Mathematical Studies SL

First examinations 2006

DIPLOMA PROGRAMME

MATHEMATICAL STUDIES SL

First examinations 2006

International Baccalaureate Organization

Buenos Aires **Cardiff** **Geneva** **New York** **Singapore**

Diploma Programme
Mathematical Studies SL

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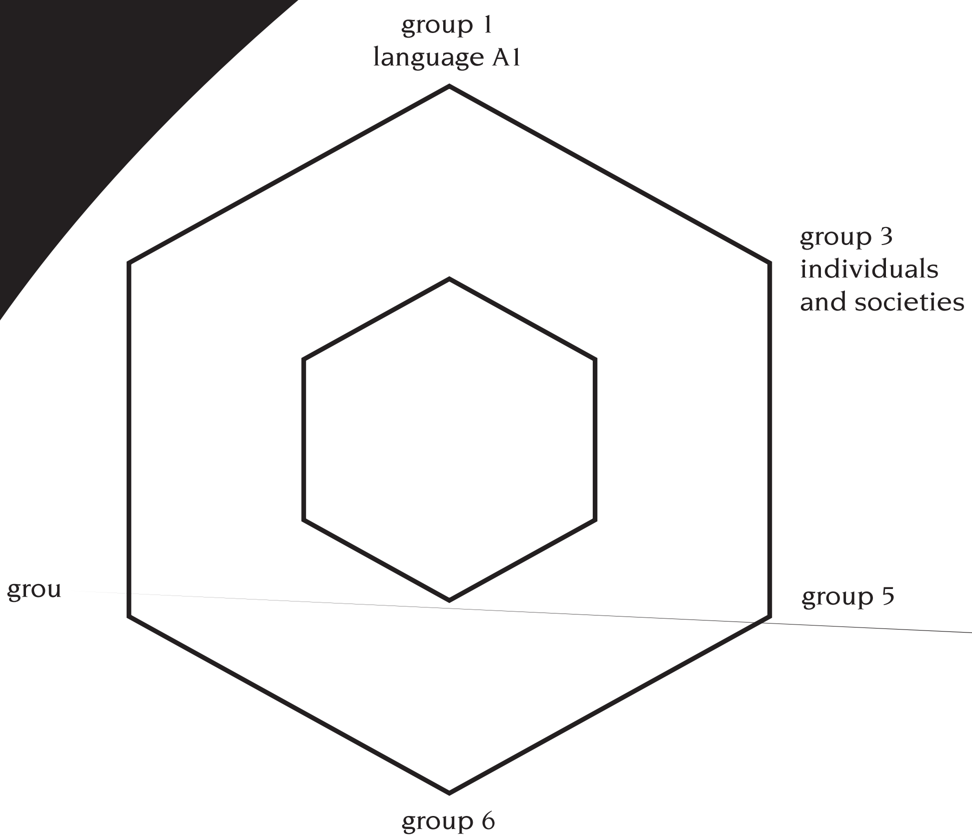

DP students are required to select one subject from each of the six subject groups. At least three and not more than four are taken at higher level (HL), the others at standard level (SL). HL courses represent 240 teaching hours; SL courses cover 150 hours. By arranging work in this fashion, students are able to explore some subjects in depth and some more broadly over the two-year period; this is a deliberate compromise between the early specialization preferred in some national systems and the breadth found in others.

Distribution requirements ensure that the science-orientated student is challenged to learn a foreign language and that the natural linguist becomes familiar with science laboratory procedures. While overall balance is maintained, flexibility in choosing HL concentrations allows the student to pursue areas of personal interest and to meet special requirements for university entrance.

Successful DP students meet three requirements in addition to the six subjects. The interdisciplinary theory of knowledge (TOK) course is designed to develop a coherent approach to learning that transcends and unifies the academic areas and encourages appreciation of other cultural perspectives. The extended essay of some 4,000 words offers the opportunity to investigate a topic of special interest and acquaints students with the independent research and writing skills expected at university. Participation in the creativity, action, service (CAS) requirement encourages students to be involved in creative pursuits, physical activities and service projects in the local, national and international contexts.

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NATURE OF THE SUBJECT

Introduction

The nature of mathematics can be summarized in a number of ways: for example, it can be seen as a well-defined body of knowledge, as an abstract system of ideas, or as a useful tool. For many people it is probably a combination of these, but there is no doubt that mathematical knowledge provides an important key to understanding the world in which we live. Mathematics can enter our lives in a number of ways: we buy produce in the market, consult a timetable, read a newspaper, time a process or estimate a length. Mathematics, for most of us, also extends into our chosen profession: artists need to learn about perspective; musicians need to appreciate the mathematical relationships within and between different rhythms; economists need to recognize trends in financial dealings; and engineers need to take account of stress patterns in physical materials. Scientists view mathematics as a language that is central to our understanding of events that occur in the natural world. Some people enjoy the challenges offered by the logical methods of mathematics and the adventure in reason that mathematical proof has to offer. Others appreciate mathematics as an aesthetic experience or even as a cornerstone of philosophy. This prevalence of mathematics in our lives provides a clear and sufficient rationale for making the study of this subject compulsory within the DP.

Summary of courses available

Because individual students have different needs, interests and abilities, there are four different courses in mathematics. These courses are designed for different types of students: those who wish to study mathematics in depth, either as a subject in its own right or to pursue their interests in areas related to mathematics; those who wish to gain a degree of understanding and competence better to understand their approach to other subjects; and those who may not as yet be aware how mathematics may be relevant to their studies and in their daily lives. Each course is designed to meet the needs of a particular group of students. Therefore, great care should be taken to select the course that is most appropriate for an individual student.

In making this selection, individual students should be advised to take account of the following types of factor.

- Their own abilities in mathematics and the type of mathematics in which they can be successful
- Their own interest in mathematics, and those particular areas of the subject that may hold the most interest for them
- Their other choices of subjects within the framework of the DP
- Their academic plans, in particular the subjects they wish to study in future
- Their choice of career

Teachers are expected to assist with the selection process and to offer advice to students about how to choose the most appropriate course from the four mathematics courses available.

Mathematical studies SL

This course is available at SL only. It caters for students with varied backgrounds and abilities. More specifically, it is designed to build confidence and encourage an appreciation of mathematics in students who do not anticipate a need for mathematics in their future studies. Students taking this course need to be already equipped with fundamental skills and a rudimentary knowledge of basic processes.

Mathematics SL

This course caters for students who already possess knowledge of basic mathematical concepts, and who are equipped with the skills needed to apply simple mathematical techniques correctly. The majority of these students will expect to need a sound mathematical background as they prepare for future studies in subjects such as chemistry, economics, psychology and business administration.

Mathematics HL

This course caters for students with a good background in mathematics who are competent in a range of analytical and technical skills. The majority of these students will be expecting to include mathematics as a major component of their university studies, either as a subject in its own right or within courses such as physics, engineering and technology. Others may take this subject because they have a strong interest in mathematics and enjoy meeting its challenges and engaging with its problems.

Further mathematics SL

This course is available at SL only. It caters for students with a good background in mathematics who have attained a high degree of competence in a range of analytical and technical skills, and who display considerable interest in the subject. Most of these students intend to study mathematics at university, either as a subject in its own right or as a major component of a related subject. The course is designed specifically to allow students to learn about a variety of branches of mathematics in depth and also to appreciate practical applications.

Mathematical studies SL—course details

This course is available at standard level (SL) only. It caters for students with varied backgrounds and abilities. More specifically, it is designed to build confidence and encourage an appreciation of mathematics in students who do not anticipate a need for mathematics in their future studies. Students taking this course need to be already equipped with fundamental skills and a rudimentary knowledge of basic processes.

