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MARKSCHEME

May 2014

MATHEMATICS
DISCRETE MATHEMATICS

Higher Level

Paper 3

9 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to Scoris instructions and the document "**Mathematics HL: Guidance for e-marking May 2014**". It is essential that you read this document before you start marking. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by Scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

Implied marks appear in ***brackets eg (M1)****, and can only be awarded if* ***correct*** *work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (****FT****) marks are awarded where an incorrect answer from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s). To award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (****MR****). A candidate should be penalized only once for a particular mis-read. Use the* ***MR*** *stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an* ***M*** *mark, but award all others so that the candidate only loses one mark.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief ***note*** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

Candidates should ***NO LONGER*** *be penalized for an accuracy error* ***(AP)***.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for ***FT***.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 3, but calculators with symbolic manipulation features (for example, TI-89) are not allowed.

Calculator notation
The Mathematics HL guide says:

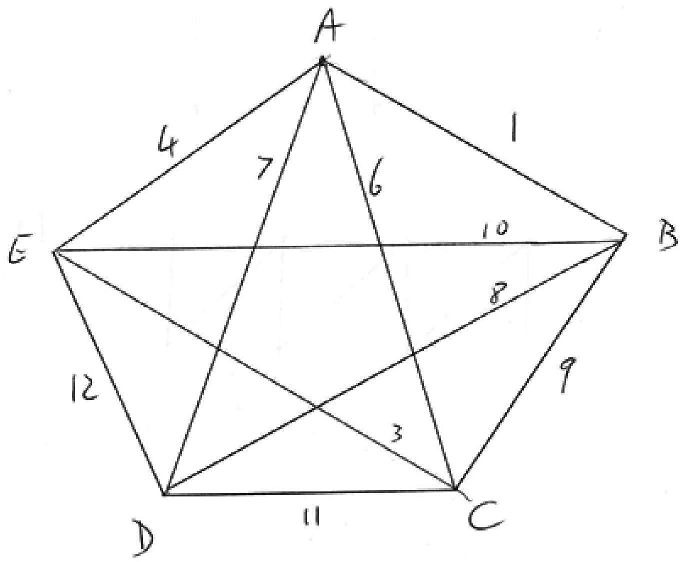
Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

1. (a)



complete graph on five vertices *A1*
weights correctly marked on graph *A1*

[2 marks]

(b) clear indication that the nearest-neighbour algorithm has been applied *M1*
$\text{DA}\,(\text{or } 7)$ *A1*
$\text{AB}\,(\text{or } 1)\ \text{BC}\,(\text{or } 9)$ *A1*
$\text{CE}\,(\text{or } 3)$, $\text{ED}\,(\text{or } 12)$, giving $\text{UB} = 32$ *A1*

[4 marks]

(c) attempt to use the vertex deletion method *M1*
minimum spanning tree is ECBD *A1*
(EC 3, BD 8, BC 9 total 20)
reconnect A with the two edges of least weight, namely AB (1)and AE (4) *M1*
lower bound is 25 *A1*

[4 marks]

Total [10 marks]

2. (a) (i) **METHOD 1**

using a relevant list of powers of 13: $(1), 13, 169, (2197)$ ***M1***

$871 = 5\times 13^2 + 2\times 13$ ***A1***

$871 = 520_{13}$ ***A1***

$1157 = 6\times 13^2 + 11\times 13$ ***A1***

$1157 = 6\text{B}0_{13}$ ***A1***

METHOD 2

attempted repeated division by 13 ***M1***

$871 \div 13 = 67$; $67 \div 13 = 5\,\text{rem}\,2$ ***A1***

$871 = 520_{13}$ ***A1***

$1157 \div 13 = 89$; $89 \div 13 = 6\,\text{rem}\,11$ ***A1***

$1157 = 6\text{B}0_{13}$ ***A1***

Note: Allow (11) for B only if brackets or equivalent are present.

(ii) $871 = 13\times 67$; $1157 = 13\times 89$ ***(M1)***

67 and 89 are primes (base 10) or they are co-prime ***A1***

So $\gcd(871, 1157) = 13$ ***AG***

Note: Must be done by hence not Euclid's algorithm on 871 and 1157.

[7 marks]

(b) let K be the set of possible remainders on division by n ***(M1)***

then $K = \{0, 1, 2, \ldots n-1\}$ has n members ***A1***

because $n < n+1\left(= n(L)\right)$ ***A1***

by the pigeon-hole principle (appearing anywhere and not necessarily mentioned by name as long as is explained) ***R1***

at least two members of L correspond to one member of K ***AG***

[4 marks]

(c) (i) form the appropriate linear combination of the equations: ***(M1)***

$2a + b - c = 7x + 7z$ ***A1***

$= 7(x+z)$ ***R1***

so 7 divides $2a + b - c$ ***AG***

(ii) modulo 2, the equations become ***M1***

$y + z = 1$

$z = 0$ ***A1***

$x = 1$

solution: $(1, 1, 0)$ ***A1***

Note: Award full mark to use of GDC (or done manually) to solve the system giving $x = -1$, $y = -3$, $z = 2$ and then converting mod 2.

