



IB DIPLOMA PROGRAMME  
PROGRAMME DU DIPLÔME DU BI  
PROGRAMA DEL DIPLOMA DEL BI

# **Mathematics**

## **Standard level**

**Specimen questions paper 1 and paper 2**

**For first examinations in 2008**





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## Introduction

The assessment model has been changed for May 2008:

- Paper 1 and paper 2 will both consist of section A, short questions answered on the paper (similar to the current paper 1), and section B, extended-response questions answered on answer sheets (similar to the current paper 2).
- Calculators will **not** be allowed on paper 1.
- Graphic display calculators (GDCs) will be **required** on paper 2.

Full details of the revised assessment model for external components can be found in the second edition of the mathematics SL guide which was sent to schools in September 2006 and is available on the online curriculum centre (OCC).

### Why are these changes being made?

Experience has shown that certain papers can be answered using the GDC very little, although some students will answer the same papers by using a GDC on almost every question. We have seen some very interesting and innovative approaches used by students and teachers, however there have been occasions when the paper setters wished to assess a particular skill or approach. The fact that candidates had a GDC often meant that it was difficult (if not impossible) to do this. The problem was exacerbated by the variety of GDCs used by students worldwide. The examining team feel that a calculator-free environment is needed in order to better assess certain knowledge and skills.

### How will these changes affect the way the course is taught?

Most teachers should not find it necessary to change their teaching in order to be able to comply with the change in the assessment structure. Rather it will give them the freedom to emphasize the analytical approach to certain areas of the course that they may have been neglecting somewhat, not because they did not deem it relevant or even essential, but because it was becoming clear that technology was “taking the upper hand” and ruling out the need to acquire certain skills.

### Are there changes to the syllabus content?

No, it should be emphasized that it is only the assessment model that is being changed. There is no intention to change the syllabus content. Neither is there any intention to reduce the role of the GDC, either in teaching or in the examination.

Any references in the subject guide to the use of a GDC will still be valid, for example, finding the inverse of a  $3 \times 3$  matrix using a GDC or obtaining the standard deviation from a GDC; this means that these will not appear on paper 1. Other examples of questions that will not appear on paper 1 are calculations of binomial coefficients in algebra, and statistics questions requiring the use of tables. In trigonometry, candidates are expected to be familiar with the characteristics of the sin, cos and tan curves, including knowledge of the ratios of  $0^\circ$ ,  $90^\circ$ ,  $180^\circ$  and so on.

### What types of questions will be asked on paper 1?

Paper 1 questions will mainly involve analytical approaches to solutions rather than requiring the use of a GDC. It is not intended to have complicated calculations with the potential for careless errors. However, questions will include some arithmetical manipulations when they are essential to the development of the question.

## **What types of questions will be asked on paper 2?**

These questions will be similar to those asked on the current papers. Students must have access to a GDC at all times, however not all questions will necessarily require the use of the GDC. There will be questions where a GDC is not needed and others where its use is optional. There will be some questions that cannot be answered without a GDC that meets the minimum requirements.

## **What is the purpose of this document**

This document is a combination of the original specimen papers (published in November 2004) and the new specimen questions (published online in October 2006). It should be noted that this is not two specimen papers but a collection of questions illustrating the types of questions that may be asked on each paper. Thus they will not necessarily reflect balanced syllabus coverage, nor the relative importance of the syllabus topics.

In order to provide teachers with information about the examinations, the rubrics for each paper and section are included below. Section A questions should be answered in the spaces provided, and Section B questions on the answer sheets provided by the IBO. Graph paper should be used if required. The answer spaces have been included with the first 2 questions of Section A on each paper.

### **Paper 1**

*Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.*

#### **Section A**

*Answer all the questions in the spaces provided. Working may be continued below the lines, if necessary.*

#### **Section B**

*Answer all the questions on the answer sheets provided. Please start each question on a new page.*

### **Paper 2**

*Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.*

#### **Section A**

*Answer all the questions in the spaces provided. Working may be continued below the lines, if necessary.*

#### **Section B**

*Answer all the questions on the answer sheets provided. Please start each question on a new page.*

## Markscheme instructions

### A. Abbreviations

**M** Marks awarded for attempting to use a correct **Method**; working must be seen.

**(M)** Marks awarded for **Method**; may be implied by **correct** subsequent working.

**A** Marks awarded for an **Answer** or for **Accuracy**: often dependent on preceding **M** marks.

**(A)** Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

**R** Marks awarded for clear **Reasoning**.

**N** Marks awarded for **correct** answers if **no** working shown.

**AG** Answer given in the question and so no marks are awarded.

### B. Using the markscheme

**Follow through (FT)** marks: Only award **FT** marks when a candidate uses an incorrect answer in a subsequent **part**. Any exceptions to this will be noted on the markscheme. Follow through marks are now the exception rather than the rule within a question or part question. Follow through marks may only be awarded to work that is seen. Do **not** award **N FT** marks. If the question becomes much simpler then use discretion to award fewer marks. If a candidate mis-reads data from the **question** apply follow-through.

**Discretionary (d)** marks: There will be rare occasions where the markscheme does not cover the work seen. In such cases, **(d)** should be used to indicate where an examiner has used discretion. It must be accompanied by a brief note to explain the decision made.

It is important to understand the difference between “**implied**” marks, as indicated by the brackets, and marks which can only be awarded for work seen - no brackets. The implied marks can only be awarded if **correct** work is seen or implied in subsequent working. Normally this would be in the next line.

Where **MI AI** are awarded on the same line, this usually means **MI** for an attempt to use an appropriate formula, **AI** for correct substitution.

As **A** marks are normally **dependent** on the preceding **M** mark being awarded, it is not possible to award **MO AI**.

As **N** marks are only awarded when there is no working, it is not possible to award a mixture of **N** and other marks.

Accept all correct alternative methods, even if not specified in the markscheme. Where alternative methods for complete questions are included, they are indicated by **METHOD 1**, **METHOD 2**, *etc.* Other alternative (part) solutions, are indicated by **EITHER...OR**. Where possible, alignment will also be used to assist examiners to identify where these alternatives start and finish.

Unless the question specifies otherwise, accept **equivalent forms**. On the markscheme, these equivalent numerical or algebraic forms will generally be written in brackets after the required answer. The markscheme indicate the required answer, by allocating full marks at that point. Once the correct answer is seen, ignore further working, unless it contradicts the answer.

Brackets will also be used for what could be described as the well-expressed answer, but which candidates may not write in examinations. Examiners need to be aware that the marks for answers should be awarded for the form preceding the brackets *e.g.* in differentiating  $f(x) = 2\sin(5x - 3)$ , the markscheme says

$$f'(x) = (2\cos(5x - 3))5 \quad (= 10\cos(5x - 3)) \quad \mathbf{AI}$$

This means that the **AI** is awarded for seeing  $(2\cos(5x - 3))5$ , although we would normally write the answer as  $10\cos(5x - 3)$ .

As this is an international examination, all **alternative forms of notation** should be accepted.

Where the markscheme specifies **M2**, **A3**, *etc.*, for an answer do NOT split the marks unless otherwise instructed.

Do **not** award full marks for a correct answer, all working must be checked.

Candidates should be penalized **once IN THE PAPER** for an accuracy error (**AP**). There are two types of accuracy error:

- **Rounding errors:** only applies to final answers not to intermediate steps.
- **Level of accuracy:** when this is not specified in the question the general rule is *unless otherwise stated in the question all numerical answers must be given exactly or to three significant figures*.







3. [Maximum mark: 7]

Consider  $f(x) = \sqrt{x-5}$ .

(a) Find

(i)  $f(11)$ ;

(ii)  $f(86)$ ;

(iii)  $f(5)$ .

[3 marks]

(b) Find the values of  $x$  for which  $f$  is undefined.

[2 marks]

(c) Let  $g(x) = x^2$ . Find  $(g \circ f)(x)$ .

[2 marks]

4. [Maximum mark: 6]

The quadratic function  $f$  is defined by  $f(x) = 3x^2 - 12x + 11$ .

(a) Write  $f$  in the form  $f(x) = 3(x-h)^2 - k$ .

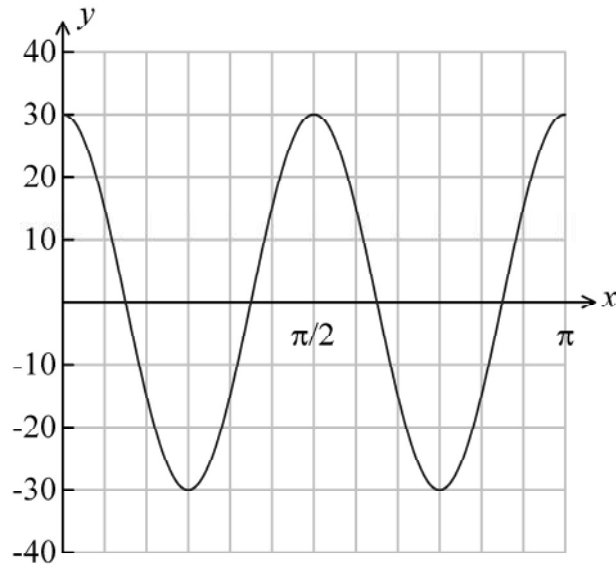
[3 marks]

(b) The graph of  $f$  is translated 3 units in the positive  $x$ -direction and 5 units in the positive  $y$ -direction. Find the function  $g$  for the translated graph, giving your answer in the form  $g(x) = 3(x-p)^2 + q$ .