The course concentrates on mathematics that can be applied to contexts related as far as possible to other subjects being studied, to common real-world occurrences and to topics that relate to home, work and leisure situations. The course includes project work, a feature unique within this group of courses: students must produce a project, a piece of written work based on personal research, guided and supervised by the teacher. The project provides an opportunity for students to carry out a mathematical investigation in the context of another course being studied, a hobby or interest of their choice using skills learned before and during the course. This process allows students to ask their own questions about mathematics and to take responsibility for a part of their own course of studies in mathematics.

The students most likely to select this course are those whose main interests lie outside the field of mathematics, and for many students this course will be their final experience of being taught formal mathematics. All parts of the syllabus have therefore been carefully selected to ensure that an approach starting with first principles can be used. As a consequence, students can use their own inherent, logical thinking skills and do not need to rely on standard algorithms and remembered formulae. Students likely to need mathematics for the achievement of further qualifications should be advised to consider an alternative mathematics course.

Because of the nature of mathematical studies, teachers may find that traditional methods of teaching are inappropriate and that less formal, shared learning techniques can be more stimulating and rewarding for students. Lessons that use an inquiry-based approach, starting with practical investigations where possible, followed by analysis of results, leading to the understanding of a mathematical principle and its formulation into mathematical language, are often most successful in engaging the interest of students. Furthermore, this type of approach is likely to assist students in their understanding of mathematics by providing a meaningful context and by leading them to understand more fully how to structure their work for the project.

AIMS

The aims of all courses in group 5 are to enable students to:

- appreciate the multicultural and historical perspectives of all group 5 courses
- enjoy the courses and develop an appreciation of the elegance, power and usefulness of the subjects
- develop logical, critical and creative thinking
- develop an understanding of the principles and nature of the subject
- employ and refine their powers of abstraction and generalization
- develop patience and persistence in problem solving
- appreciate the consequences arising from technological developments
- transfer skills to alternative situations and to future developments
- communicate clearly and confidently in a variety of contexts.

Internationalism

One of the aims of this course is to enable students to appreciate the multiplicity of cultural and historical perspectives of mathematics. This includes the international dimension of mathematics. Teachers can exploit opportunities to achieve this aim by discussing relevant issues as they arise and making reference to appropriate background information. For example, it may be appropriate to encourage students to discuss:

- differences in notation
- the lives of mathematicians set in a historical and/or social context
- the cultural context of mathematical discoveries
- the ways in which specific mathematical discoveries were made and the techniques used to make them
- how the attitudes of different societies towards specific areas of mathematics are demonstrated
- the universality of mathematics as a means of communication.

OBJECTIVES

Having followed any one of the mathematics courses in group 5, students are expected to know and use mathematical concepts and principles. In particular, students must be able to:

- read, interpret and solve a given problem using appropriate mathematical terms
- organize and present information and data in tabular, graphical and/or diagrammatic forms
- know and use appropriate notation and terminology
- formulate a mathematical argument and communicate it clearly
- select and use appropriate mathematical strategies and techniques
- demonstrate an understanding of both the significance and the reasonableness of results
- recognize patterns and structures in a variety of situations, and make generalizations
- recognize and demonstrate an understanding of the practical applications of mathematics
- use appropriate technological devices as mathematical tools
- demonstrate an understanding of and the appropriate use of mathematical modelling.

SYLLABUS OUTLINE

Mathematical Studies SL — Total 150 hrs

The course consists of the study of eight topics.

Requirements

All topics are compulsory. Students must study all the sub-topics in each of the topics in the syllabus as listed in this guide. Students are also required to be familiar with the topics listed as presumed knowledge (PK).

Syllabus content — 130 hrs

Topic 1—Introduction to the graphic display calculator	3 hrs
Topic 2—Number and algebra	14 hrs
Topic 3—Sets, logic and probability	20 hrs
Topic 4—Functions	24 hrs
Topic 5—Geometry and trigonometry	20 hrs
Topic 6—Statistics	24 hrs
Topic 7—Introductory differential calculus	15 hrs
Topic 8—Financial mathematics	10 hrs

Project — 20 hrs

The project is an individual piece of work involving the collection of information or the generation of measurements, and the analysis and evaluation of the information or measurements.

SYLLABUS DETAILS

Format of the syllabus

The syllabus to be taught is presented as three columns.

- **Content**: the first column lists, under each topic, the sub-topics to be covered.
- **Amplifications/exclusions**: the second column contains more explicit information on specific sub-topics listed in the first column. This helps to define what is required and what is not required in terms of preparing for the examination.
- **Teaching notes**: the third column provides useful suggestions for teachers. These suggestions are not compulsory.

Course of study

Teachers are required to teach all the sub-topics listed for the eight topics in the syllabus.

The topics in the syllabus do not need to be taught in the order in which they appear in this guide. Teachers should therefore construct a course of study that is tailored to the needs of their students and that integrates the areas covered by the syllabus, and, where necessary, the presumed knowledge.

Integration of project work

Work leading to the completion of the project must be fully integrated into the course of study. Full details of how to do this are given in the section on internal assessment.

Time allocation

The recommended teaching time for standard level courses is 150 hours. For mathematical studies SL, it is expected that 20 hours will be spent on work for the project. The time allocations given in this guide are approximate, and are intended to suggest how the remaining 130 hours allowed for the teaching of the syllabus might be allocated. However, the exact time spent on each topic depends on a number of factors, including the background knowledge and level of preparedness of each student. Teachers should therefore adjust these timings to correspond to the needs of their students.

Time has been allocated in each section of the syllabus to allow for the teaching of topics requiring the use of a graphic display calculator (GDC).

Use of calculators

Students are expected to have access to a GDC at all times during the course. The minimum requirements are reviewed as technology advances, and updated information will be provided to schools. It is expected that teachers and schools monitor calculator use with reference to the calculator policy. Regulations covering the types of calculators allowed are provided in the *Vade Mecum*. Further information and advice is provided in the teacher support material.

Mathematical studies SL information booklet

Because each student is required to have access to a clean copy of this booklet during the examination, it is recommended that teachers ensure students are familiar with the contents of this document from the beginning of the course. The booklet is provided by IBCA and is published separately.

Teacher support materials

A variety of teacher support materials will accompany this guide. These materials will include suggestions to help teachers integrate the use of graphic display calculators into their teaching, guidance for teachers on the marking of projects, and specimen examination papers and markschemes. These will be distributed to all schools.

External assessment guidelines

It is recommended that teachers familiarize themselves with the section on external assessment guidelines, as this contains important information about the examination papers. In particular, students need to be familiar with notation the IBO uses and the command terms, as these will be used without explanation in the examination papers.

Presumed knowledge

General

Students are not required to be familiar with all the topics listed as presumed knowledge (PK) **before** they start this course. However, they should be familiar with these topics before they take the **examinations**, because questions assume knowledge of them. Teachers must therefore ensure that any topics included in PK that are unknown to their students at the start of the course are included at an early stage.

Students must be familiar with SI (*Système International*) units of length, mass and time, and their derived units.

Topics

Number and algebra

Basic use of the four operations of arithmetic, using integers, decimals and simple fractions, including order of operations.

- *Examples*: $2(3+4\times7)=62$; $2\times3+4\times7=34$.

Prime numbers, factors and multiples.

Simple applications of ratio, percentage and proportion.

Basic manipulation of simple algebraic expressions including factorization and expansion.

- *Examples*: $ab+ac=a(b+c)$; $(x+1)(x+2)=x^2+3x+2$.

Rearranging formulae.

- *Example*: $A=\frac{1}{2}bh \Rightarrow h=\frac{2A}{b}$.

Evaluating formulae by substitution.

- *Example*: If $x = -3$, then $x^2 - 2x + 3 = 18$.

Solving linear equations in one variable.

