International Baccalaureate®
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# MARKSCHEME

# May 2013

# MATHEMATICAL STUDIES

# Standard Level

# Paper 2

**Paper 2 Markscheme**
**Instructions to Examiners**

**Notes:**     **If in doubt about these instructions or any other marking issues, contact your team leader for clarification.**

**1     Abbreviations**

*M*     Marks awarded for **Method**

*A*     Marks awarded for an **Answer** or for **Accuracy**

*R*     Marks awarded for clear **Reasoning**

*G*     Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, when no working shown.

*AG*     **Answer Given** in the question and consequently, marks not awarded.

**ft**     Marks that can be awarded as **follow through** from previous results in the question.

**2     Method of Marking**

(a)     All marking must be done in scoris using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.

(b)     If a question part is completely correct use the number tick annotations to award full marks. If a part is completely wrong use the *A0* annotation, otherwise full annotations must be shown.

(c)     Working crossed out by the candidate should not be awarded any marks.

(d)     Where candidates have written two solutions to a question, only the first solution should be marked.

(e)     If correct working results in a correct answer but then further working is developed, full marks may **not** always be awarded. Full marks **will** be awarded if the candidate shows correct working leading to the correct answer. See also section 4(c).

**Example:**     Calculate the gradient of the line passing through the points $(5, 3)$ and $(0, 9)$.

| Markscheme | Candidates' Scripts | Marking |
|---|---|---|
| $\dfrac{9-3}{0-5}$ **(M1)** <br><br> *Award **(M1)** for correct substitution in gradient formula* <br><br><br> $= -\dfrac{6}{5}$ **(A1)** | (i)  $\dfrac{9-3}{0-5} = -\dfrac{6}{5}$ <br><br> Gradient is $= -\dfrac{6}{5}$ <br><br> *(There is clear understanding of the gradient.)* <br><br> $y = -\dfrac{6}{5}x + 9$ <br><br><br> (ii)  $\dfrac{9-3}{0-5} = -\dfrac{6}{5}$ <br><br> $y = -\dfrac{6}{5}x + 9$ <br><br> *(There is confusion about what is required.)* | **(M1)** <br><br> **(A1)** <br><br><br><br><br><br><br> **(M1)** <br><br> **(A0)** |

**3    Follow-through (ft) Marks**

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with **'(ft)'**.

(a)    Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.

(b)    If an answer resulting from follow through is extremely unrealistic (*e.g.* negative distances or incorrect by large order of magnitude) then the final *A* mark should not be awarded.

(c)    If a question is transformed by an error into a **different**, **much simpler question** then follow through may not apply.

(d)    To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.

(e)    The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**

(f)    Inadvertent use of radians will be penalised the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

**Example:**    Finding angles and lengths using trigonometry

| Markscheme | Candidates' Scripts | Marking |
|---|---|---|
| (a) $\dfrac{\sin A}{3} = \dfrac{\sin 30}{4}$    *(M1)(A1)* <br><br> *Award (M1) for substitution in sine rule formula, (A1) for correct substitutions.* <br><br> $A = 22.0°\,(22.0243\ldots)\,(A1)(G2)$ | (a) $\dfrac{\sin A}{4} = \dfrac{\sin 30}{3}$ <br><br><br><br><br> $A = 41.8°$ | *(M1)(A0)* <br><br> *(use of sine rule but with wrong values)* <br><br><br> *(A0)* <br> *(**Note:** the 2$^{nd}$ (A1) here was not marked (ft) and cannot be awarded because there was an earlier error in the **same** question part.)* |
| (b) $x = 7\tan(22.0243\ldots°)$   *(M1)* <br> $\quad = 2.83\,(2.83163\ldots)$   *(A1)(ft)* | (b)   case (i)   $x = 7\tan 41.8°$ <br> $\qquad\qquad\qquad = 6.26$ <br> ***but***   case (ii)    6.26 | *(M1)* <br> *(A1)(ft)* <br> *(G0)* <br> *since no working shown* |

**4      Using the Markscheme**

(a)  *A* marks are **dependent** on the preceding *M* mark being awarded, it is **not** possible to award *(M0)(A1)*.  Once an *(M0)* has been awarded, all subsequent *A* marks are lost in that part of the question, even if calculations are performed correctly, until the next *M* mark.
The only exception to this will be for an answer where the accuracy is specified in the question – see section 5.

(b)  *A* marks are **dependent** on the *R* mark being awarded, it is **not** possible to award *(A1)(R0)*.  Hence the *(A1)* cannot be awarded for an answer which is correct when no reason or the wrong reason is given.

(c)  In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working.  Answers which are correct but not supported by adequate working will **not always receive full marks**, these unsupported answers are designated *G* in the mark scheme as an alternative to the full marks.  Example *(M1)(A1)(A1)(G2)*.

