FURTHER MATHEMATICS

Overall Grade Boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 34	35 - 47	48 - 60	61 - 73	74 - 100

Standard level paper one

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 15	16 - 20	21 - 26	27 - 33	34 - 39	40 - 60

General comments

On the whole the paper was found approachable and only a few candidates produced poor papers. The question coverage allowed candidates to display their knowledge of the program and their abilities in Mathematics.

The areas of the programme and examination that appeared difficult for candidates

- Samples taken from a given distribution
- Restrictions on the validity of integrals. The finer points of limits
- Algebraic manipulation of a differential equation to some standard form
- Confusion of the properties of equivalence relations and groups
- Algebraic proofs in number and graph theory
- Geometrical theorems

The areas of the programme and examination in which candidates appeared well prepared

- Standard statistical operations
- c^2 tests
- The first and easier parts of questions
- Basic geometry

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

a)i) Very few mistakes were made in this question, although sometimes variance and standard deviation were confused. Why both variance and standard deviation are needed might be something that teachers could explore.

ii) Again there were no serious problems although some candidates fail to show all the important parameters such as degrees of freedom.

b) This was found to be relatively straightforward except for using the correct variance of 144. It would be useful here to make clear the distinction between the sum of random variables and a multiple of a random variable.

Question 2

a) This was found to be the most difficult question on the paper. Whilst the question looked straightforward, indiscriminate use of the infinity symbol showed a lack of appreciation of the subtleties involved in the question. Many of the refinements required in each section were not considered. Knowledge of improper integrals was very poor. Perhaps integration is seen too easily as the application of a number of rules without much thought.

b) Generally this was quite well done. However, many candidates did not realize that it could be solved using an integrating factor and used substitution or variables separable. Whilst these last two approaches could work, the algebra involved soon became unmanageable. This led to many mistakes.

Question 3

a) This was not difficult but a surprising number of candidates were unable to do it. Care with notation and logic were lacking.

b) The question was at first straightforward but some candidates mixed up the properties of an equivalence relation with those of a group. The idea of an equivalence class is still not clearly understood by many candidates so that some were missing.

Question 4

a) A ‘verbal’ argument rather than a symbolic approach was often taken, leading to some doubt about the validity of the ‘proofs’. This discursive approach in trying to prove propositions often leads down blind alleys to confusion.

b) This was reasonably well done although clear proofs were not abundant and the same comments mentioned in (a) apply. In (ii) the word ‘Hence’ was sometimes ignored. Candidates should realize that such ‘signpost’ words are there to help them.

Question 5

The diagrams that some candidates drew were not always helpful to them and sometimes served to confuse what was required and make the problem harder than it was. Candidates were asked to use Ptolemy’s theorem but some ignored this request. Lengths of various segments were often written down without any evidence of where they came from.

Recommendation and guidance for the teaching of future candidates

- Knowledge of proof is still very poor with too many candidates. This applies particularly to the discrete and sets and groups topics.
- Facility with the application of geometrical theorems needs to be improved.
- With further mathematics students it is to be expected that they appreciate the finer points of those elements of the course that are common to both Higher Level mathematics and further mathematics.
- A secure knowledge of the process of finding limits and the techniques involved can be expected.
- The weaker candidates entered should perhaps be encouraged to make sure they have a thorough knowledge of at least 60% of the programme rather than trying to master all of it.

Standard level paper two

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 34	35 - 41	42 - 58	59 - 75	76 - 92	93 - 120

General Comments

As with paper 1, this paper had an acceptable number of parts that should have been within the capabilities of any student contemplating further mathematics. It covered a large percentage of the program and thus should have given candidates the opportunity to demonstrate what they could do.