- *Examples*: $3(x+6) - 4(x-1) = 0$; $\frac{6x}{5} + 4 = 7$.

Solving systems of linear equations in two variables.

- *Example*: $3x + 4y = 13$, $\frac{1}{3}x - 2y = -1$.

Evaluating exponential expressions with integer values.

- *Examples*: $a^b, b \in \mathbb{Z}$; $2^{-4} = \frac{1}{16}$.

Order relations $<$, $\leq$, $>$, $\geq$ and their properties.

Intervals on the real number line.

- *Example*: $2 < x \leq 5, x \in \mathbb{R}$.

Geometry and trigonometry

Basic geometric concepts: point, line, plane, angle.

Simple two-dimensional shapes and their properties, including perimeters and areas of circles, triangles, quadrilaterals and compound shapes.

The (x, y) coordinate plane.

Sine, cosine and tangent of acute angles.

Pythagoras' theorem.

Statistics

The collection of data and its representation in bar charts, pie charts and pictograms.

Financial mathematics

Basic use of commonly accepted world currencies.

- *Examples*: Swiss franc (CHF); United States dollar (US$); British pound sterling (UK£); euro (€); Japanese yen (JPY); Australian dollar (AUD).

Syllabus content

Topic 1—Introduction to the graphic display calculator 3 hrs

Aims

The aim of this section is to introduce the numerical, graphical and listing facilities of the graphic display calculator (GDC).

Details

	Content	Amplifications/exclusions	Teaching notes
1.1	Arithmetic calculations, use of the GDC to graph a variety of functions. Appropriate choice of "window"; use of "zoom" and "trace" (or equivalent) to locate points to a given accuracy. Explanations of commonly used buttons. Entering data in lists.	Extensive use of the GDC is expected throughout the course. A time allowance has been made in individual sections to allow for this extensive use.	See teacher support material.

Topic 2—Number and algebra 14 hrs

Aims

The aim of this section is to introduce students to some basic elements and concepts of mathematics. A clear understanding of these is essential for further work in the course.

Details

	Content	Amplifications/exclusions	Teaching notes
2.1	The sets of natural numbers, $\mathbb{N}$; integers, $\mathbb{Z}$; rational numbers, $\mathbb{Q}$; and real numbers, $\mathbb{R}$.	Not required: proof of irrationality, for example, of $\sqrt{2}$.	
2.2	Approximation: decimal places; significant figures. Percentage errors.	Included: an awareness of the errors that can result from premature rounding.	
	Estimation.	Included: the ability to recognize whether the results of calculations are reasonable, including reasonable values of, for example, lengths, angles and areas.	For example, lengths cannot be negative.
2.3	Expressing numbers in the form $a \times 10^k$ where $1 \le a < 10$ and $k \in \mathbb{Z}$.	Awareness and use of scientific mode on the GDC.	Work should include examples on very large and very small numbers in scientific, economic and other applications.
	Operations with numbers expressed in the form $a \times 10^k$ where $1 \le a < 10$ and $k \in \mathbb{Z}$.	All answers should be written in the form $a \times 10^k$ where $1 \le a < 10$ $(k \in \mathbb{Z})$. It is not acceptable to write down calculator displays in an examination.	
2.4	SI (*Système International*) and other basic units of measurement: for example, gram (g), metre (m), second (s), litre (l), metre per second ($m\ s^{-1}$), Celsius and Fahrenheit scales.	Included: conversion between different units.	Link with the form of the notation in 2.3, for example, $5\,\text{km} = 5 \times 10^6$ mm.

Topic 2—Number and algebra (continued)

	Content	Amplifications/exclusions	Teaching notes
2.5	Arithmetic sequences and series, and their applications.	Included: simple interest as an application. Link with simple interest 8.2.	
	Use of the formulae for the n^{th} term and the sum of the first n terms.		The formulae can be verified using numerical examples. Students may use a GDC for calculations, but they will be expected to identify clearly the first term and the common difference.
2.6	Geometric sequences and series, and their applications.	Included: compound interest as an application. Link with compound interest 8.3.	
	Use of the formulae for the n^{th} term and the sum of n terms.	Not required: use of logarithms to find n, given the sum of a series; sums to infinity.	The formulae can be verified using numerical examples. Students may use a GDC for calculations, but they will be expected to identify clearly the first term and the common ratio.
2.7	Solutions of pairs of linear equations in two variables by use of a GDC.	Included: revision of analytical methods.	
	Solutions of quadratic equations: by factorizing; by use of a GDC.	Optional: knowledge of quadratic formula. Standard terminology, such as zeros or roots and factors should be taught.	Link with quadratic functions in 4.3.

Topic 3—Sets, logic and probability 20 hrs

Aims

The aims of this section are to enable students to understand the concept of a set and to use appropriate notation, to enable them to translate between verbal and symbolic statements, to introduce the principles of logic to analyse these statements, and to enable students to analyse random events.

Details

	Content	Amplifications/exclusions	Teaching notes
3.1	Basic concepts of set theory: subsets; intersection; union; complement.	Discuss notation for set relations.	$\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ and sets of prime numbers, multiples and factors can be used as examples.
3.2	Venn diagrams and simple applications.	Included: diagrams with up to three subsets of the universal set. Not required: knowledge of de Morgan's laws.	
3.3	Sample space: event, A; complementary event, A'.		Alternative notations for the complement of a set are recognized.
3.4	Basic concepts of symbolic logic: definition of a proposition; symbolic notation of propositions.		
3.5	Compound statements: implication, $\Rightarrow$; equivalence, $\Leftrightarrow$; negation, $\neg$; conjunction, $\wedge$; disjunction, $\vee$; exclusive disjunction, $\underline{\vee}$. Translation between verbal statements, symbolic form and Venn diagrams. Knowledge and use of the "exclusive disjunction" and the distinction between it and "disjunction".	Included: an emphasis on analogies between sets and logic, for example, $(A \cap B)$ and $(a \wedge b)$.	

Topic 3—Sets, logic and probability (continued)

	Content	Amplifications/exclusions	Teaching notes
3.6	Truth tables: the use of truth tables to provide proofs for the properties of connectives; concepts of logical contradiction and tautology.	A maximum of three propositions will be used in truth tables.	Truth tables can be used to illustrate the associative and distributive properties of connectives, and for variations of implication and equivalence statements, for example, $\neg q \Rightarrow \neg p$.
3.7	Definition of implication: converse; inverse; contrapositive. Logical equivalence.		
3.8	Equally likely events. Probability of an event A given by $\mathrm{P}(A) = \frac{n(A)}{n(U)}$. Probability of a complementary event, $\mathrm{P}(A') = 1 - \mathrm{P}(A)$.		In general, probability should be introduced and taught in a practical way using coins, dice, playing cards and other examples to demonstrate random behaviour.
3.9	Venn diagrams; tree diagrams; tables of outcomes. Solution of problems using "with replacement" and "without replacement".		Examples: cards, dice and other simple cases of random selection.
3.10	Laws of probability. Combined events: $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$. Mutually exclusive events: $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$. Independent events: $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$. Conditional probability: $\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$.	Students should be encouraged to use the most appropriate method in solving individual questions. In examinations: no questions involving playing cards will be set.	Experiments using, for example, coins, dice and packs of cards can enhance understanding of experimental relative frequency versus theoretical probability. Teachers should emphasize that some problems of probability might be more easily solved with the aid of a Venn diagram or tree diagram.

Topic 4—Functions 24 hrs

Aims

The aim of this section is to develop understanding of some of the functions that can be applied to practical situations. Extensive use of a GDC is to be encouraged in this section.