[6 marks]

continued…

Question 2 continued

(d) (i) separate consideration of even and odd n ***M1***
$\text{even}^2 - \text{even} + \text{odd}$ is odd ***A1***
$\text{odd}^2 - \text{odd} + \text{odd}$ is odd ***A1***
all elements of P are odd ***AG***

Note: Allow other methods *eg*, $n^2 - n = n(n-1)$ which must be even.

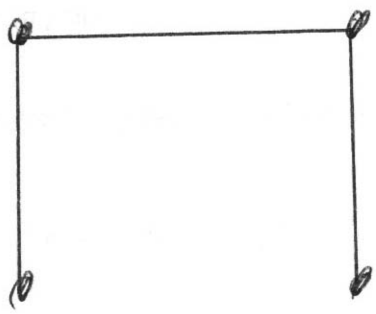
(ii) the list is $[41, 41, 43, 47, 53, 61]$ ***A1***

(iii) $41^2 - 41 + 41 = 41^2$ divisible by 41 ***A1***
but is not a prime ***R1***
the statement is disproved (by counterexample) ***AG***

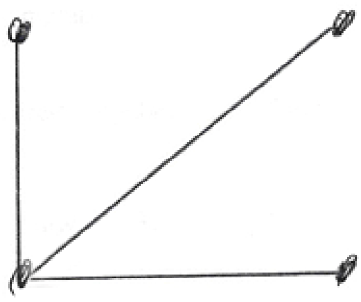
[6 marks]

Total [23 marks]

3. (a) (i)



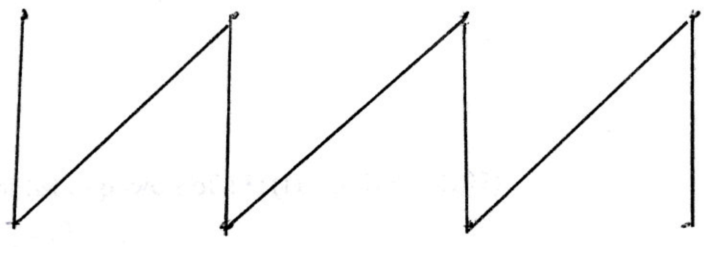
or



A1

Note: Or equivalent not worrying about the orientation.

(ii)



A1

Note: Other trees are possible, but must clearly come from the bipartite graph, so, for example, a straight line graph is not acceptable unless the bipartite nature is clearly shown *eg*, with black and white vertices.

[2 marks]

continued…

Question 3 continued

(b) graph is simple implies maximum degree is $n-1$ ***A1***
graph is connected implies minimum degree is 1 ***A1***
by a pigeon-hole principle two vertices must have the same degree ***R1***

[3 marks]

(c) if the graph is not a tree it contains a cycle ***A1***
remove one edge of this cycle ***M1***
the graph remains connected ***A1***
repeat until there are no cycles ***M1***
the final graph is connected and has no cycles ***A1***
so is a tree ***AG***

Note: Allow other methods *eg*, induction, reference to Kruskal's algorithm.

[5 marks]

Total [10 marks]

4. (a) (i) use of auxiliary equation or recognition of a geometric sequence ***(M1)***
$u_n = (-2)^n u_0$ or $= \mathrm{A}(-2)^n$ or $u_1(-2)^{n-1}$ ***A1***

(ii) substitute suggested solution ***M1***
$An + B + 2\left(A(n-1) + B\right) = 3n - 2$ ***A1***
equate coefficients of powers of n and attempt to solve ***(M1)***
$A = 1$, $B = 0$ ***A1***
(so particular solution is $u_n = n$)

(iii) use of general solution = particular solution + homogeneous solution ***(M1)***
$u_n = \mathrm{C}(-2)^n + n$ ***A1***
attempt to find C using $u_1 = 7$ ***M1***
$u_n = -3(-2)^n + n$ ***A1***

[10 marks]

(b) the auxiliary equation is $r^2 - 2r + 2 = 0$ ***A1***
solutions: $r_1, r_2 = 1 \pm \mathrm{i}$ ***A1***
general solution of the recurrence: $u_n = \mathrm{A}(1+\mathrm{i})^n + \mathrm{B}(1-\mathrm{i})^n$ or trig form ***A1***
attempt to impose initial conditions ***M1***
$A = B = 1$ or corresponding constants for trig form ***A1***
$u_n = 2^{\left(\frac{n}{2}+1\right)} \times \cos\left(\frac{n\pi}{4}\right)$ ***A1A1***

[7 marks]

Total [17 marks]