The areas of the programme and examination that appeared difficult for candidates

- Difference between Eulerian circuits and trails
- Proofs and results from graph theory
- Power of a point involving two circles
- Symmetry of a confidence interval and when to merge cells in the chi squared test
- Degrees of freedom
- Proof of isomorphism

The areas of the programme and examination in which candidates appeared well prepared

- Properties of graphs and use of adjacency matrices
- Co-ordinate geometry of the circle. Confidence intervals for proportions
- Fermat's little theorem. Solution of Diophantine equations

- Intervals of convergence
- Cayley tables. Subgroups and generators
- Organization of successive differentiation using l'Hôpital's rule.
- Using one Maclaurin series to find a related series
- Clear proofs for isomorphism of groups and the significance of Ü in a proof

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Parts (a) to (c) and (e) did not prove unusually difficult and were answered well. Part (d) proved more problematic since there was confusion between the conditions to be satisfied for there to be a circuit and a trail. There is a difference between 'there are two odd vertices' and 'there are exactly two odd vertices'. As noted elsewhere on paper 1, appreciation of the restrictions as well as the applications of results in mathematics should both be emphasized. A final note on this question is that not all of the matrix in part (e) needed to be shown.

Question 2

a) This proof is really something that should not have presented much difficulty but the appearance in (i) of a logical progression from the beginning to the conclusion of the proof was a rarity. In (ii) the circle equations were usually rearranged correctly but the centres and radii were not stated so that subsequent lines of the solution seemed to appear from nowhere.

b) This was not a difficult question but again too many candidates often left gaps in their solutions perhaps thinking that what they were doing was obvious and needed no written support.

Question 3

a) This part was usually well done.

b)i) Interval symmetry was often missed which was a pity since this provided a nice way to get to the answer.

b)ii) This was not so well done since the question was a little unusual. This again shows that some students seem to expect questions (perhaps especially those on statistics) to all be the same.

c) The B(5, 0.35) was often missed off the hypotheses thereby losing the mark. Combining the last two columns was often forgotten and some candidates had difficulty deciding on the number of degree of freedom.

Question 4

a) Some creative ways of doing this part involved more work than four marks merited although there were many solutions that were less simple than that in the mark scheme.

b)i) Various ways were used and accepted.

b)ii) Alternative valid solutions were found and in general this part was found to be within the reach of most candidates.

Question 5

a) There was some confusion in differentiating twice using l'Hopital's Rule but the confusion was made worse by not taking care to write legibly.

b) This was in general well done but some students did not bother to test the end points.

c)i) This was generally well done with various approaches being used.

c)ii) This part was often done by using the differentiation all over again instead of using part (ii) again demonstrating a lack of appreciation of where time and effort can be saved in answering questions and ignoring the word 'Hence'.

c)iii) Candidates often managed to work their way through this question but with lack of clarity as to where $\frac{p}{6}$ came from.

Question 6

a)i) This was routine start to the question, but some candidates thought that commutativity was necessary as a group property.

a)ii) Showing why 1 and 5 were generators would have been appropriate since this is needed for the cyclic property of the group.

a)ii) This did not prove difficult for most candidates.

b) and c) There were some long, confused arguments that did not lead anywhere. Candidates often do not appreciate the significance of 'if' and 'only'.

Recommendation and guidance for the teaching of future candidates

Much of the routine work was done in an adequate fashion. It is to the conditions, restrictions, limitations, refinements, etc. involved with various mathematical techniques to which teachers should direct their students' attention. Often it seems that students have hardly scratched the surface of a theorem and have not looked at its application in different situations or its expression in different forms. For example, Fermat's little theorem can be stated in at least two different ways (one a congruence and one not) and has a simple corollary that might be useful in solving problems. Another example would be that students found an interval of convergence but did not show too much concern as to whether or not the end points were included in the interval.

I would suggest that teachers might give their students questions that demanded more than the application of standard routines and to go further into the theoretical background of some of the topics in the course. This is particularly important in statistics where there is a tendency to expect only five or six standard questions. An important aspect of this course is the appreciation of proof. This applies in all the topics so that time is not wasted looking at where theorems come from in geometry, the theoretical underpinning of the use of l'Hôpital's rule, the notion of infinity, the derivation of the many results in statistics (even if the numerical work can be done on a calculator), even why an arithmetic sequence is so called, etc.

There were also too many algebraic errors and clumsy ways of organizing algebraic processes and some students make it very difficult for the examiner to be able to read their work. It seems obvious but candidates should be encouraged to write legibly and not assume that examiners will fill in the gaps left in proofs and calculations.