Details

	Content	Amplifications/exclusions	Teaching notes
4.1	Concept of a function as a mapping. Domain and range. Mapping diagrams.	Examples should include functions defined on the sets $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$ and $\mathbb{R}$ as domains. In examinations: if the domain is $\mathbb{R}$ then the statement $x \in \mathbb{R}$ will be omitted.	In the notation, the letters *f* and *x* can each be replaced by any other letter, for example, $g(x), h(y), k(t)$.
4.2	Linear functions and their graphs, for example, $f: x \mapsto mx + c$.		Teachers should illustrate with examples from real-world problems, such as temperature conversion graphs and car hire charges. Link with equation of a line in 5.2.
4.3	The graph of the quadratic function: $f(x) = ax^2 + bx + c$. Properties of symmetry; vertex; intercepts.	Axis of symmetry, $x = -\frac{b}{2a}$. Properties should be illustrated with a GDC.	The form of the equation of the axis of symmetry may initially be found by investigation. Link with the quadratic equations in 2.7.
4.4	The exponential expression: $a^b; b \in \mathbb{Q}$ Graphs and properties of exponential functions. $f(x) = a^x$; $f(x) = a^{\lambda x}$; $f(x) = ka^{\lambda x} + c$; $k, a, c, \lambda \in \mathbb{Q}$. Growth and decay; basic concepts of asymptotic behaviour.	In examinations: students will be expected to use graphical methods, including GDCs, to solve problems.	Real-world examples, such as population growth, radioactive decay, and the cooling of a liquid can be used. Link with compound interest in 8.3.

Topic 4—Functions (continued)

	Content	Amplifications/exclusions	Teaching notes
4.5	Graphs and properties of the sine and cosine functions: $f(x) = a\sin bx + c$; $f(x) = a\cos bx + c$; $a, b, c, \in \mathbb{Q}$. Amplitude and period.	In examinations: students will be expected to use graphical methods to solve problems. GDCs should be in degree mode.	Examples of periodic phenomena may include, for example, tides, length of day and rotating wheels.
4.6	Accurate graph drawing.	Students are expected to draw accurate graphs of all the previous functions.	
4.7	Use of a GDC to sketch and analyse some simple, unfamiliar functions.	Examples: $\frac{1}{x+c}$ and higher polynomials. Students need to recognize and identify horizontal and vertical asymptotes only.	
4.8	Use of a GDC to solve equations involving simple combinations of some simple, unfamiliar functions.	Examples: $x - 2 = \frac{1}{x}$ and $5x = 3^x$.	

Topic 5—Geometry and trigonometry 20 hrs

Aims

The aims of this section are to develop the ability to draw clear diagrams, to represent information given in two dimensions, and to develop the ability to apply geometric and trigonometric techniques to problem solving.

Details

	Content	Amplifications/exclusions	Teaching notes
5.1	Coordinates in two dimensions: points; lines; midpoints. Distances between points.		
5.2	Equation of a line in two dimensions: the forms $y = mx + c$ and $ax + by + d = 0$. Gradient; intercepts. Points of intersection of lines; parallel lines; perpendicular lines.	Included: for lines with gradients, m_1 and m_2; for parallel lines, knowledge of $m_1 = m_2$; for perpendicular lines, $m_1 = -\frac{1}{m_2}$.	Link with linear functions in 4.2.
5.3	Right-angled trigonometry. Use of the ratios of sine, cosine and tangent.	In examinations: problems incorporating Pythagoras' theorem will be set. Use of the inverse trigonometric functions on a GDC is expected, but a detailed understanding of the functions themselves is not expected.	Example: solve $\sin x = 0.7$.

Topic 5—Geometry and trigonometry (continued)

	Content	Amplifications/exclusions	Teaching notes
5.4	The sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.	Not required: radian measure.	In all areas of this section, students should be encouraged to draw sufficient, well-labelled diagrams to support their solutions.
	The cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$; $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$.	In examinations: students will not be asked to derive the sine and cosine rules. The ambiguous case could be taught, but will not be examined.	
	Area of a triangle: $\frac{1}{2}ab\sin C$.		
	Construction of labelled diagrams from verbal statements.		
5.5	Geometry of three-dimensional shapes: cuboid; prism; pyramid; cylinder; sphere; hemisphere; cone.	Included: surface area and volume of these shapes.	
	Lengths of lines joining vertices with vertices, vertices with midpoints and midpoints with midpoints; sizes of angles between two lines and between lines and planes.	Included: only right prisms and square-based right pyramids.	

Topic 6—Statistics 24 hrs

Aims

The aims of this section are to introduce concepts that will prove useful in further studies of inferential statistics, and to develop techniques to describe and analyse sets of data

Details

	Content	Amplifications/exclusions	Teaching notes
6.1	Classification of data as discrete or continuous.		Student, school and/or community data can be used.
6.2	Simple discrete data: frequency tables; frequency polygons.		
6.3	Grouped discrete or continuous data: frequency tables; mid-interval values; upper and lower boundaries. Frequency histograms. Stem and leaf diagrams (stem plots).	A frequency histogram uses equal class intervals.	
6.4	Cumulative frequency tables for grouped discrete data and for grouped continuous data; cumulative frequency curves. Box and whisker plots (box plots). Percentiles; quartiles.		
6.5	Measures of central tendency. For simple discrete data: mean; median; mode. For grouped discrete and continuous data: approximate mean; modal group; 50^{th} percentile.		Students should use mid-interval values to estimate the mean of group data. They may link the median to the 50^{th} percentile and to cumulative diagrams. Students should be familiar with sigma notation (Σ).

Topic 6—Statistics (continued)

	Content	Amplifications/exclusions	Teaching notes
6.6	Measures of dispersion: range; interquartile range; standard deviation.	Included: an awareness of the concept of dispersion and an understanding of the significance of the numerical value of the standard deviation, s_n. Students are expected to use a GDC to calculate standard deviations. Students should understand the concept of population and sample. They should also be aware that the population mean, μ, and the population standard deviation, σ, are generally unknown, and that the sample mean, $\bar{x}$, and the sample standard deviation, s_n, serve only as estimates of these quantities.	Initially, the calculation of standard deviation from first principles should be demonstrated. Students should be aware that the IBO notation may differ from the notation on their GDCs.
6.7	Scatter diagrams; line of best fit, by eye, passing through the mean point. Bivariate data: the concept of correlation. Pearson's product–moment correlation coefficient: use of the formula $r = \frac{s_{xy}}{s_x s_y}$. Interpretation of positive, zero and negative correlations.	In examinations: the value of s_{xy} will be given if required. s_x represents the standard deviation of the variable *X*; s_{xy} represents the covariance of the variables *X* and *Y*. A GDC can be used to calculate *r* when raw data is given.	

Topic 6—Statistics (continued)

	Content	Amplifications/exclusions	Teaching notes
6.8	The regression line for y on x: use of the formula $y-\bar{y}=\frac{s_{xy}}{s_x^{2}}(x-\bar{x})$. Use of the regression line for prediction purposes.	Understanding of outliers is expected. Students should be aware that the regression line is less reliable when extended far beyond the region of the data. A GDC can be used to calculate the equation of the regression line when raw data is given.	y is the dependent variable.
6.9	The χ^2 test for independence: formulation of null and alternative hypotheses; significance levels; contingency tables; expected frequencies; use of the formula $\chi_{calc}^{2}=\sum\frac{(f_o-f_e)^2}{f_e}$; degrees of freedom; use of tables for critical values; p-values.	Included: h by k contingency tables where $h,k\le 4$. In examinations: questions on various commonly used significance levels (1%, 5%, 10%) will be set. The GDC can be used to calculate the χ^2 value when raw data is given. Not required: Yates' continuity correction. p-values will be used to deal with both the upper and lower one-tailed tests, but not with two-tailed tests.	