[3 marks]

5. [Maximum mark: 6]

The graph of a function of the form  $y = p \cos qx$  is given in the diagram below.



- (a) Write down the value of  $p$ . [2 marks]
- (b) Calculate the value of  $q$ . [4 marks]

6. [Maximum mark: 7]

Given that  $\frac{\pi}{2} \leq \theta \leq \pi$  and that  $\cos \theta = -\frac{12}{13}$ , find

- (a)  $\sin \theta$ ; [3 marks]
- (b)  $\cos 2\theta$ ; [3 marks]
- (c)  $\sin(\theta + \pi)$ . [1 mark]

7. [Maximum mark: 6]

- (a) Given that  $2 \sin^2 \theta + \sin \theta - 1 = 0$ , find the two values for  $\sin \theta$ . [4 marks]
- (b) Given that  $0^\circ \leq \theta \leq 360^\circ$  and that one solution for  $\theta$  is  $30^\circ$ , find the other two possible values for  $\theta$ . [2 marks]

8. [Maximum mark: 5]

Let  $A = \begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix}$ .

(a) Find  $A^2$ . [2 marks]

(b) Let  $B = \begin{pmatrix} -3 & 4 \\ 2 & 1 \end{pmatrix}$ . Solve the matrix equation  $3X + A = B$ . [3 marks]

9. [Maximum mark: 6]

Let  $M = \begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix}$ , and  $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ . Given that  $M^2 - 6M + kI = O$ , find  $k$ .

10. [Maximum mark: 6]

(a) Given  $A = \begin{pmatrix} 7 & 8 \\ 2 & 3 \end{pmatrix}$ , find  $A^{-1}$ . [2 marks]

(b) **Hence**, solve the system of simultaneous equations.

$$7x + 8y = 1$$

$$2x + 3y = 1$$

[4 marks]

11. [Maximum mark: 6]

Consider the points A(5, 8), B(3, 5) and C(8, 6).

(a) Find

(i)  $\vec{AB}$ ;

(ii)  $\vec{AC}$ .

[3 marks]

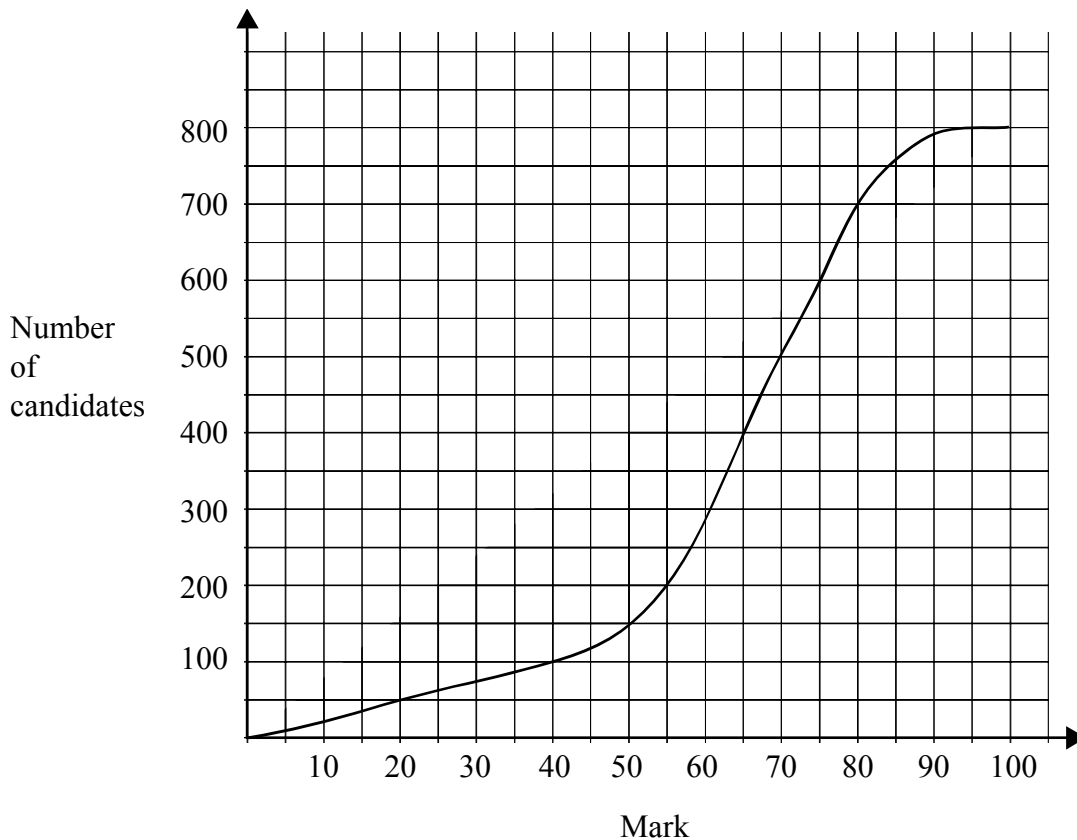
(b) (i) Find  $\vec{AB} \cdot \vec{AC}$ .

(ii) Find the sine of the angle between  $\vec{AB}$  and  $\vec{AC}$ .

[3 marks]

12. [Maximum mark: 6]

A test marked out of 100 is written by 800 students. The cumulative frequency graph for the marks is given below.



- (a) Write down the number of students who scored 40 marks or less on the test. [2 marks]
- (b) The middle 50 % of test results lie between marks  $a$  and  $b$ , where  $a < b$ . Find  $a$  and  $b$ . [4 marks]

13. [Maximum mark: 6]

A random variable  $X$  is distributed normally with a mean of 100 and a variance of 100.

- (a) Find the value of  $X$  that is 1.12 standard deviations **above** the mean. [4 marks]
- (b) Find the value of  $X$  that is 1.12 standard deviations **below** the mean. [2 marks]

14. [Maximum mark: 7]

In a game a player rolls a biased four-faced die. The probability of each possible score is shown below.

|             |               |               |                |     |
|-------------|---------------|---------------|----------------|-----|
| Score       | 1             | 2             | 3              | 4   |
| Probability | $\frac{1}{5}$ | $\frac{2}{5}$ | $\frac{1}{10}$ | $x$ |

- (a) Find the value of  $x$ . [2 marks]
- (b) Find  $E(X)$ . [3 marks]
- (c) The die is rolled twice. Find the probability of obtaining two scores of 3. [2 marks]

15. [Maximum mark: 6]

Find the equation of the tangent to the curve  $y = e^{2x}$  at the point where  $x = 1$ .  
Give your answer in terms of  $e^2$ .

16. [Maximum mark: 7]

- (a) Find  $\int_1^2 (3x^2 - 2) dx$ . [4 marks]
- (b) Find  $\int_0^1 2e^{2x} dx$ . [3 marks]

17. [Maximum mark: 6]

The velocity  $v \text{ ms}^{-1}$  of a moving body at time  $t$  seconds is given by  $v = 50 - 10t$ .

- (a) Find its acceleration in  $\text{ms}^{-2}$ . [2 marks]
- (b) The initial displacement  $s$  is 40 metres. Find an expression for  $s$  in terms of  $t$ . [4 marks]

**Section B questions**

**18.** [Maximum mark: 13]

Solve the following equations.

(a)  $\log_x 49 = 2$  [3 marks]

(b)  $\log_2 8 = x$  [2 marks]

(c)  $\log_{25} x = -\frac{1}{2}$  [3 marks]

(d)  $\log_2 x + \log_2(x - 7) = 3$  [5 marks]

**19.** [Maximum mark: 15]

Let  $f(x) = 2x^2 - 12x + 5$ .

(a) Express  $f(x)$  in the form  $f(x) = 2(x - h)^2 - k$ . [3 marks]

(b) Write down the vertex of the graph of  $f$ . [2 marks]

(c) Write down the equation of the axis of symmetry of the graph of  $f$ . [1 mark]

(d) Find the  $y$ -intercept of the graph of  $f$ . [2 marks]

(e) The  $x$ -intercepts of  $f$  can be written as  $\frac{p \pm \sqrt{q}}{r}$ , where  $p, q, r \in \mathbb{Z}$ .  
Find the value of  $p$ , of  $q$ , and of  $r$ . [7 marks]



**20.** [Maximum mark: 14]

Let  $f(x) = \frac{1}{x}$ ,  $x \neq 0$ .

- (a) Sketch the graph of  $f$ . [2 marks]

The graph of  $f$  is transformed to the graph of  $g$  by a translation of  $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ .

- (b) Find an expression for  $g(x)$ . [2 marks]

- (c) (i) Find the intercepts of  $g$ .

(ii) Write down the equations of the asymptotes of  $g$ .

- (iii) Sketch the graph of  $g$ . [10 marks]

**21.** [Maximum mark: 10]

A spring is suspended from the ceiling. It is pulled down and released, and then oscillates up and down. Its length,  $l$  centimetres, is modelled by the function  $l = 33 + 5 \cos((720t)^\circ)$ , where  $t$  is time in seconds after release.

