigour needed in a proof by induction. Some candidates made some progress with part (b) but a number did not recognise that the integration was connected to their answers in part (a). Part (c) was found to be the hardest question on the paper with very few candidates making significant progress. The fact that this was a question testing parts of different options in the same question may have been unusual for candidates.

Question 9

Although many weaker candidates made no significant beginning to this question or did not attempt it at all, stronger students successfully used Menelaus and Ceva's theorems successfully to gain the correct answer to part (a). The more unusual nature of part (b) caused problems for all but the strongest students and many made life difficult for themselves by not using properly labelled diagrams. It was pleasing to see, however, a number of strong students gain full marks on this question.

Recommendations and guidance for the teaching of future candidates

- Students need to cover the entire syllabus.
- Students should be encouraged to pay attention to mathematical notation and accuracy.
- Students need to be aware that they need to use properly labelled diagram when doing geometry questions.
- Teachers should emphasise the importance of students setting out their procedures in a logical fashion and that all relevant working needs to be shown.
- Students need to practice papers of a similar style in order that they understand the need to balance their time.
- Students need to be aware that question may test different aspects of different options within a single question.