Topic 7 – Introductory differential calculus 15 hrs

Aims

The aim of this section is to introduce the concept of the gradient of the graph of a function, which is fundamental to the study of differential calculus, so that students can apply the concept of the derivative of a function to solving practical problems.

Details

	Content	Amplifications/exclusions	Teaching notes
7.1	Gradient of the line through two points, P and Q, that lie on the graph of a function. Behaviour of the gradient of the line through two points, P and Q, on the graph of a function as Q approaches P. Tangent to a curve.	Not required: formal treatment of limits. Included: solving problems involving a particular function for given values of h and x. The derivative as the gradient function; $f'(x)=\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$; $f'(x)=\frac{dy}{dx}$. In examinations: questions on differentiation from first principles will not be set.	The concept of a limit should be introduced using numerical and graphical investigations.
7.2	The principle that $f(x)=ax^n \Rightarrow f'(x)=anx^{n-1}$ $\Rightarrow f''(x)=an(n-1)x^{n-2}$. The derivative of functions of the form $f(x)=ax^n+bx^{n-1}+\ldots, n\in\mathbb{Z}$.	Included: negative integer values for n.	
7.3	Gradients of curves for given values of x. Values of x where $f'(x)$ is given. Equation of the tangent at a given point.	Not required: equation of the normal.	

Topic 7—Introductory differential calculus (continued)

	Content	Amplifications/exclusions	Teaching notes
7.4	Increasing and decreasing functions. Graphical interpretation of $f'(x) > 0$, $f'(x) = 0$, $f'(x) < 0$.		
7.5	Values of x where the gradient of a curve is 0 (zero): solution of $f'(x) = 0$.		
	Local maximum and minimum points.	Included: the concept of a function changing from increasing to decreasing and vice versa as a test for local maxima and minima. Awareness of points of inflexion with zero gradient is to be encouraged, but will not be examined.	

Topic 8 – Financial mathematics 10 hr

Aims

The aim of this section is to build a firm understanding of the concepts underlying certain financial transactions. Students can use any correct method (for example, iterative processes and finding successive approximations) that is valid for obtaining a solution to a problem in this section.

Details

	Content	Amplifications/exclusions	Teaching notes
8.1	Currency conversions.	Included: currency transactions involving commission.	
8.2	Simple interest: use of the formula $I = \frac{Crn}{100}$ where C = capital, r = % rate, n = number of time periods, I = interest.	In examinations: questions that ask students to derive the formula will not be set.	Link with arithmetic sequences in 2.5.
8.3	Compound interest: use of the formula $I = C \times \left(1 + \frac{r}{100}\right)^n - C$. Depreciation. The value of r can be positive or negative.	In examinations: questions that ask students to derive the formula will not be set. Included: the use of iterative methods, successive approximation methods or a GDC to find n (the number of time periods). Not required: use of logarithms. Compound interest can be calculated yearly, half-yearly, quarterly, monthly or daily.	Link with geometric sequences 2.6 and exponential functions 4.4.
8.4	Construction and use of tables: loan and repayment schemes; investment and saving schemes; inflation.		

ASSESSMENT OUTLINE

First examinations 2006

Mathematical studies SL

External assessment 3 hrs 80%

Written papers

Paper 1 1 hr 30 mins 40%

15 compulsory short-response questions based on the whole syllabus

Paper 2 1 hr 30 mins 40%

5 compulsory extended-response questions based on the whole syllabus

Internal assessment 20%

Project

The project is an individual piece of work involving the collection of information or the generation of measurements, and the analysis and evaluation of the information or measurements.

ASSESSMENT DETAILS

External assessment details 3 hrs 80%

General

Paper 1 and paper 2

These papers are externally set and externally marked. Together they contribute 80% of the final mark for the course. These papers are designed to allow students to demonstrate what they know and what they can do.

Calculators

For both examination papers, students must have access to a GDC at all times. Regulations covering the types of calculator allowed are provided in the *Vade Mecum*.

Mathematical studies SL information booklet

Each student must have access to a clean copy of the information booklet during the examination. One copy of this booklet is provided by IBCA as part of the examination papers mailing.

Awarding of marks

Marks may be awarded for method, accuracy, answers and reasoning, including interpretation.

In paper 1, full marks are awarded for each correct answer irrespective of the presence or absence of working.

In paper 2, full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations (in the form of, for example, diagrams, graphs or calculations). Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. All students should therefore be advised to show their working.

Paper 1 1 hr 30 mins 40%

This paper consists of **15** compulsory short-response questions.

Syllabus coverage

- Knowledge of **all** topics is required for this paper. However, not all topics are necessarily assessed in every examination session.
- The intention of this paper is to test students' knowledge across the breadth of the syllabus. However, it should not be assumed that the separate topics are given equal emphasis.

Question type

- A small number of steps is needed to solve each question.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.

Mark allocation

- This paper is worth **90** marks, representing **40%** of the final mark.
- Questions of varying levels of difficulty are set. Each question is worth **6** marks.

Paper 2 — 1 hr 30 mins — 40%

This paper consists of **5** compulsory extended-response questions.

Syllabus coverage

- Knowledge of **all** topics is required for this paper. However, not all topics are necessarily assessed in every examination session.
- Individual questions may require knowledge of more than one topic.
- The intention of this paper is to test students' knowledge of the syllabus in depth. The range of syllabus topics tested in this paper may be narrower than that tested in paper 1.
- To provide appropriate syllabus coverage of each topic, questions in this section are likely to contain two or more unconnected parts.

Question type

- Questions require extended responses involving sustained reasoning.
- Individual questions may develop a single theme or be divided into unconnected parts.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.
- Normally, each question reflects an incline of difficulty, from relatively easy tasks at the start of a question to relatively difficult tasks at the end of a question. The emphasis is on problem solving.

Mark allocation

- This paper is worth **90** marks, representing **40%** of the final mark.
- Questions in this section may be unequal in terms of length and level of difficulty. Therefore, individual questions may not necessarily be worth the same number of marks. The exact number of marks allocated to each question is indicated at the start of each question.

Guidelines

Notation

Of the various notations in use, the IBO has chosen to adopt a system of notation based on the recommendations of the International Organization for Standardization (ISO). This notation is used in the examination papers for this course without explanation. If forms of notation other than those listed in this guide are used on a particular examination paper, they are defined within the question in which they appear.

Because students are required to recognize, though not necessarily use, IBO notation in examinations, it is recommended that teachers introduce students to this notation at the earliest opportunity. Students are **not** allowed access to information about this notation in the examinations.

Students must always use correct mathematical notation, not calculator notation.

$\mathbb{N}$	the set of positive integers and zero, $\{0, 1, 2, 3, ...\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, ...\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, ...\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \mid x \in \mathbb{Q}, x > 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \mid x \in \mathbb{R}, x > 0\}$
$\{x_1, x_2, ...\}$	the set with elements $x_1, x_2, ...$
$n(A)$	the number of elements in the finite set A
$\{x \mid \quad \}$	the set of all x such that
$\in$	is an element of
$\notin$	is not an element of
$\varnothing$	the empty (null) set
U	the universal set
$\cup$	union
$\cap$	intersection
$\subset$	is a proper subset of
$\subseteq$	is a subset of
A'	the complement of the set A
$p \wedge q$	conjunction: p and q
$p \vee q$	disjunction: p or q (or both)
$p \underline{\vee} q$	exclusive disjunction: p or q (**not** both)
$\neg p$	negation: not p