**Example:**  Using trigonometry to calculate an angle in a triangle.

| Markscheme | Candidates' Scripts | Marking |
|---|---|---|
| (a)  $\dfrac{\sin A}{3} = \dfrac{\sin 30}{4}$  *(M1)(A1)*<br><br>*Award (M1) for substitution in sine rule formula, (A1) for correct substitutions.*<br><br><br>$A = 22.0°$ (22.0243…) *(A1)(G2)* | (i)  $\dfrac{\sin A}{3} = \dfrac{\sin 30}{4}$<br><br>$A = 22.0°$<br><br>(ii)  $A = 22.0°$<br>***Note: G** marks are used only if no working has been shown and the answer is correct.* | *(M1)(A1)*<br><br>*(A1)*<br><br><br>*(G2)* |

(d)  **Alternative methods** may not always be included.  Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.
Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' *etc*.

(e)  Unless the question specifies otherwise, accept **equivalent forms**.  For example: $\dfrac{\sin\theta}{\cos\theta}$ for $\tan\theta$.
On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.
Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:
the 3 significant figure answer worked through from full calculator display;
the exact value (for example $\sqrt{3}$ if applicable);
the full calculator display in the form 2.83163… as in the example above.
Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

(f)     As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7; 1·7; 1,7 .

Different descriptions of an interval:  $3 < x < 5$ ; (3, 5);  ] 3, 5 [ .

Different forms of notation for set properties (*e.g.* complement):  $A'$ ;  $\bar{A}$ ;  $A^c$ ;  $U - A$ ; $(A$ ; $U \setminus A$.

Different forms of logic notation: $\neg p$ ;  $p'$ ;  $\tilde{p}$ ;  $\bar{p}$ ;  $\sim p$.

$$p \Rightarrow q \,;\ p \rightarrow q \,;\ q \Leftarrow p \,.$$

(g)     Discretionary marks:  There will be very rare occasions where the markscheme does not cover the work seen.  In such cases the annotation DM should be used to indicate where an examiner has used discretion.  Discretion should be used sparingly and if there is doubt and exception should be raised through scoris to the team leader.

**T**here will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

**5      Accuracy of Answers**

**Incorrect accuracy should be penalized once only in each question according to the rules below.**

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1.      If the candidate's unrounded answer is seen and would round to the required 3 sf answer, then award *(A1)* and ignore subsequent rounding.

2.      If the candidate's unrounded answer is **not** seen then award *(A1)* if the answer given is **correctly** rounded to 2 or more significant figures, otherwise *(A0)*.

**Note:** If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.

3.      If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples following.

| | If candidates final answer is given … | | | | |
|---|---|---|---|---|---|
| | Exact or correct to 3 or more sf | **Incorrect to 3 sf** | Correct to 2 sf[3] | Incorrect to 2 sf | Correct or incorrect to 1 sf |
| Unrounded answer seen[1] | Award the final *(A1)* irrespective of correct or incorrect rounding | | | | |
| Unrounded answer not seen[2] | *(A1)* | *(A0)* | *(A1)* | *(A0)* | *(A0)* |
| Treatment of subsequent parts | As per MS | Treat as follow through, only if working is seen[3] | | | |

**Examples:**

| Markscheme | Candidates' Scripts | Marking |
|---|---|---|
| 9.43 (9.43398…) *(A1)* | (i) 9.43398… is seen followed by 9; 9.4; 9.43; 9.434 *etc*. (correctly rounded) | *(A1)* |
| | (ii) 9.43398… is seen followed by 9.433; 9.44 *etc*. (incorrectly rounded) | *(A1)* |
| | (iii) 9.4 | *(A1)* |
| | (iv) 9 | *(A0)* *(correct to 1 sf)* |
| | (v) 9.3 | *(A0)* *(incorrectly rounded to 2 sf)* |
| | (vi) 9.44 | *(A0)* *(incorrectly rounded to 3 sf)* |

| Markscheme | Candidates' Scripts | Marking |
|---|---|---|
| 7.44 (7.43798…) *(A1)* | (i) 7.43798… is seen followed by 7; 7.4; 7.44; 7.438 *etc*. (correctly rounded) | *(A1)* |
| | (ii) 7.43798… is seen followed by 7.437; 7.43 *etc*. (incorrectly rounded) | *(A1)* |
| | (iii) 7.4 | *(A1)* |
| | (iv) 7 | *(A0)* *(correct to 1 sf)* |
| | (v) 7.5 | *(A0)* *(incorrectly rounded to 2 sf)* |
| | (vi) 7.43 | *(A0)* *(incorrectly rounded to 3 sf)* |

**Example:** ABC is a right angled triangle with angle $ABC = 90°$, $AC = 32$ cm and $AB = 30$ cm. Find (a) the length of BC, (b) The area of triangle ABC.