- (a) Find the length of the spring after 1 second. [2 marks]

- (b) Find the minimum length of the spring. [3 marks]

- (c) Find the first time at which the length is 33 cm. [3 marks]

- (d) What is the period of the motion? [2 marks]

22. [Maximum mark: 16]

Two lines  $L_1$  and  $L_2$  are given by  $\mathbf{r}_1 = \begin{pmatrix} 9 \\ 4 \\ -6 \end{pmatrix} + s \begin{pmatrix} -2 \\ 6 \\ 10 \end{pmatrix}$  and  $\mathbf{r}_2 = \begin{pmatrix} 1 \\ 20 \\ 2 \end{pmatrix} + t \begin{pmatrix} -6 \\ 10 \\ -2 \end{pmatrix}$ .

(a) Let  $\theta$  be the acute angle between  $L_1$  and  $L_2$ . Show that  $\cos \theta = \frac{52}{140}$ . [5 marks]

(b) (i) P is the point on  $L_1$  when  $s = 1$ . Find the position vector of P.

(ii) Show that P is also on  $L_2$ . [8 marks]

(c) A third line  $L_3$  has direction vector  $\begin{pmatrix} 6 \\ x \\ -30 \end{pmatrix}$ . If  $L_1$  and  $L_3$  are parallel,

find the value of  $x$ . [3 marks]

23. [Maximum mark: 9]

The heights of trees in a forest are normally distributed with mean height 17 metres. One tree is selected at random. The probability that a selected tree has a height greater than 24 metres is 0.06.

(a) Find the probability that the tree selected has a height less than 24 metres. [2 marks]

(b) The probability that the tree has a height less than  $D$  metres is 0.06. Find the value of  $D$ . [3 marks]

(c) A woodcutter randomly selects 200 trees. Find the expected number of trees whose height lies between 17 metres and 24 metres. [4 marks]

**24.** [Maximum mark: 10]

The probability of obtaining heads on a biased coin is  $\frac{1}{3}$ .

(a) Sammy tosses the coin three times. Find the probability of getting

(i) three heads;

(ii) two heads and one tail.

[5 marks]

(b) Amir plays a game in which he tosses the coin 12 times.

(i) Find the expected number of heads.

(ii) Amir wins \$ 10 for each head obtained, and loses \$ 6 for each tail.  
Find his expected winnings.

[5 marks]

**25.** [Maximum mark: 14]

Let  $g(x) = x^3 - 3x^2 - 9x + 5$ .

(a) Find the two values of  $x$  at which the tangent to the graph of  $g$  is horizontal.

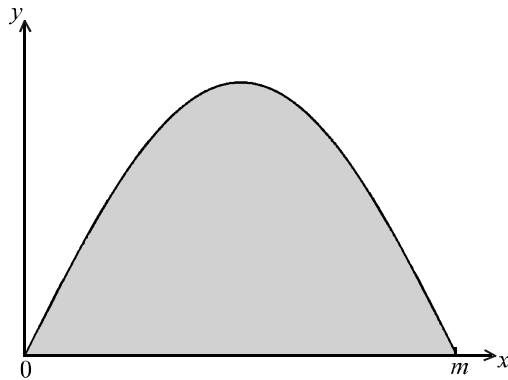
[8 marks]

(b) For each of these values, determine whether it is a maximum or a minimum.

[6 marks]

26. [Maximum mark: 10]

The diagram below shows part of the graph of  $y = \sin 2x$ . The shaded region is between  $x = 0$  and  $x = m$ .



- (a) Write down the period of this function. [2 marks]
- (b) Hence or otherwise write down the value of  $m$ . [2 marks]
- (c) Find the area of the shaded region. [6 marks]
-

## Paper 1 markscheme

### Section A

1. (a) (i) attempt to set up equations *(M1)*  
 $-37 = u_1 + 20d$  and  $-3 = u_1 + 3d$  *A1*  
 $-34 = 17d$   
 $d = -2$  *A1*      *N2*
- (ii)  $-3 = u_1 - 6 \Rightarrow u_1 = 3$  *A1*      *N1*
- (b)  $u_{10} = 3 + 9 \times -2 = -15$  *(A1)*  
 $S_{10} = \frac{10}{2}(3 + (-15))$  *M1*  
 $= -60$  *A1*      *N2*  
*[7 marks]*
2. (a)  $u_1 = 1, u_2 = -1, u_3 = -3$  *A1A1A1*      *N3*
- (b) Evidence of using appropriate formula *M1*  
correct values  $S_{20} = \frac{20}{2}(2 \times 1 + 19 \times -2)$  ( $= 10(2 - 38)$ ) *A1*  
 $S_{20} = -360$  *A1*      *N1*  
*[6 marks]*
3. (a) (i)  $\sqrt{6}$  *A1*      *N1*
- (ii) 9 *A1*      *N1*
- (iii) 0 *A1*      *N1*
- (b)  $x < 5$  *A2*      *N2*
- (c)  $(g \circ f)(x) = (\sqrt{x-5})^2$  *(M1)*  
 $= x - 5$  *A1*      *N2*  
*[7 marks]*

4. (a) For a reasonable attempt to complete the square, (or expanding) *(M1)*  
e.g.  $3x^2 - 12x + 11 = 3(x^2 - 4x + 4) + 11 - 12$   
 $f(x) = 3(x - 2)^2 - 1$  (accept  $h = 2, k = 1$ ) *A1A1* *N3*

(b) **METHOD 1**

Vertex shifted to  $(2 + 3, -1 + 5) = (5, 4)$  *M1*  
so the new function is  $3(x - 5)^2 + 4$  (accept  $p = 5, q = 4$ ) *A1A1* *N2*

**METHOD 2**

$g(x) = 3((x - 3) - h)^2 + k + 5 = 3((x - 3) - 2)^2 - 1 + 5$  *M1*  
 $= 3(x - 5)^2 + 4$  (accept  $p = 5, q = 4$ ) *A1A1* *N2*  
*[6 marks]*

5. (a)  $p = 30$  *A2* *N2*

(b) **METHOD 1**

Period  $= \frac{2\pi}{q}$  *(M2)*  
 $= \frac{\pi}{2}$  *(A1)*  
 $\Rightarrow q = 4$  *A1* *N4*

**METHOD 2**

Horizontal stretch of scale factor  $= \frac{1}{q}$  *(M2)*  
scale factor  $= \frac{1}{4}$  *(A1)*  
 $\Rightarrow q = 4$  *A1* *N4*  
*[6 marks]*

6. (a) Evidence of using Pythagoras **(M1)**  
*e.g.* diagram,  $\sin^2 x + \cos^2 x = 1$   
 Correct calculation **(A1)**  
*e.g.*  $5, \sqrt{1 - \frac{144}{169}}$   
 $\sin \theta = \frac{5}{13}$  **A1** **N3**
- (b) Evidence of using formula for  $\cos 2\theta$  **(M1)**  
*e.g.*  $\cos 2\theta = 2\cos^2 \theta - 1$   
 Correct substitution/calculation **A1**  
*e.g.*  $2\left(-\frac{12}{13}\right)^2 - 1$   
 $\cos 2\theta = \frac{119}{169}$  **A1** **N2**
- (c)  $\sin(\theta + \pi) = -\sin \theta = -\frac{5}{13}$  **A1** **N1**  
**[7 marks]**
7. (a) Attempt to factorise **(M1)**  
 correct factors  $(2\sin \theta - 1)(\sin \theta + 1) = 0$  **A1**  
 $\sin \theta = \frac{1}{2}, \sin \theta = -1$  **A1A1** **N2**
- (b) other solutions are  $150^\circ, 270^\circ$  **A1A1** **N1N1**  
**[6 marks]**
8. (a) Attempt to multiply *e.g.*  $\begin{pmatrix} 1+0 & -2-6 \\ 0+0 & 0+9 \end{pmatrix}$  **(M1)**  
 $A^2 = \begin{pmatrix} 1 & -8 \\ 0 & 9 \end{pmatrix}$  **A1** **N2**
- (b)  $3X + \begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} -3 & 4 \\ 2 & 1 \end{pmatrix}$  **(M1)**  
 $3X = \begin{pmatrix} -4 & 6 \\ 2 & -2 \end{pmatrix}$  **(A1)**  
 $X = \frac{1}{3} \begin{pmatrix} -4 & 6 \\ 2 & -2 \end{pmatrix}$  **A1** **N2**  
**[5 marks]**

9. 
$$\begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix} - 6 \begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix} + k \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad (A1)$$

$$M^2 = \begin{pmatrix} 7 & -6 \\ -18 & 19 \end{pmatrix} \quad A2$$

$$6M = \begin{pmatrix} 12 & -6 \\ -18 & 24 \end{pmatrix} \quad A1$$

$$\begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix} + \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad A1$$

$$k = 5 \quad A1 \quad N2$$

**[6 marks]**

10. (a)  $\det A = 5 \quad (A1)$

$$A^{-1} = \frac{1}{5} \begin{pmatrix} 3 & -8 \\ -2 & 7 \end{pmatrix} \quad A1 \quad N2$$

