



**MATHEMATICS**  
**STANDARD LEVEL**  
**PAPER 1**

Wednesday 7 May 2008 (afternoon)

1 hour 30 minutes

Candidate session number

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**INSTRUCTIONS TO CANDIDATES**

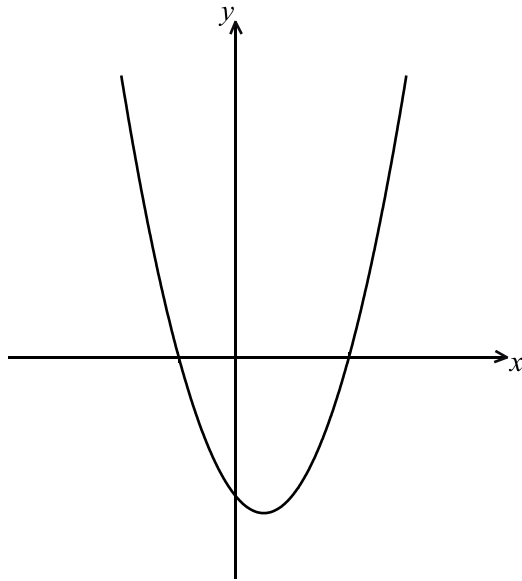
- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.





2. [Maximum mark: 6]

The following diagram shows part of the graph of  $f$ , where  $f(x) = x^2 - x - 2$ .



(a) Find both  $x$ -intercepts.

[4 marks]

(b) Find the  $x$ -coordinate of the vertex.

[2 marks]

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3. [Maximum mark: 6]

Let  $M = \begin{pmatrix} 2 & 1 \\ 2 & -1 \end{pmatrix}$ .

(a) Write down the determinant of  $M$ . [1 mark]

(b) Write down  $M^{-1}$ . [2 marks]

(c) Hence solve  $M \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$ . [3 marks]

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4. [Maximum mark: 6]

(a) Given that  $\cos A = \frac{1}{3}$  and  $0 \leq A \leq \frac{\pi}{2}$ , find  $\cos 2A$ .

[3 marks]

(b) Given that  $\sin B = \frac{2}{3}$  and  $\frac{\pi}{2} \leq B \leq \pi$ , find  $\cos B$ .

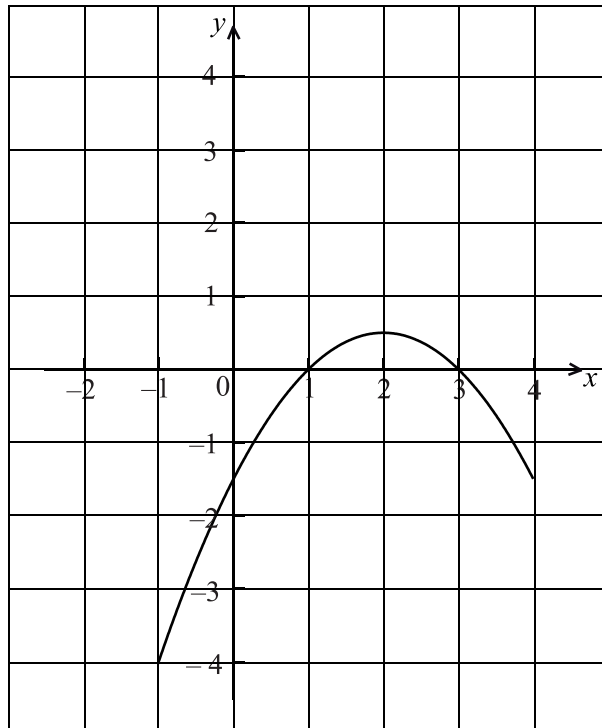
[3 marks]

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5. [Maximum mark: 6]

Part of the graph of a function  $f$  is shown in the diagram below.



(a) On the same diagram sketch the graph of  $y = -f(x)$ . [2 marks]

(b) Let  $g(x) = f(x+3)$ .

(i) Find  $g(-3)$ .

(ii) Describe **fully** the transformation that maps the graph of  $f$  to the graph of  $g$ . [4 marks]

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6. [Maximum mark: 7]

There are 20 students in a classroom. Each student plays only one sport. The table below gives their sport and gender.