$p \Rightarrow q$	implication: if p then q
$p \Leftarrow q$	implication: if q then p
$p \Leftrightarrow q$	equivalence: p is equivalent to q
$a^{1/n}$, $\sqrt[n]{a}$	a to the power $\frac{1}{n}$, n^{th} root of a (if $a \geq 0$ then $\sqrt[n]{a} \geq 0$)
$a^{-n} = \frac{1}{a^n}$	a to the power $-n$, reciprocal of a^n
$a^{1/2}$, $\sqrt{a}$	a to the power $\frac{1}{2}$, square root of a (if $a \geq 0$ then $\sqrt{a} \geq 0$)
$\approx$	is approximately equal to
$>$	is greater than
$\geq$	is greater than or equal to
$<$	is less than
$\leq$	is less than or equal to
$\not>$	is not greater than
$\not<$	is not less than
u_n	the n^{th} term of a sequence or series
d	the common difference of an arithmetic sequence
r	the common ratio of a geometric sequence
S_n	the sum of the first n terms of a sequence, $u_1 + u_2 + ... + u_n$
$\sum_{i=1}^{n} u_i$	$u_1 + u_2 + ... + u_n$
$f : A \rightarrow B$	f is a function under which each element of set A has an image in set B
$f : x \mapsto y$	f is a function under which x is mapped to y
$f(x)$	the image of x under the function f
f^{-1}	the inverse function of the function f
$\lim_{x \to a} f(x)$	the limit of $f(x)$ as x tends to a

$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$f'(x)$	the derivative of $f(x)$ with respect to x
sin, cos, tan	the circular functions
$\mathrm{A}(x, y)$	the point A in the plane with Cartesian coordinates x and y
$[\mathrm{AB}]$	the line segment with end points A and B
AB	the length of $[\mathrm{AB}]$
(AB)	the line containing points A and B
$\hat{\mathrm{A}}$	the angle at A
$\mathrm{C}\hat{\mathrm{A}}\mathrm{B}$	the angle between $[\mathrm{CA}]$ and $[\mathrm{AB}]$
$\Delta\mathrm{ABC}$	the triangle whose vertices are A, B and C
$\mathrm{P}(A)$	probability of event A
$\mathrm{P}(A')$	probability of the event "not A"
$\mathrm{P}(A \mid B)$	probability of the event A given the event B
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
$\overline{x}$	sample mean
s_n	standard deviation of the sample $s_n = \sqrt{\frac{\sum_{i=1}^{n} f_i (x_i - \overline{x})^2}{n}}$
r	Pearson's product-moment correlation coefficient
χ^2	chi-squared

Glossary of command terms

The following command terms are used without explanation on examination papers. Teachers should familiarize themselves and their students with the terms and their meanings. This list is not exhaustive. Other command terms may be used, but it should be assumed that they have their usual meaning (for example, "explain" and "estimate"). The terms included here are those that sometimes have a meaning in mathematics that is different from the usual meaning.

Further clarification and examples can be found in the teacher support material.

Write down	Obtain the answer(s), usually by extracting information. Little or no calculation is required. Working does not need to be shown.
Calculate	Obtain the answer(s) showing all relevant working. "Find" and "determine" can also be used.
Find	Obtain the answer(s) showing all relevant working. "Calculate" and "determine" can also be used.
Determine	Obtain the answer(s) showing all relevant working. "Find" and "calculate" can also be used.
Differentiate	Obtain the derivative of a function.
Integrate	Obtain the integral of a function.
Solve	Obtain the solution(s) or root(s) of an equation.
Draw	Represent by means of a labelled, **accurate** diagram or graph, using a pencil. A ruler (straight edge) should be used for straight lines. Diagrams should be drawn to scale. Graphs should have points correctly plotted (if appropriate) and joined in a straight line or smooth curve.
Sketch	Represent by means of a diagram or graph, labelled if required. A sketch should give a general idea of the required shape of the diagram or graph. A sketch of a graph should include relevant features such as intercepts, maxima, minima, points of inflexion and asymptotes.
Plot	Mark the position of points on a diagram.
Compare	Describe the similarities and differences between two or more items.
Deduce	Show a result using known information.
Justify	Give a valid reason for an answer or conclusion.
Show that	Obtain the required result (possibly using information given) without the formality of proof. "Show that" questions should not generally be "analysed" using a calculator.
Hence	Use the preceding work to obtain the required result.
Hence or otherwise	It is suggested that the preceding work is used, but other methods could also receive credit.

Weighting of objectives

Some objectives can be linked more easily to the different types of assessment. In particular, some will be assessed more appropriately in the internal assessment (as indicated in the following section) and only minimally in the examination papers.

Objective	**Percentage weighting**
Know and use mathematical concepts and principles.	15%
Read, interpret and solve a given problem using appropriate mathematical terms.	15%
Organize and present information and data in tabular, graphical and/or diagrammatic forms.	12%
Know and use appropriate notation and terminology (internal assessment).	5%
Formulate a mathematical argument and communicate it clearly.	10%
Select and use appropriate mathematical strategies and techniques.	15%
Demonstrate an understanding of both the significance and the reasonableness of results (internal assessment).	5%
Recognize patterns and structures in a variety of situations, and make generalizations (internal assessment).	3%
Recognize and demonstrate an understanding of the practical applications of mathematics (internal assessment).	3%
Use appropriate technological devices as mathematical tools (internal assessment).	15%
Demonstrate an understanding of and the appropriate use of mathematical modelling (internal assessment).	2%

Internal assessment details 20%

The purpose of the project

The specific purposes of the project are to:

- develop students' personal insight into the nature of mathematics and to develop their ability to ask their own questions about mathematics
- encourage students to initiate and sustain a piece of work in mathematics
- enable students to acquire confidence in developing strategies for dealing with new situations and problems
- provide opportunities for students to develop individual skills and techniques and to allow students with varying abilities, interests and experiences to achieve a sense of personal satisfaction in studying mathematics
- enable students to experience mathematics as an integrated organic discipline rather than fragmented and compartmentalized skills and knowledge
- enable students to see connections and applications of mathematics to other areas of interest
- provide opportunities for students to show, with confidence, what they know and what they can do.

Objectives

The project is internally assessed by the teacher and externally moderated by the IBO. Assessment criteria have been developed to address collectively all the group 5 objectives. In developing these criteria, particular attention has been given to the objectives listed here, which are most satisfactorily assessed without the time constraints imposed by written examinations.

Where appropriate in the project, students are expected to:

- organize and present information and data in tabular, graphical and/or diagrammatic forms
- know and use appropriate notation and terminology
- recognize patterns and structures in a variety of situations, and make generalizations.
- recognize and demonstrate an understanding of the practical applications of mathematics
- use appropriate technological devices as mathematical tools
- demonstrate an understanding of and the appropriate use of mathematical modelling.

Teaching and learning strategies

The following teaching and learning strategies are implied by the aims of the project.

- Students need to be provided with opportunities, as an integral part of the course, to experiment, to explore, to generate hypotheses and to ask questions.
- Students should be provided with experience of taking initiatives to develop their own mathematics in the classroom.
- Students should be encouraged to become active learners of mathematics, inside and outside the classroom.
- Topics in the syllabus should be organized to allow an active inquiry approach.
- Students should be encouraged to identify and reflect on the variety of mathematical processes that may be used to solve problems.
- Teachers should aim to offer advice about the choice of topics and titles, bearing in mind that the most valuable projects are those that reflect the student’s interests and enthusiasms.

Requirements

The project is a piece of written work based on personal research involving the collection, analysis and evaluation of data.

Each project must contain:

- a title
- a statement of the task
- measurements, information or data which has been collected and/or generated
- an analysis of the measurements, information or data
- an evaluation of the analysis
- a bibliography and footnotes, as appropriate.

Students can choose from a wide variety of project types, for example, modelling, investigations, applications and statistical surveys. Historical projects that reiterate facts but have little mathematical content are not appropriate and should be actively discouraged.