(b) Set up matrix equation  $\begin{pmatrix} 7 & 8 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (M1)$

premultiplying by  $A^{-1} \quad M1$

$$A^{-1} \begin{pmatrix} 7 & 8 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = A^{-1} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{5} \begin{pmatrix} -5 \\ 5 \end{pmatrix} \quad \left( \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right) \quad A1$$

$x = -1, y = 1 \quad A1 \quad N0$

**[6 marks]**

11. (a) (i) evidence of combining vectors  $(M1)$

$e.g. \vec{AB} = \vec{OB} - \vec{OA}$

$$\vec{AB} = \begin{pmatrix} -2 \\ -3 \end{pmatrix} \quad A1 \quad N2$$

(ii) 
$$\vec{AC} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad A1 \quad N1$$

(b) (i) 
$$\vec{AB} \cdot \vec{AC} = (-2)(3) + (-3)(-2) = 0 \quad A1 \quad N1$$

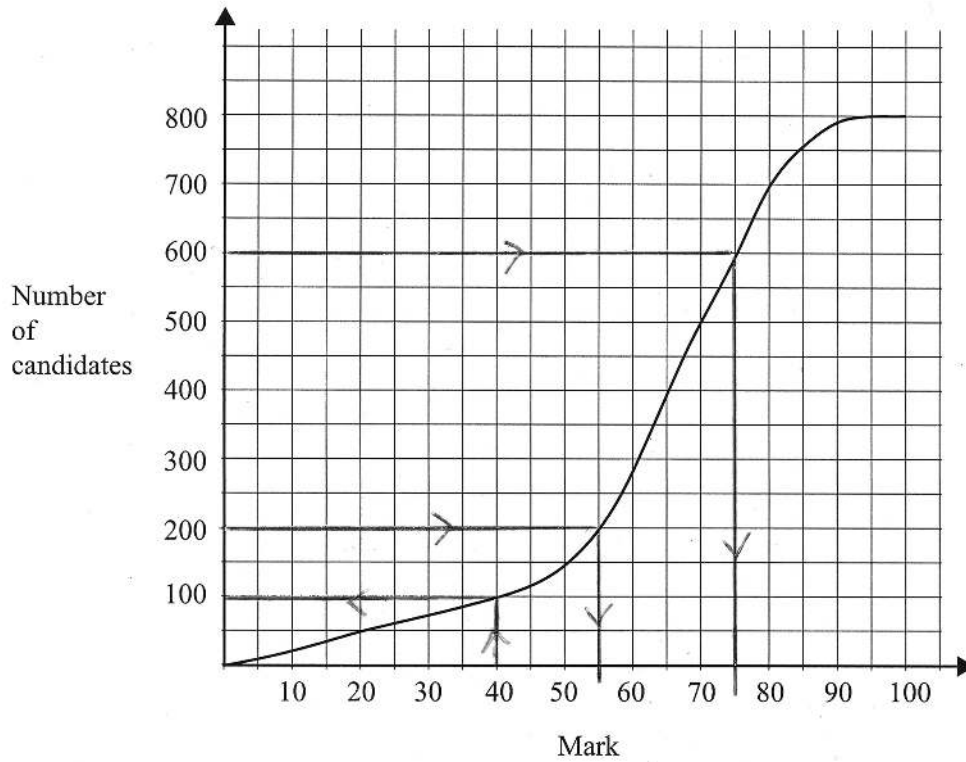
(ii) scalar product = 0  $\Rightarrow$  perpendicular,  $\theta = 90^\circ$   $(R1)$

$\sin \theta = 1 \quad A1 \quad N2$

**[6 marks]**



12.



- (a) Lines on graph  
100 students score 40 marks or fewer.

(M1)  
A1 N2

- (b) Identifying 200 and 600  
Lines on graph  
 $a = 55$ ,  $b = 75$

A1  
(M1)  
A1A1 N1N1  
[6 marks]

**13. METHOD 1**

- (a)  $\sigma = 10$  (A1)  
 $1.12 \times 10 = 11.2$  (A1)  
 $11.2 + 100$  (M1)  
 $x = 111.2$  (A1) N2
- (b)  $100 - 11.2$  (M1)  
 $= 88.8$  (A1) N2
- [6 marks]**

**METHOD 2**

- (a)  $\sigma = 10$  (A1)  
 Evidence of using standardisation formula (M1)  
 $\frac{x - 100}{10} = 1.12$  (A1)  
 $x = 111.2$  (A1) N2
- (b)  $\frac{100 - x}{10} = 1.12$  (A1)  
 $x = 88.8$  (A1) N2
- [6 marks]**

- 14. (a)** For summing to 1 (M1)  
 e.g.  $\frac{1}{5} + \frac{2}{5} + \frac{1}{10} + x = 1$   
 $x = \frac{3}{10}$  (A1) N2
- (b) For evidence of using  $E(X) = \sum x f(x)$  (M1)  
 Correct calculation (A1)  
 e.g.  $\frac{1}{5} \times 1 + 2 \times \frac{2}{5} + 3 \times \frac{1}{10} + 4 \times \frac{3}{10}$   
 $E(X) = \frac{25}{10} (= 2.5)$  (A1) N2
- (c)  $\frac{1}{10} \times \frac{1}{10}$  (M1)  
 $\frac{1}{100}$  (A1) N2
- [7 marks]**

15. Attempt to differentiate *(M1)*  
 $\frac{dy}{dx} = 2e^{2x}$  *A1*  
At  $x=1$   $\frac{dy}{dx} = 2e^2$  *A1*  
 $y = e^2$  *A1*  
Equation of tangent is  $y - e^2 = 2e^2(x - 1)$  ( $y = 2e^2x - e^2$ ) *M1A1* *N2*  
*[6 marks]*

16. (a)  $\int_1^2 (3x^2 - 2) dx = [x^3 - 2x]_1^2$  *A1A1*  
 $= (8 - 4) - (1 - 2)$  *(A1)*  
 $= 5$  *A1* *N2*
- (b)  $\int_0^1 2e^{2x} dx = [e^{2x}]_0^1$  *A1*  
 $= e^2 - e^0$  *(A1)*  
 $= e^2 - 1$  *A1* *N2*  
*[7 marks]*

17. (a)  $a = \frac{dv}{dt}$  *(M1)*  
 $= -10 \text{ (ms}^{-2}\text{)}$  *A1* *N2*
- (b)  $s = \int v dt$  *(M1)*  
 $= 50t - 5t^2 + c$  *A1*  
 $40 = 50(0) - 5(0) + c \Rightarrow c = 40$  *A1*  
 $s = 50t - 5t^2 + 40$  *A1* *N2*

**Note:** Award *(M1)* and the first *A1* in part (b) if  $c$  is missing, but do **not** award the final 2 marks.

*[6 marks]*

**Section B**

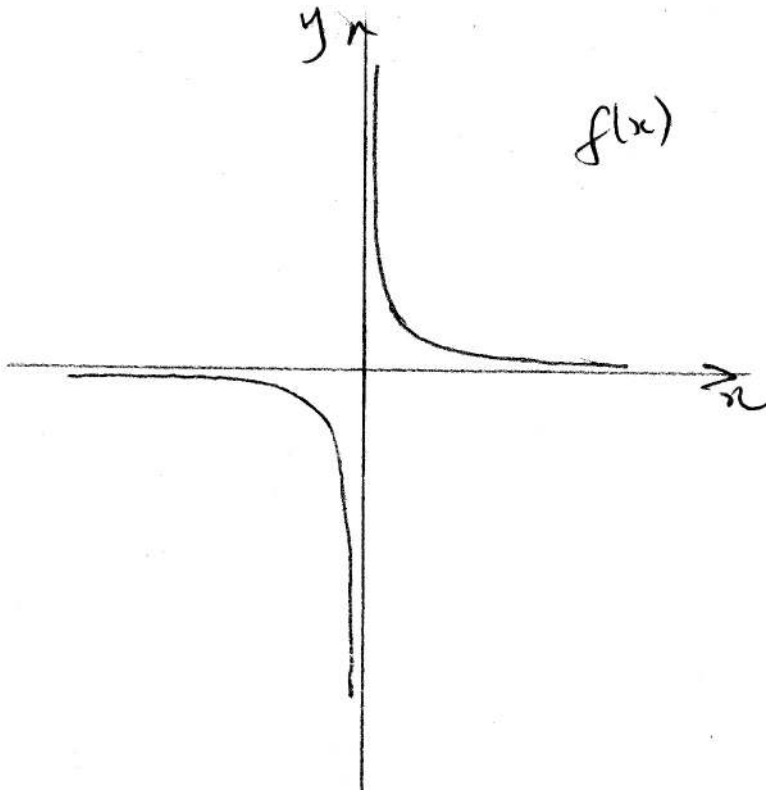
18. (a)  $x^2 = 49$  (M1)  
 $x = \pm 7$  (A1)  
 $x = 7$  A1 N3
- (b)  $2^x = 8$  (M1)  
 $x = 3$  A1 N2
- (c)  $x = 25^{-\frac{1}{2}}$  (M1)  
 $x = \frac{1}{\sqrt{25}}$  (A1)  
 $x = \frac{1}{5}$  A1 N3
- (d)  $\log_2(x(x-7)) = 3$  (M1)  
 $\log_2(x^2 - 7x) = 3$   
 $2^3 = 8 \quad (8 = x^2 - 7x)$  (A1)  
 $x^2 - 7x - 8 = 0$  A1  
 $(x-8)(x+1) = 0 \quad (x = 8, x = -1)$  (A1)  
 $x = 8$  A1 N3