	Football	Tennis	Hockey
Female	5	3	3
Male	4	2	3

- (a) One student is selected at random.
  - (i) Calculate the probability that the student is a male or is a tennis player.
  - (ii) Given that the student selected is female, calculate the probability that the student does not play football. [4 marks]
  
- (b) Two students are selected at random. Calculate the probability that neither student plays football. [3 marks]

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7. [Maximum mark: 7]

Let  $\int_1^5 3f(x) dx = 12$ .

(a) Show that  $\int_5^1 f(x) dx = -4$ . [2 marks]

(b) Find the value of  $\int_1^2 (x + f(x)) dx + \int_2^5 (x + f(x)) dx$ . [5 marks]

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**SECTION B**

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

8. [Maximum mark: 13]

Consider the points  $A(1, 5, 4)$ ,  $B(3, 1, 2)$  and  $D(3, k, 2)$ , with  $(AD)$  perpendicular to  $(AB)$ .

(a) Find

(i)  $\vec{AB}$ ;

(ii)  $\vec{AD}$ , giving your answer in terms of  $k$ . [3 marks]

(b) Show that  $k = 7$ . [3 marks]

The point  $C$  is such that  $\vec{BC} = \frac{1}{2}\vec{AD}$ .

(c) Find the position vector of  $C$ . [4 marks]

(d) Find  $\cos \hat{ABC}$ . [3 marks]

9. [Maximum mark: 14]

Let  $f : x \mapsto \sin^3 x$ .

(a) (i) Write down the range of the function  $f$ .

(ii) Consider  $f(x) = 1$ ,  $0 \leq x \leq 2\pi$ . Write down the number of solutions to this equation. Justify your answer. [5 marks]

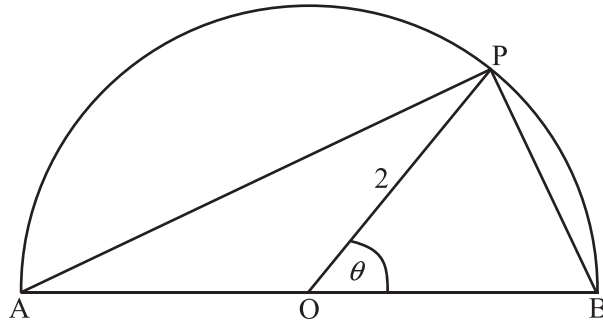
(b) Find  $f'(x)$ , giving your answer in the form  $a \sin^p x \cos^q x$  where  $a, p, q \in \mathbb{Z}$ . [2 marks]

(c) Let  $g(x) = \sqrt{3} \sin x (\cos x)^{\frac{1}{2}}$  for  $0 \leq x \leq \frac{\pi}{2}$ . Find the volume generated when the curve of  $g$  is revolved through  $2\pi$  about the  $x$ -axis. [7 marks]



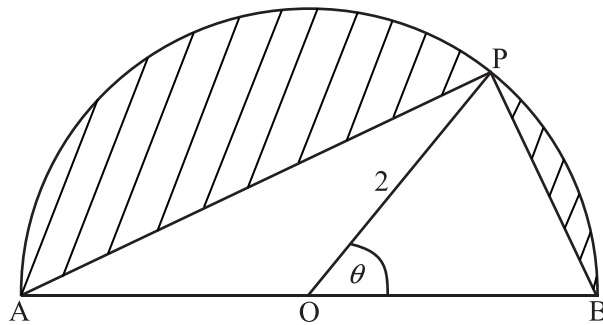
10. [Maximum mark: 18]

The following diagram shows a semicircle centre O, diameter [AB], with radius 2. Let P be a point on the circumference, with  $\widehat{POB} = \theta$  radians.



- (a) Find the area of the triangle OPB, in terms of  $\theta$ . [2 marks]
- (b) Explain why the area of triangle OPA is the same as the area triangle OPB. [3 marks]

Let  $S$  be the total area of the two segments shaded in the diagram below.



- (c) Show that  $S = 2(\pi - 2 \sin \theta)$ . [3 marks]
- (d) Find the value of  $\theta$  when  $S$  is a local minimum, justifying that it is a minimum. [8 marks]
- (e) Find a value of  $\theta$  for which  $S$  has its greatest value. [2 marks]