Teachers should give guidance to students in choosing appropriate areas of study for their projects. As much as possible, these areas of study should relate to the students' own interests and, while mathematical in nature, they may be based in contexts such as sport, art, music, the environment, health, travel, trade and commerce.

In developing their projects, students should make use of mathematics learned as part of the course. The level of sophistication of the mathematics should be similar to that suggested by the syllabus. It is not expected that students produce work that is outside the mathematical studies SL syllabus, and they are not penalized if they produce such work.

Introduction of the project

Project work should be incorporated into the course so that students are given the opportunity to learn the skills needed for the completion of a successful project.

Time in class can therefore be used:

- for general discussion of areas of study for project work, such as: how data can be collected or measurements generated; where data can be collected; how much data should be collected; different ways of displaying data; what steps should be taken to analyse the data; how data should be evaluated.
- to give students the opportunity to mark projects from previous years, using the assessment criteria.

Management of the project

Time allocation

The *Vade Mecum* states that a standard level course requires at least **150** teaching hours. In mathematical studies, **20** of these hours should be allocated to work connected with the project. This allows time for teachers to explain to students the requirements of the project, to discuss fully the assessment criteria and to allow class time for the development of project work.

Planning

Students need to start planning their projects as early as possible in the course. Deadlines, preferably reached by agreement between students and teachers, need to be firmly established. There needs to be a date for submission of the project title and a brief outline description, a date for the completion of data collection/generation, a date for the submission of the first draft and, of course, a date for project completion.

Discussion

Time should be allocated for discussion between the teacher and the student(s) and/or discussion between students on particular aspects of projects belonging to individual students.

Presentations

Time in class can be used for oral/visual presentations by individual students on particular aspects of their projects or on their projects as a whole.

Length

The project should not normally exceed **2,000** words, excluding diagrams, graphs, appendices and bibliography. However, it is the quality of the mathematics and the processes used and described that is important, rather than the number of words written.

Guidance

- Work set by the teacher is not appropriate for a project.
- All students should be familiar with the requirements of the project and the criteria by which it is assessed. In particular, teachers should discuss with their students the levels of achievement expected for Criterion G, Commitment.
- It should be made clear to students that all work connected with the project, including the writing of the project, should be their own. It is therefore helpful if teachers try to encourage in students a sense of responsibility for their own learning so that they accept a degree of ownership and take pride in their own work.
- Group work should not be used for projects. Each project should be based on different data collected or measurements generated.
- It must be emphasized that students are expected to consult the teacher throughout the process. The teacher is expected to give appropriate guidance at all stages of the project by, for example, directing students into more productive routes of inquiry, making suggestions for suitable sources of information, and providing advice on the content and clarity of a project in the writing-up stage.
- Teachers are encouraged to indicate to students the existence of errors but should not explicitly correct these errors.

Authenticity

Teachers must ensure that each project is the student's own work. If the teacher views the project at each stage of its development, this serves as a very effective way of ensuring that the project is the intellectual property of the student and also acts as a safeguard against plagiarism. In this way, by monitoring all stages of the development, a teacher can ensure that the project is the authentic, personal work of the student.

All external sources quoted or used must be fully referenced by use of a full bibliography and footnotes.

If in doubt, authenticity may be checked by one or more of the following methods:

- discussion with the student
- asking the student to explain the methods used and to summarize the results/conclusions
- asking the student to replicate part of the analysis using different data
- asking the student to produce the sources used.

It is also appropriate for teachers to request that each project be signed on completion to indicate that it is the student's own work.

Assessment strategies

The assessment strategies used in the classroom should:

- match the teaching and learning strategies listed in this guide
- enable students to gain new insights into the nature of mathematics
- enable students to assess the quality of their own work.

Assessment at each stage of the project can be monitored by the teacher, by class discussion of the project by other students, and by self-assessment by the owner of the project (that is, the student). Assessment should be criterion-referenced and formative, thereby enabling the student to modify, extend and improve a project throughout the course of its development. The clear intent is to encourage dialogue between teacher and student without compromising the integrity of the project as the student's own work.

Internal assessment criteria

The project is internally assessed by the teacher and externally moderated by the IBO using assessment criteria that relate to the objectives for group 5 mathematics.

Form of the assessment criteria

Each project should be assessed against the following seven criteria:

Criterion A	Introduction
Criterion B	Information/measurement
Criterion C	Mathematical processes
Criterion D	Interpretation of results
Criterion E	Validity
Criterion F	Structure and communication
Criterion G	Commitment.

Applying the assessment criteria

The method of assessment used is criterion referenced, not norm referenced. That is, the method of assessing each project judges students by their performance in relation to identified assessment criteria and not in relation to the work of other students.

Each project submitted for mathematical studies SL is assessed against the seven criteria A to G. For each assessment criterion, different levels of achievement are described that concentrate on positive achievement. The description of each achievement level represents the minimum requirement for that level to be achieved.

The aim is to find, for each criterion, the level descriptor that conveys most adequately the achievement levels attained by the student.

Read the description of each achievement level, starting with level 0, until one is reached that describes a level of achievement that has not been reached. The level of achievement gained by the student is therefore the preceding one and it is this that should be recorded.

For example, if, when considering successive achievement levels for a particular criterion, the description for level 3 does not apply then level 2 should be recorded.

For each criterion, whole numbers only may be recorded; fractions and decimals are not acceptable.

The highest achievement levels do not imply faultless performance and teachers should not hesitate to use the extremes, including zero, if they are appropriate descriptions of the work to be assessed.

The whole range of achievement levels should be awarded as appropriate. For a particular piece of work, a student who attains a high achievement level in relation to one criterion may not necessarily attain high achievement levels in relation to other criteria.

A student who attains a particular level of achievement in relation to one criterion does not necessarily attain similar levels of achievement in relation to the others. Teachers should not assume that the overall assessment of the students produces any particular distribution of scores.

It is recommended that the assessment criteria be available to students at all times.

The final mark

The final mark for each project is the sum of the scores for each criterion.

The maximum possible final mark is 20.

Achievement levels

Criterion A: Introduction

In this context, the word "task" is defined as "what the student is going to do"; the word "plan" is defined as "how the student is going to do it". A statement of the task should appear at the beginning of each project. It is expected that each project has a clear title.

Achievement level	
0	The student does not produce a clear statement of the task. *There is no evidence in the project of any statement of what the student is going to do or has done.*
1	The student produces a clear statement of the task. *For this level to be achieved the task should be stated explicitly.*
2	The student produces a title, a clear statement of the task and a clear description of the plan. *The plan need not be highly detailed, but must describe how the task will be performed.*

Criterion B: Information/Measurement

In this context, generated measurements include those that have been generated by computer, by observation, by investigation, by prediction from a mathematical model or by experiment. Mathematical information includes geometrical figures and data that is collected empirically or assembled from outside sources. This list is not exclusive and mathematical information does not solely imply data for statistical analysis.

Achievement level	
0	The student does not collect relevant information or generate relevant measurements. *No attempt has been made to collect any relevant information or generate any relevant measurements.*
1	The student collects relevant information or generates relevant measurements. *This achievement level can be awarded even if a fundamental flaw exists in the instrument used to collect the information, for example, a faulty questionnaire or an interview conducted in an invalid way.*
2	The relevant information collected, or set of measurements generated by the student, is organized in a form appropriate for analysis or is sufficient in both quality and quantity. *A satisfactory attempt has been made to structure the information/measurements ready for the process of analysis, or the information/measurements are adequate in both quantity and quality.*
3	The relevant information collected, or set of measurements generated by the student, is organized in a form appropriate for analysis and is sufficient in both quality and quantity. *This level cannot be achieved if the measurements/information are too sparse (that is, insufficient in quantity) or too simple (for example, one-dimensional) as clearly it does not lend itself to being structured. It should therefore be recognized that within this descriptor there are assumptions about the quantity and, more importantly, the quality (in terms of depth and breadth) of information or measurements generated.*

Criterion C: Mathematical processes

When presenting diagrams, students are expected to use rulers where necessary and not merely sketch. A freehand sketch would not be considered a correct mathematical process. When technology is used the student would be expected to show a clear understanding of the mathematical processes used.