[13 marks]

19. (a) Evidence of completing the square *(M1)*  
 $f(x) = 2(x^2 - 6x + 9) + 5 - 18$  *(A1)*  
 $= 2(x - 3)^2 - 13$  (accept  $h = 3, k = 13$ ) *A1*      *N3*
- (b) Vertex is  $(3, -13)$  *A1A1*      *N2*
- (c)  $x = 3$  (must be an equation) *A1*      *N1*
- (d) evidence of using fact that  $x = 0$  at  $y$ -intercept *(M1)*  
 $y$ -intercept is  $(0, 5)$  (accept 5) *A1*      *N2*
- (e) **METHOD 1**  
evidence of using  $y = 0$  at  $x$ -intercept *(M1)*  
e.g.  $2(x - 3)^2 - 13 = 0$   
evidence of solving this equation *(M1)*  
e.g.  $(x - 3)^2 = \frac{13}{2}$  *A1*  
 $(x - 3) = \pm \sqrt{\frac{13}{2}}$   
 $x = 3 \pm \sqrt{\frac{13}{2}} = 3 \pm \frac{\sqrt{26}}{2}$  *A1*  
 $x = \frac{6 \pm \sqrt{26}}{2}$   
 $p = 6, q = 26, r = 2$  *A1A1A1*      *N4*
- METHOD 2**  
evidence of using  $y = 0$  at  $x$ -intercept *(M1)*  
e.g.  $2x^2 - 12x + 5 = 0$   
evidence of using the quadratic formula *(M1)*  
 $x = \frac{12 \pm \sqrt{12^2 - 4 \times 2 \times 5}}{2 \times 2}$  *A1*  
 $x = \frac{12 \pm \sqrt{104}}{4} \left( = \frac{6 \pm \sqrt{26}}{2} \right)$  *A1*  
 $p = 12, q = 104, r = 4$  (or  $p = 6, q = 26, r = 2$ ) *A1A1A1*      *N4*

**[15 marks]**

20. (a)



*A1A1*

*N2*

**Note:** Award *A1* for the left branch, and *A1* for the right branch.

(b)  $g(x) = \frac{1}{x-2} + 3$

*A1A1*

*N2*

(c) (i) Evidence of using  $x=0$   $\left(g(0) = -\frac{1}{2} + 3\right)$

*(M1)*

$y = \frac{5}{2}$  (= 2.5)

*A1*

evidence of solving  $y=0$   $(1+3(x-2)=0)$

*M1*

$1+3x-6=0$

*(A1)*

$3x=5$

$x = \frac{5}{3}$

*A1*

Intercepts are  $x = \frac{5}{3}$ ,  $y = \frac{5}{2}$  (accept  $\left(\frac{5}{3}, 0\right)$   $\left(0, \frac{5}{2}\right)$ )

*N3*

(ii)  $x=2$   
 $y=3$

*A1*

*N1*

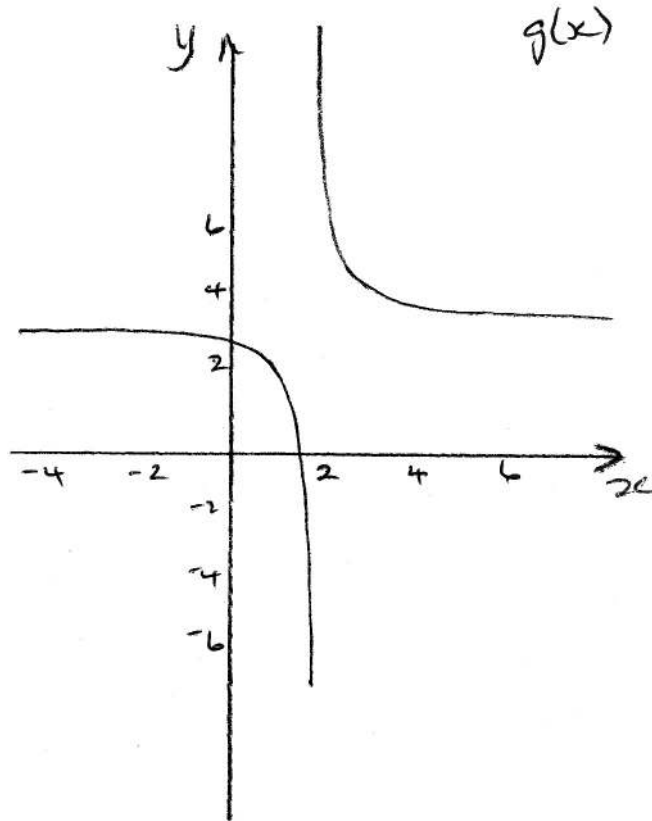
*A1*

*N1*

*continued ...*

Question 20 (c) continued

(iii)



A1A1A1

N3

**Note:** Award **A1** for the shape (both branches), **A1** for the correct behaviour close to the asymptotes, and **A1** for the intercepts at approximately  $\left(\frac{5}{3}, 0\right)$   $\left(0, \frac{5}{2}\right)$ .

[14 marks]

21. (a) When  $t = 1$ ,  $l = 33 + 5 \cos 720$  **(M1)**  
 $l = 33 + 5 = 38$  **A1** **N2**
- (b) Minimum when  $\cos = -1$  **(M1)**  
 $l_{\min} = 33 - 5$  **(M1)**  
 $= 28$  **A1** **N3**
- (c)  $33 = 33 + 5 \cos 720t$  ( $0 = 5 \cos 720t$ ) **M1**  
 $720t = 90$  **A1**  
 $t = \frac{90}{720} \left( = \frac{1}{8} \right)$  **A1** **N1**
- (d) Evidence of dividing into 360 **(M1)**  
 $\text{period} = \frac{360}{720} \left( = \frac{1}{2} \right)$  **A1** **N2**

[10 marks]

22. (a) Using direction vectors  $\mathbf{u} = \begin{pmatrix} -2 \\ 6 \\ 10 \end{pmatrix}$  and  $\mathbf{v} = \begin{pmatrix} -6 \\ 10 \\ -2 \end{pmatrix}$  **(M1)**
- $|\mathbf{u}| = \sqrt{4 + 36 + 100} = \sqrt{140}$ ,  $|\mathbf{v}| = \sqrt{36 + 100 + 4} = \sqrt{140}$  **A1A1**
- $\mathbf{u} \cdot \mathbf{v} = 12 + 60 - 20 = 52$  **A1**
- $\cos \theta = \frac{52}{\sqrt{140}\sqrt{140}}$  **A1**
- $= \frac{52}{140}$  **AG** **N0**
- (b) (i) For substituting  $s = 1$  **(M1)**  
 Correct calculations **(A1)**  
 $9 + 1(-2) = 7$ ,  $4 + 1(6) = 10$ ,  $-6 + 1(10) = 4$
- position vector of P is  $\begin{pmatrix} 7 \\ 10 \\ 4 \end{pmatrix}$  **A1** **N3**
- (ii) For substituting into the equation  $\begin{pmatrix} 7 \\ 10 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 20 \\ 2 \end{pmatrix} + t \begin{pmatrix} -6 \\ 10 \\ -2 \end{pmatrix}$  **(M1)**
- For one correct equation **A1**  
*e.g.*  $7 = 1 - 6t$   
 Solving gives  $t = -1$  **A1**
- verify for second coordinate,  $10 = 20 + (-1)(10)$  **A1**  
 verify for third coordinate,  $4 = 2 + (-1)(-2)$  **A1**  
 Thus, P is also on  $L_2$ . **AG** **N0**
- (c)  $k \begin{pmatrix} -2 \\ 6 \\ 10 \end{pmatrix} = \begin{pmatrix} 6 \\ x \\ -30 \end{pmatrix}$  **(M1)**
- $-2k = 6$   
 $k = -3$  **A1**  
 $x = -3 \times 6 = -18$  **A1** **N2**
- [16 marks]**



- 23 (a) Evidence of using the complement *e.g.*  $1 - 0.06$   
 $p = 0.94$  (M1)  
 A1 N2
- (b) For evidence of using symmetry (M1)  
 Distance from the mean is 7 (A1)  
*e.g.* diagram,  $D = \text{mean} - 7$   
 $D = 10$  A1 N2
- (c)  $P(17 < H < 24) = 0.5 - 0.06$  (M1)  
 $= 0.44$  A1
- E (trees) =  $200 \times 0.44$  (M1)  
 $= 88$  A1 N2  
 [9 marks]
24. (a) (i) Attempt to find  $P(3H) = \left(\frac{1}{3}\right)^3$  (M1)  
 $= \frac{1}{27}$  A1 N2
- (ii) Attempt to find  $P(2H, 1T)$  (M1)  
 $= 3\left(\frac{1}{3}\right)^2 \frac{2}{3}$  A1  
 $= \frac{2}{9}$  A1 N2
- (b) (i) Evidence of using  $np$   $\left(\frac{1}{3} \times 12\right)$  (M1)  
 expected number of heads = 4 A1 N2
- (ii) 4 heads, so 8 tails (A1)  
 E (winnings) =  $4 \times 10 - 8 \times 6$  (= 40 - 48) (M1)  
 $= -\$ 8$  A1 N1  
 [10 marks]

