International Baccalaureate®
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FURTHER MATHEMATICS

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 13	14 – 27	28 – 38	39 – 49	50 – 61	62 – 71	72 – 100

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 7	8 – 14	15 – 23	24 – 29	30 – 36	37 – 42	43 – 60

The areas of the programme and examination that appeared difficult for candidates

- Questions on equivalence relations which involve matrices continue to cause problems for many candidates.
- Some candidates find difficulty with proving results in number theory.

The areas of the programme and examination in which candidates appeared well prepared

- Candidates are generally able to solve straightforward problems on statistical inference.
- Questions on graph theory are well answered in general.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

This question was well answered by many candidates. A common but fairly minor error in terms of loss of marks was confusion between the two hypotheses.

Question 2

This question was not well done in general, again illustrating that questions involving both matrices and equivalence relations tend to cause problems for candidates. A common error was to assume, incorrectly, that $\boldsymbol{A}R\boldsymbol{B}$ and $\boldsymbol{B}R\boldsymbol{C}$ $\Rightarrow A = BX$ and $B = CX$, not realizing that a different 'x' is required each time. In proving that *R* is an equivalence relation, consideration of the determinant is necessary in this question although many candidates neglected to do this.

Question 3

Many candidates wrote down the adjacency matrix correctly and then proceeded to raise it to the fourth power, although at least one candidate actually enumerated, correctly, the 13 walks. Part (b) was reasonably well done with many candidates realising that, although the degrees of the vertices were the same, their relative positions meant that the two graphs were not isomorphic.

Question 4

Solutions to this question were extremely disappointing in general. Many candidates spotted that there was a periodicity in the values of the polynomial as far as $n = 4$ and even $n = 8$ but then the majority simply assumed that this would continue without any need for proof. This is equivalent to noting that $2^{2n+1} - 1$ is prime for $n =$ 1, 2 and 3 and simply assuming that this will be true for larger values of *n*.

Question 5

Solutions were often disappointing with some candidates even unable to find the integrating factor because of an inability to integrate $2\tan x$. Some candidates who found the integrating factor correctly were then unable to integrate $\sin x \sec^2 x$ and others omitted the constant of integration. Some of the candidates who obtained the correct expression for y failed to realise that the quickest way to find the maximum value was to plot the graph of y on their calculator.

Question 6

Most candidates solved (a) correctly although some used similar triangles instead of the more obvious tangent-secant theorem. Although (b) and then (c) were fairly well signposted, many candidates were unable to cope with the required algebra.

Recommendation and guidance for the teaching of future candidates

- Students need to cover the entire syllabus. It would appear from the scripts that a number of students had not done this.
- More attention should be paid to proof in the context of number theory.
- Candidates need to be aware that many of the questions will involve a certain amount of core work and they need to be prepared for that.

Standard level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 17	18 – 35	36 – 46	47 – 59	60 – 73	74 – 86	87 – 120

The areas of the programme and examination that appeared difficult for the candidates

- Candidates continue to find difficulty with geometry problems, particularly those which require an initial addition to a given diagram.
- Candidates need to realise that the normal distribution should be used when the variance is known and the *t*-distribution when it is not.
- Candidates need to be more familiar with the use of the E-operator.

The areas of the programme and examination in which candidates appeared well prepared

On the whole candidates appeared to have been reasonably well prepared for questions on group theory and graph theory

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

This question was well answered by many candidates. The most common error in (a) was confusing associativity with commutativity. Many wholly correct or almost wholly correct answers to part (b) were seen. Those who did make errors in part (b) were usually unable to fully justify the properties of a group, could not explain why the group was cyclic or could not relate subgroups to Lagrange's theorem. Some candidates made errors in calculating the orders of the elements.

Question 2

Part A was reasonably well done by many candidates. It would appear from the scripts that, in general, candidates find Menelaus' Theorem more difficult to apply than Ceva's Theorem, probably because the choice of transversal is not always obvious. Few fully correct answers were seen to part B with most candidates unable to identify which cyclic quadrilaterals should be used.

Question 3

This question was attempted by the majority of students with at least partial success by most. Most candidates were able to give a partial explanation of the condition for a graph to have an Eulerian trail and not an Eulerian circuit, but few were able to provide the required detail. Most candidates were able to write down an Eulerian trail in G. Many candidates successfully applied Kruskal's algorithm and Dijkstra's algorithm, but a number of candidates did not appreciate the significance of the order of adding edges in Kruskal's algorithm, and the explanations of Dijkstra's algorithm were sometimes poor.

Question 4

Most candidates attempted (a) although some used the normal distribution instead of the t-distribution. Many candidates were unable even to start (b) and many of those who did filled several pages of algebra with factors such as $n/(n-1)$ prominent. Few candidates realised that the solution required only a few lines.

Question 5

Most candidates solved (a)(i) satisfactorily, but many did not realise that a conditional probability was required in (a)(ii). Only a few candidates gave a satisfactory explanation of the result in (b)(i) although some candidates who failed with that went on to make a reasonable attempt at (b)(ii). Few candidates gave a complete solution to (b)(iii) with the double use of integration by parts causing problems. It was disappointing to note that no candidate realised that T is the sum of two exponential random variables each having mean 10.

Question 6

Most candidates attempted (a), although in many cases the explanations were poor and unconvincing. It was pleasing to see that some candidates who were unable to do part (a) moved on and made a reasonable attempt at (b) and (c). Attempts at (c) were often disappointing with candidates not realising that, in this case, the sum to infinity lies between any two successive partial sums.

Recommendation and guidance for the teaching of future candidates

- Students need to cover the entire syllabus. It would appear from the scripts that a number of students had not done this.
- More attention should be paid to solving geometry problems.