Achievement level

0 The student does not attempt to carry out any mathematical processes.

This would include students who have copied processes from a book with no attempt being made to use their own collected/generated information.

Projects consisting of only historical accounts, for example, will achieve this level.

1 The student carries out simple mathematical processes.

Simple processes are considered to be those that the average mathematical studies student could carry out easily, for example, percentages, areas of plane shapes, linear and quadratic functions (graphing and analysing), bar charts, pie charts, mean and standard deviation, simple probability. This level does not require the representation to be comprehensive, nor does it demand the calculations to be without error.

2 The simple mathematical processes are mostly or completely correct, or the student makes an attempt to use at least one sophisticated process.

Examples of sophisticated processes are volumes of pyramids and cones, analysis of trigonometric and exponential functions, optimization, statistical tests and compound probability. For this level to be achieved it is not required that the calculations for the sophisticated process(es) be without error.

3 The student carries out at least one sophisticated process, and all the processes used are mostly or completely accurate.

The key word in this descriptor is "accurate". It is accepted that not all calculations need to be checked before awarding this achievement level; random checking of some calculations is sufficient. A small number of isolated mistakes should not disqualify a student from achieving this level.

However, incorrect use of formulae, or consistent mistakes in using data, would disqualify the student from achieving this level.

4 The student carries out at least one sophisticated process; the processes used are mostly or completely accurate and all the processes used are relevant.

For this level to be achieved the mathematical processes must be appropriate and used in a meaningful way.

5 The student accurately carries out a number of relevant sophisticated processes.

To achieve this level the student would be expected to have carried out a range of meaningful mathematical processes. The processes may all relate to a single area of mathematics, for example, geometry. Measurements, information or data that are limited in scope would not allow the student to achieve this level.

Criterion D: Interpretation of results

Use of the terms "interpretations" and "conclusions" refers very specifically to statements about what the mathematics used tells us after it has been used to process the original information or data. Wider discussion of limitations and validity of the processes is assessed elsewhere.

Achievement level	
0	The student does not produce any interpretations or conclusions. *For the student to be awarded this level there must be no evidence of interpretation or conclusions anywhere in the project, or a completely false interpretation is given without reference to any of the results obtained.*
1	The student produces at least one interpretation or conclusion. *Only minimal evidence of interpretations or conclusions is required for this level. This level can be achieved by recognizing the need to interpret the results and attempting to do so, but reaching only false conclusions.*
2	The student produces at least one interpretation and/or conclusion that is consistent with the mathematical processes used. *For this level to be achieved at least one interpretation and/or conclusion is required. A "follow through" procedure should be used and, consequently, it is irrelevant here whether the processes are either correct or appropriate; the only requirement is consistency.*
3	The student produces a comprehensive discussion of interpretations and conclusions that are consistent with the mathematical processes used. *To achieve this level the student would be expected to produce a meaningful discussion of the results obtained and the conclusions drawn. In this context, the word "comprehensive" should be taken to mean thorough and detailed discussion of interpretations based on the level of understanding reasonably to be expected from a student for mathematical studies SL.* *This achievement level cannot be awarded if the project is a very simple one, with few opportunities for substantial interpretation. This level would not be achieved with too many incorrect interpretations or conclusions present.*

Criterion E: Validity

An important distinction is drawn between interpretations and conclusions, and validity. Validity addresses the questions as to whether appropriate mathematics was used to deal with the information collected and whether the mathematics used has any limitations in its applicability within the project. Any limitations or qualifications of the conclusions and interpretations should also be judged within this criterion. The considerations here are independent of whether the particular interpretations and conclusions reached are correct or adequate.

Achievement level	
0	The student does not comment on the mathematical processes used or the interpretations/conclusions made. *There is no attempt to evaluate (as opposed to interpret) the project to assess the validity of the mathematical processes or model used.*
1	The student has made an attempt to comment on either the mathematical processes used or the interpretations/conclusions made. *The student shows an awareness of the possibility that some or all of the results may have a limited validity and makes an attempt to discuss the reasons for such limitations.* *Statements merely acknowledging the need for more information/measurements, but with no further evaluation, belong in this achievement level. If it is believed that validity is not an issue, this must be stated with at least some reasonable justification for the belief.*
2	The student has made a serious attempt to comment on both the mathematical processes used and the interpretations/conclusions made. *There is significant discussion of the validity of the techniques used, recognition of any limitations that might apply and at least one realistic suggestion for improvement. A statement such as "I should have used more information/measurements" without further clarification, is not sufficient to earn full marks for this criterion. If the student considers that validity is not an issue, this must be fully justified, and can only achieve this achievement level if the argument is reasonable.* *If the discussion of validity is clearly worth achievement level 1 and is then supplemented with sensible suggestions for extension of the project, this can also assist in the achievement of this level though such suggestions alone are not adequate if there is no discussion of validity.*

Criterion F: Structure and communication

The term "structure" should be taken primarily as referring to the organization of the information, calculations and interpretations in such a way as to present the project as a logical sequence of thought and activities starting with the task and the plan, and finishing with the conclusions and limitations.

The term "communication" refers primarily to the correct and effective use of mathematical notation and sensible choice of diagrammatic and tabular representations. It is not expected that spelling, grammar and syntax are perfect and these features are not judged in assigning a level for this criterion. Nevertheless, teachers are strongly encouraged to correct and assist students with the linguistic aspects of their work. Projects that are very poor linguistically are also less likely to excel in the areas that are important in this criterion.

Achievement level	
0	The student has made no attempt to structure the project. *It is not expected that many students will be awarded this level.*
1	The student has made some attempt to structure the project or has used appropriate notation and terminology. *There must be a logical development to the project or the appropriate notation and terminology must be used correctly.*
2	The student has made some attempt to structure the project and has used appropriate notation and terminology. *There must be a logical development to the project and the appropriate notation and terminology must be used correctly.*
3	The student has produced a project that is well structured and communicated in a coherent manner. *To achieve this level the project would be expected to read well, and contain footnotes and a bibliography, as appropriate.*

Criterion G: Commitment

The project should be an ongoing process involving consultation between student and teacher. The student should be aware of the expectations of the teacher from the beginning of the process and each achievement level awarded should be justified by a written comment from the teacher at the time of marking. The examples given below for each criterion level are teacher orientated and each teacher should use discretion when judging the levels.

Achievement level

0 The student showed little or no commitment.

For example, the student did not participate in class discussions on project work, did not submit the required work in progress, and/or missed many deadlines.

1 The student showed satisfactory commitment.

For example, the student participated in class discussions on project work, kept to most deadlines, had some discussion initiated by the teacher but did not necessarily exploit all the available opportunities for the development or improvement of the project.

2 The student showed full commitment.

For example, the student participated fully in class discussions on project work, took initiatives both in discussion with the teacher and/or the rest of the class and in subsequent work of a more independent nature and/or demonstrated a full understanding of all the steps in the development of his/her project.

To obtain the highest achievement level for this criterion the student should have excelled in several areas such as those listed below. This list is not exhaustive and teachers are encouraged to add their own expectations.

The student:

- *actively participated at all stages of the development of the project*
- *demonstrated a full understanding of the concepts associated with his/her project*
- *participated in class activities on project work*
- *demonstrated initiative*
- *demonstrated perseverance*
- *showed insight*
- *prepared well to meet deadlines set by the teacher.*