25. (a) Attempt to differentiate *(M1)*  
 $g'(x) = 3x^2 - 6x - 9$  *A1A1A1*
- for setting derivative equal to zero *M1*  
 $3x^2 - 6x - 9 = 0$
- Solving *A1*  
*e.g.*  $3(x-3)(x+1) = 0$
- $x = 3 \quad x = -1$  *A1A1*      *N3*
- (b) **METHOD 1**
- $g'(x < -1)$  is positive,  $g'(x > -1)$  is negative *A1A1*  
 $g'(x < 3)$  is negative,  $g'(x > 3)$  is positive *A1A1*
- min when  $x = 3$ , max when  $x = -1$  *A1A1*      *N2*
- METHOD 2**
- Evidence of using second derivative *(M1)*  
 $g''(x) = 6x - 6$  *A1*  
 $g''(3) = 12$  (or positive),  $g''(-1) = -12$  (or negative) *A1A1*
- min when  $x = 3$ , max when  $x = -1$  *A1A1*      *N2*
- [14 marks]*
26. (a) period =  $\frac{2\pi}{2} = \pi$  *M1A1*      *N2*
- (b)  $m = \frac{\pi}{2}$  *A2*      *N2*
- (c) Using  $A = \int_0^{\frac{\pi}{2}} \sin 2x \, dx$  *(M1)*
- Integrating correctly,  $A = \left[ -\frac{1}{2} \cos 2x \right]_0^{\frac{\pi}{2}}$  *A1*
- Substituting,  $A = -\frac{1}{2} \cos \pi - \left( -\frac{1}{2} \cos 0 \right)$  *(M1)*
- Correct values,  $A = -\frac{1}{2}(-1) - \left( -\frac{1}{2}(1) \right) \left( = \frac{1}{2} + \frac{1}{2} \right)$  *A1A1*
- $A = 1$  *A1*      *N2*
- [10 marks]*
-

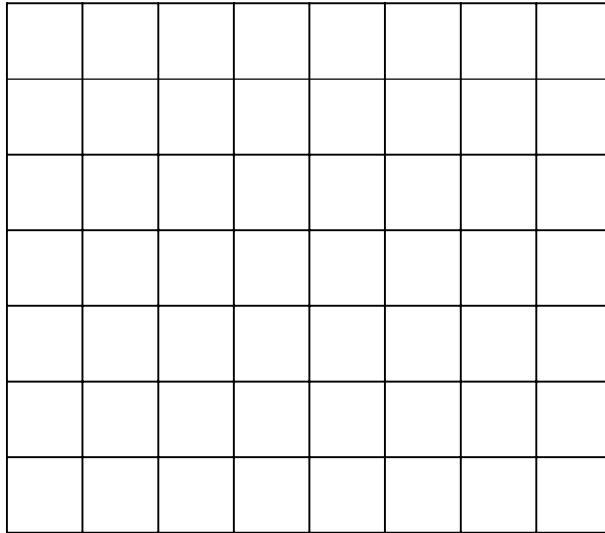




3. [Maximum mark: 6]

The function  $f$  is defined by  $f(x) = \frac{3}{\sqrt{9-x^2}}$ , for  $-3 < x < 3$ .

(a) On the grid below, sketch the graph of  $f$ .



[2 marks]

(b) Write down the equation of each vertical asymptote.

[2 marks]

(c) Write down the range of the function  $f$ .

[2 marks]

4. [Maximum mark: 6]

The functions  $f$  and  $g$  are defined by  $f : x \mapsto 3x$ ,  $g : x \mapsto x + 2$ .

(a) Find an expression for  $(f \circ g)(x)$ .

[2 marks]

(b) Find  $f^{-1}(18) + g^{-1}(18)$ .

[4 marks]

5. [Maximum mark: 7]

A triangle has its vertices at  $A(-1, 3)$ ,  $B(3, 6)$  and  $C(-4, 4)$ .

(a) Show that  $\vec{AB} \cdot \vec{AC} = -9$ .

[3 marks]

(b) Find  $\hat{BAC}$ .

[4 marks]

6. [Maximum mark: 6]

(a) Write down the inverse of the matrix  $A = \begin{pmatrix} 1 & -3 & 1 \\ 2 & 2 & -1 \\ 1 & -5 & 3 \end{pmatrix}$ . [2 marks]

(b) **Hence** solve the simultaneous equations

$$\begin{aligned} x - 3y + z &= 1 \\ 2x + 2y - z &= 2 \\ x - 5y + 3z &= 3 \end{aligned}$$

[4 marks]

7. [Maximum mark: 6]

A factory makes calculators. Over a long period, 2 % of them are found to be faulty. A random sample of 100 calculators is tested.

(a) Write down the expected number of faulty calculators in the sample. [1 mark]

(b) Find the probability that three calculators are faulty. [2 marks]

(c) Find the probability that more than one calculator is faulty. [3 marks]

8. [Maximum mark: 6]

The speeds of cars at a certain point on a straight road are normally distributed with mean  $\mu$  and standard deviation  $\sigma$ . 15 % of the cars travelled at speeds greater than  $90 \text{ km h}^{-1}$  and 12 % of them at speeds less than  $40 \text{ km h}^{-1}$ . Find  $\mu$  and  $\sigma$ .

9. [Maximum mark: 6]

The function  $f$  is given by  $f(x) = 2 \sin(5x - 3)$ .

(a) Find  $f''(x)$ . [4 marks]

(b) Write down  $\int f(x) dx$ . [2 marks]

**Section B questions**

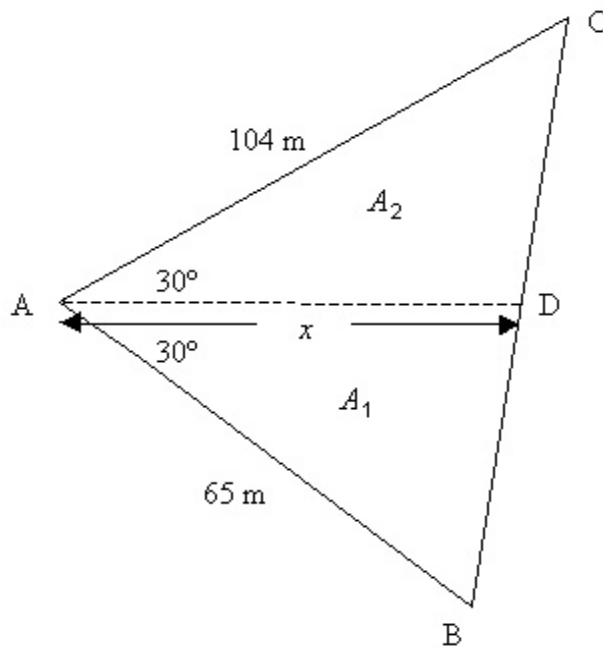
**10.** [Maximum mark: 18]

A farmer owns a triangular field ABC. One side of the triangle, [AC], is 104 m, a second side, [AB], is 65 m and the angle between these two sides is  $60^\circ$ .

(a) Use the cosine rule to calculate the length of the third side of the field. [3 marks]

(b) Given that  $\sin 60^\circ = \frac{\sqrt{3}}{2}$ , find the area of the field in the form  $p\sqrt{3}$  where  $p$  is an integer. [3 marks]

Let D be a point on [BC] such that [AD] bisects the  $60^\circ$  angle. The farmer divides the field into two parts  $A_1$  and  $A_2$  by constructing a straight fence [AD] of length  $x$  metres, as shown on the diagram below.



(c) (i) Show that the area of  $A_1$  is given by  $\frac{65x}{4}$ .  
(ii) Find a similar expression for the area of  $A_2$ .  
(iii) **Hence**, find the value of  $x$  in the form  $q\sqrt{3}$ , where  $q$  is an integer. [7 marks]

(d) (i) Explain why  $\sin \hat{A}DC = \sin \hat{A}DB$ .  
(ii) Use the result of part (i) and the sine rule to show that  $\frac{BD}{DC} = \frac{5}{8}$ . [5 marks]

11. [Maximum mark: 15]

In this question, distance is in kilometres, time is in hours.

A balloon is moving at a constant height with a speed of  $18 \text{ km h}^{-1}$ , in the direction of the vector  $\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$ .

At time  $t = 0$ , the balloon is at point B with coordinates  $(0, 0, 5)$ .

(a) Show that the position vector  $\mathbf{b}$  of the balloon at time  $t$  is given by

$$\mathbf{b} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} + t \begin{pmatrix} 10.8 \\ 14.4 \\ 0 \end{pmatrix}. \quad [6 \text{ marks}]$$

At time  $t = 0$ , a helicopter goes to deliver a message to the balloon. The position vector  $\mathbf{h}$  of the helicopter at time  $t$  is given by

$$\mathbf{h} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 49 \\ 32 \\ 0 \end{pmatrix} + t \begin{pmatrix} -48 \\ -24 \\ 6 \end{pmatrix}.$$

(b) (i) Write down the coordinates of the starting position of the helicopter.

(ii) Find the speed of the helicopter. [4 marks]

(c) The helicopter reaches the balloon at point R.

(i) Find the time the helicopter takes to reach the balloon.

(ii) Find the coordinates of R. [5 marks]



12. [Maximum mark: 19]

Bag A contains 2 red balls and 3 green balls. Two balls are chosen at random from the bag without replacement. Let  $X$  denote the number of red balls chosen. The following table shows the probability distribution for  $X$ .

|            |                |                |                |
|------------|----------------|----------------|----------------|
| $X$        | 0              | 1              | 2              |
| $P(X = x)$ | $\frac{3}{10}$ | $\frac{6}{10}$ | $\frac{1}{10}$ |

- (a) Calculate  $E(X)$ , the mean number of red balls chosen. [3 marks]

Bag B contains 4 red balls and 2 green balls. Two balls are chosen at random from bag B.

- (b) (i) Draw a tree diagram to represent the above information, including the probability of each event.  
(ii) Hence find the probability distribution for  $Y$ , where  $Y$  is the number of red balls chosen. [8 marks]

A standard die with six faces is rolled. If a 1 or 6 is obtained, two balls are chosen from bag A, otherwise two balls are chosen from bag B.

- (c) Calculate the probability that two red balls are chosen. [5 marks]  
(d) Given that two red balls are obtained, find the conditional probability that a 1 or 6 was rolled on the die. [3 marks]

13. [Maximum mark: 13]

The function  $f$  is defined by  $f : x \mapsto -0.5x^2 + 2x + 2.5$ .

Let  $N$  be the normal to the curve at the point where the graph intercepts the  $y$ -axis.

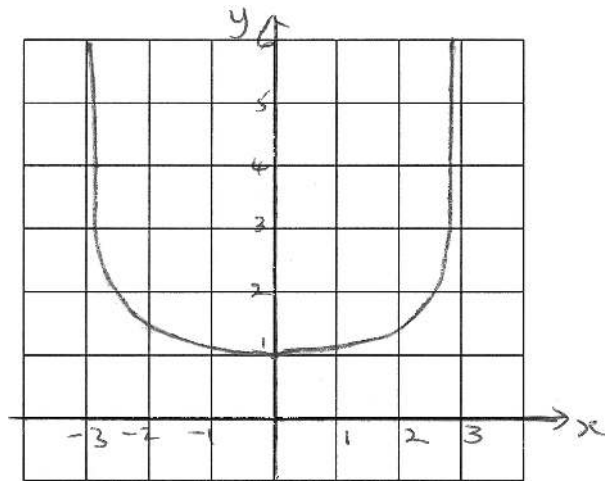
- (a) Show that the equation of  $N$  may be written as  $y = -0.5x + 2.5$ . [4 marks]  
(b) Find the coordinates of the other point of intersection of the normal and the curve. [5 marks]  
(c) Let  $R$  be the region enclosed between the curve and  $N$ . Find the area of  $R$ . [4 marks]
-

## Paper 2 markscheme

### Section A

1. (a) Recognizing an AP *(M1)*  
 $u_1 = 15 \quad d = 2 \quad n = 20$  *(A1)*  
 substituting into  $u_{20} = 15 + (20 - 1) \times 2$  *M1*  
 $= 53$  (that is, 53 seats in the 20th row) *A1*      *N2*
- (b) Substituting into  $S_{20} = \frac{20}{2}(2(15) + (20 - 1)2)$  (or into  $\frac{20}{2}(15 + 53)$ ) *M1*  
 $= 680$  (that is, 680 seats in total) *A1*      *N2*  
**[6 marks]**
2. (a)  $5000(1.063)^n$  *A1*      *N1*
- (b) Value = \$ 5000(1.063)<sup>5</sup> (= \$ 6786.3511...) *A1*      *N1*  
 $= \$ 6790$  to 3 s.f. (accept \$ 6786, or \$ 6786.35)
- (c) (i)  $5000(1.063)^n > 10\,000$  or  $(1.063)^n > 2$  *A1*      *N1*
- (ii) Attempting to solve the inequality  $n \log(1.063) > \log 2$  *(M1)*  
 $n > 11.345\dots$  *(A1)*  
 12 years *A1*      *N3*
- Note:** Candidates are likely to use TABLE or LIST on a GDC to find  $n$ .  
 A good way of communicating this is suggested below.
- Let  $y = 1.063^x$  *(M1)*  
 When  $x = 11$ ,  $y = 1.9582$ , when  $x = 12$ ,  $y = 2.0816$  *(A1)*  
 $x = 12$  *i.e.* 12 years *A1*      *N3*  
**[6 marks]**

3 (a)



*A1A1 N2*

**Note:** Award *A1* for the general shape and *A1* for the y-intercept at 1.

(b)  $x = 3, x = -3$

*A1A1 N1N1*

(c)  $y \geq 1$

*A2 N2*  
*[6 marks]*

4. (a)  $(f \circ g) : x \mapsto 3(x+2) (= 3x+6)$

*A2 N2*

(b) **METHOD 1**

Evidence of finding inverse functions

*M1*

e.g.  $f^{-1}(x) = \frac{x}{3} \quad g^{-1}(x) = x - 2$

$f^{-1}(18) = \frac{18}{3} (= 6)$

*(A1)*

$g^{-1}(18) = 18 - 2 (= 16)$

*(A1)*

$f^{-1}(18) + g^{-1}(18) = 6 + 16 = 22$

*A1 N3*

**METHOD 2**

Evidence of solving equations

*M1*

e.g.  $3x = 18, x + 2 = 18$

$x = 6, x = 16$

*(A1)(A1)*

$f^{-1}(18) + g^{-1}(18) = 6 + 16 = 22$

*A1 N3*

*[6 marks]*

5. (a) Finding **correct** vectors,  $\vec{AB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$   $\vec{AC} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$  **A1A1**
- Substituting correctly in the scalar product  $\vec{AB} \cdot \vec{AC} = 4(-3) + 3(1)$  **A1**  
 $= -9$  **AG** **N0**
- (b)  $|\vec{AB}| = 5$   $|\vec{AC}| = \sqrt{10}$  **(A1)(A1)**  
 Evidence of using scalar product formula **M1**  
*e.g.*  $\cos \hat{BAC} = \frac{-9}{5\sqrt{10}} = -0.569$  (3 s.f.)  
 $\hat{BAC} = 2.47$ (radians),  $125^\circ$  **A1** **N3**  
**[7 marks]**
6. (a)  $A^{-1} = \begin{pmatrix} 0.1 & 0.4 & 0.1 \\ -0.7 & 0.2 & 0.3 \\ -1.2 & 0.2 & 0.8 \end{pmatrix}$  **A2** **N2**
- (b) For recognizing that the equations may be written as  $A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$  **(M1)**
- for attempting to calculate  $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1.2 \\ 0.6 \\ 1.6 \end{pmatrix}$  **M1**
- $x = 1.2, y = 0.6, z = 1.6$  (accept row or column vectors) **A2** **N3**  
**[6 marks]**

7. (a)  $X \sim B(100, 0.02)$   
 $E(X) = 100 \times 0.02 = 2$  *AI* *N1*

(b)  $P(X = 3) = \binom{100}{3} (0.02)^3 (0.98)^{97}$  *(M1)*  
 $= 0.182$  *AI* *N2*

(c) **METHOD 1**

$P(X > 1) = 1 - P(X \leq 1) = 1 - (P(X = 0) + P(X = 1))$  *M1*  
 $= 1 - ((0.98)^{100} + 100(0.02)(0.98)^{99})$  *(M1)*  
 $= 0.597$  *AI* *N2*

**METHOD 2**

$P(X > 1) = 1 - P(X \leq 1)$  *(M1)*  
 $= 1 - 0.40327$  *(A1)*  
 $= 0.597$  *AI* *N2*

**Note:** Award marks as follows for finding  $P(X \geq 1)$ , if working shown.

$P(X \geq 1)$  *A0*  
 $= 1 - P(X \leq 2) = 1 - 0.67668$  *M1(FT)*  
 $= 0.323$  *A1(FT)* *N0*  
*[6 marks]*

8.  $X \sim N(\mu, \sigma^2)$   
 $P(X > 90) = 0.15$  and  $P(X < 40) = 0.12$  *(M1)*  
 Finding standardized values 1.036, -1.175 *A1A1*  
 Setting up the equations  $1.036 = \frac{90 - \mu}{\sigma}$ ,  $-1.175 = \frac{40 - \mu}{\sigma}$  *(M1)*  
 $\mu = 66.6$ ,  $\sigma = 22.6$  *A1A1* *N2N2*  
*[6 marks]*

9. (a) Using the chain rule *(M1)*  
 $f'(x) = (2 \cos(5x - 3)) 5 (= 10 \cos(5x - 3))$  *AI*  
 $f''(x) = -(10 \sin(5x - 3)) 5$   
 $= -50 \sin(5x - 3)$  *A1A1* *N2*

**Note:** Award *AI* for  $\sin(5x - 3)$ , *AI* for  $-50$ .

(b)  $\int f(x) dx = -\frac{2}{5} \cos(5x - 3) + c$  *A1A1* *N2*

**Note:** Award *AI* for  $\cos(5x - 3)$ , *AI* for  $-\frac{2}{5}$ .

*[6 marks]*

**Section B**

10. (a) using the cosine rule  $a^2 = b^2 + c^2 - 2bc \cos \hat{A}$  **(M1)**
- substituting correctly  $BC^2 = 65^2 + 104^2 - 2(65)(104)\cos 60^\circ$  **A1**
- $= 4225 + 10\,816 - 6760 = 8281$
- $\Rightarrow BC = 91 \text{ m}$  **A1**      **N2**
- 
- (b) finding the area, using  $\frac{1}{2}bc \sin \hat{A}$  **(M1)**
- substituting correctly, area  $= \frac{1}{2}(65)(104)\sin 60^\circ$  **A1**
- $= 1690\sqrt{3}$  (accept  $p = 1690$ ) **A1**      **N2**
- 
- (c) (i)  $A_1 = \left(\frac{1}{2}\right)(65)(x)\sin 30^\circ$  **A1**
- $= \frac{65x}{4}$  **AG**      **N0**
- 
- (ii)  $A_2 = \left(\frac{1}{2}\right)(104)(x)\sin 30^\circ$  **M1**
- $= 26x$  **A1**      **N1**
- 
- (iii) stating  $A_1 + A_2 = A$  or substituting  $\frac{65x}{4} + 26x = 1690\sqrt{3}$  **(M1)**
- simplifying  $\frac{169x}{4} = 1690\sqrt{3}$  **A1**
- $x = \frac{4 \times 1690\sqrt{3}}{169}$  **A1**
- $\Rightarrow x = 40\sqrt{3}$  (accept  $q = 40$ ) **A1**      **N2**
- 
- (d) (i) Recognizing that supplementary angles have equal sines  
e.g.  $\hat{ADC} = 180^\circ - \hat{ADB} \Rightarrow \sin \hat{ADC} = \sin \hat{ADB}$  **R1**
- 
- (ii) using sin rule in  $\triangle ADB$  and  $\triangle ADC$  **(M1)**
- substituting correctly  $\frac{BD}{\sin 30^\circ} = \frac{65}{\sin \hat{ADB}} \Rightarrow \frac{BD}{65} = \frac{\sin 30^\circ}{\sin \hat{ADB}}$  **A1**
- and  $\frac{DC}{\sin 30^\circ} = \frac{104}{\sin \hat{ADC}} \Rightarrow \frac{DC}{104} = \frac{\sin 30^\circ}{\sin \hat{ADC}}$  **M1**
- since  $\sin \hat{ADB} = \sin \hat{ADC}$
- $\frac{BD}{65} = \frac{DC}{104} \Rightarrow \frac{BD}{DC} = \frac{65}{104}$  **A1**
- $\Rightarrow \frac{BD}{DC} = \frac{5}{8}$  **AG**      **N0**

[18 marks]

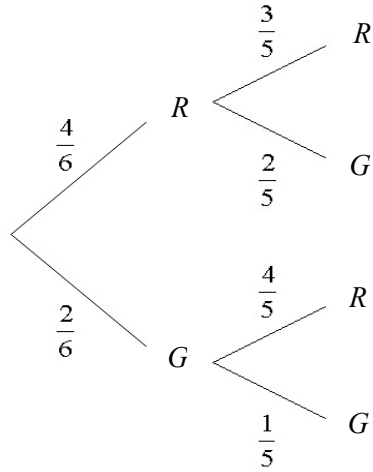
11. (a) Attempting to find unit vector ( $e_b$ ) in the direction of  $b$  (M1)
- Correct values =  $\frac{1}{\sqrt{3^2 + 4^2 + 0^2}} \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$  A1
- =  $\begin{pmatrix} 0.6 \\ 0.8 \\ 0 \end{pmatrix}$  A1
- Finding direction vector for  $b$ ,  $v_b = 18 \times e_b$  (M1)
- $b = \begin{pmatrix} 10.8 \\ 14.4 \\ 0 \end{pmatrix}$  A1
- Using vector representation  $b = b_0 + tv_b$  (M1)
- =  $\begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} + t \begin{pmatrix} 10.8 \\ 14.4 \\ 0 \end{pmatrix}$  AG N0
- (b) (i)  $t = 0 \Rightarrow (49, 32, 0)$  A1 N1
- (ii) Finding magnitude of velocity vector (M1)
- Substituting correctly  $v_h = \sqrt{(-48)^2 + (-24)^2 + 6^2}$  A1
- =  $54 \text{ (km h}^{-1}\text{)}$  A1 N2
- (c) (i) At R,  $\begin{pmatrix} 10.8t \\ 14.4t \\ 5 \end{pmatrix} = \begin{pmatrix} 49 - 48t \\ 32 - 24t \\ 6t \end{pmatrix}$  A1
- $t = \frac{5}{6} (= 0.833)$ (hours) A1 N1
- (ii) For substituting  $t = \frac{5}{6}$  into expression for  $b$  or  $h$  (M1)
- $(9, 12, 5)$  A2 N3

[15 marks]

12. (a) Using  $E(X) = \sum_0^2 x P(X = x)$  (M1)

Substituting correctly  $E(X) = 0 \times \frac{3}{10} + 1 \times \frac{6}{10} + 2 \times \frac{1}{10}$  A1  
 $= 0.8$  A1 N2

(b) (i)



A1A1A1 N3

**Note:** Award **A1** for each complementary pair of probabilities, i.e.  $\frac{4}{6}$  and  $\frac{2}{6}$ ,  $\frac{3}{5}$  and  $\frac{2}{5}$ ,  $\frac{4}{5}$  and  $\frac{1}{5}$ .

(ii)  $P(Y = 0) = \frac{2}{5} \times \frac{1}{5} = \frac{2}{30}$  A1

$P(Y = 1) = P(RG) + P(GR) \left( = \frac{4}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{4}{5} \right)$  M1  
 $= \frac{16}{30}$  A1

$P(Y = 2) = \frac{4}{6} \times \frac{3}{5} = \frac{12}{30}$  (A1)

For forming a distribution M1

|            |                |                 |                 |
|------------|----------------|-----------------|-----------------|
| $y$        | 0              | 1               | 2               |
| $P(Y = y)$ | $\frac{2}{30}$ | $\frac{16}{30}$ | $\frac{12}{30}$ |

N4

continued...



Question 12 continued

(c)  $P(\text{Bag A}) = \frac{2}{6} \left( = \frac{1}{3} \right)$  (A1)

$P(\text{Bag B}) = \frac{4}{6} \left( = \frac{2}{3} \right)$  (A1)

For summing  $P(A \cap RR)$  and  $P(B \cap RR)$  (M1)

Substituting correctly  $P(RR) = \frac{1}{3} \times \frac{1}{10} + \frac{2}{3} \times \frac{12}{30}$  (A1)

$= 0.3$  (A1) (N3)

(d) For recognising that  $P(1 \text{ or } 6 | RR) = P(A | RR) = \frac{P(A \cap RR)}{P(RR)}$  (M1)

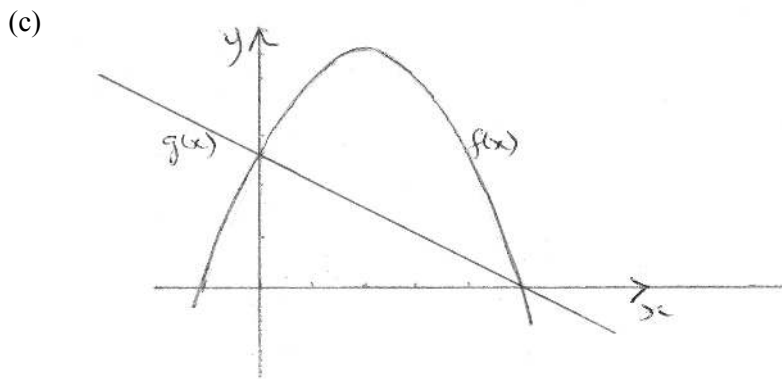
$= \frac{1}{30} \div \frac{27}{90}$  (A1)

$= 0.111$  (A1) (N2)

[19 marks]

13. (a) Curve intersects  $y$ -axis when  $x = 0$  (A1)  
 Gradient of tangent at  $y$ -intercept = 2 A1  
 $\Rightarrow$  gradient of  $N = -\frac{1}{2}$  ( $= -0.5$ ) A1  
 Finding  $y$ -intercept, 2.5 A1  
 Therefore, equation of  $N$  is  $y = -0.5x + 2.5$  AG N0

- (b)  $N$  intersects curve when  $-0.5x^2 + 2x + 2.5 = -0.5x + 2.5$  A1  
 Solving equation (M1)  
*e.g.* sketch, factorising  
 $\Rightarrow x = 0$  or  $x = 5$  A1  
 Other point when  $x = 5$  (R1)  
 $x = 5 \Rightarrow y = 0$  (so other point  $(5, 0)$ ) A1 N2



Using appropriate method, with subtraction/correct expression, **correct** limits **M1A1**

*e.g.*  $\int_0^5 f(x) dx - \int_0^5 g(x) dx, \int_0^5 (-0.5x^2 + 2.5x) dx$

Area = 10.4

A2 N2  
**[13 marks]**