| Markscheme | Candidates' Scripts | Marking |
|---|---|---|
| (a)    $BC = \sqrt{32^2 - 30^2}$    **(M1)**<br><br>*Award **(M1)** for correct substitution in Pythagoras' formula*<br><br>$= 11.1 \left(\sqrt{124}, 11.1355...\right)$ (cm) **(A1)**<br><br><br>(b)    $\text{Area} = \dfrac{1}{2} \times 30 \times 11.1355...$    **(M1)**<br><br>*Award **(M1)** for correct substitution in area of triangle formula*<br><br>$= 167 (167.032...) (\text{cm}^2)$    **(A1)(ft)** | (a)    $BC = \sqrt{32^2 - 30^2}$           **(M1)**<br><br>       $11$ (cm)                      **(A1)**<br>              *(2 sf answer only seen, but correct)*<br><br>(b)    case (i)    $\text{Area} = \dfrac{1}{2} \times 30 \times 11$    **(M1)**<br>                                 *(working shown)*<br><br>                    $= 165 (\text{cm}^2)$        **(A1)(ft)**<br><br>       case (ii)        $= 165 (\text{cm}^2)$    **(M0)(A0)(ft)**<br>       *(No working shown, the answer 11 is treated as a ft, so no marks awarded here)* | |

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

*e.g.* Chi-squared, correlation coefficient, mean

| Markscheme | Candidates' Scripts | Marking |
|---|---|---|
| Chi-squared<br><br>$7.68\,(7.67543…)$    **(A2)** | (a)    7.7 | **(G2)** |
| | (b)    7.67 | **(G1)** |
| | (c)    7.6 | **(G1)** |
| | (d)    8 | **(G0)** |
| | (e)    7 | **(G0)** |
| | (e)    7.66 | **(G0)** |

Regression line

| Markscheme | Candidates' Scripts | Marking |
|---|---|---|
| $y = 0.888x + 13.5$ **(A2)** <br> ($y = 0.887686\ldots x + 13.4895\ldots$) <br> If an answer is not in the form of an equation award at most **(A1)(A0)**. | (a) $\quad y = 0.89x + 13$ | **(G2)** <br> *(both accepted)* |
| | (b) $\quad y = 0.88x + 13$ | **(G1)** <br> *(one rounding error)* |
| | (c) $\quad y = 0.88x + 14$ | **(G1)** <br> *(rounding error repeated)* |
| | (d) (i) $\quad y = 0.9x + 13$ | |
| | (ii) $\quad y = 0.8x + 13$ | **(G1)** <br> *(1 sf not accepted)* |
| | (e) $\quad 0.88x + 13$ | **(G0)** <br> *(one rounding error and not an equation)* |

Maximum/minimum/points of intersection

| Markscheme | Candidates' Scripts | Marking |
|---|---|---|
| $(2.06, 4.49)$ **(A1)(A1)** <br> $(2.06020\ldots, 4.49253\ldots)$ | (a) $(2.1, 4.5)$ | **(A1)(A1)** <br> *(both accepted)* |
| | (b) $(2.0, 4.4)$ | **(A1)** <br> *(same rounding error twice)* |
| | (c) $(2.06, 4.4)$ | **(A1)** <br> *(one rounding error)* |
| | (d) $(2, 4.4)$ | **(A0)** <br> *(1sf not accepted, one rounding error)* |

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. Exact answers such as $\dfrac{1}{4}$ can be written as decimals to fewer than three significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected.

Ratios of $\pi$ and answers taking the form of square roots of integers or any rational power of an integer (*e.g.* $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$ ,) may be accepted as exact answers. All other powers (*e.g.* of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

**If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy**. In **all** such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding **(M0)**.

**6      Level of accuracy in finance questions**

The accuracy level required for answers will be specified in all questions involving money.  This will usually be either whole units or two decimal places.  The first answer not given to the specified level of accuracy will not be awarded the final *A* mark.  The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

**Example:**   A financial question demands accuracy correct to 2 dp.

| Markscheme | Candidates' Scripts | Marking |
|---|---|---|
| $231.62 (231.6189)   *(A1)* | (i)     231.6 | *(A0)* |
| | (ii)    232 | *(A0)* |
| | | *(Correct rounding to incorrect level)* |
| | (iii)   231.61 | *(A0)* |
| | (iv)    232.00 | *(A0)* |
| | | *(Parts (iii) and (iv) are both incorrect rounding to correct level)* |

**7      Units in answers**

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme.  The first correct answer with no units or incorrect units will not be awarded the final *A* mark.  The markscheme will give clear instructions to ensure that only one mark per paper can be lost for lack of units or incorrect units.

The units are considered only when the numerical answer is awarded *(A1)* under the accuracy rules given in Section 5.

**Example:**

| Markscheme | Candidates' Scripts | Marking |
|---|---|---|
| (a)     37000 m$^2$   *(A1)* | (a)     36000 m$^2$ | *(A0)* |
| | | *(Incorrect answer so units not considered)* |
| (b)     3200 m$^3$    *(A1)* | (b)     3200 m$^2$ | *(A0)* |
| | | *(Incorrect units)* |

**If no method is shown and the answer is correct but with incorrect or missing units award G marks with a one mark penalty.**

**8      Graphic Display Calculators**

Candidates will often be obtaining solutions directly from their calculators.  They must use mathematical notation, not calculator notation.  No method marks can be awarded for incorrect answers supported only by calculator notation.  The comment 'I used my GDC' cannot receive a method mark.

**QUESTION 1**

(a)   $U$



*(A1)(A1)(A1)(A1)*   *[4 marks]*
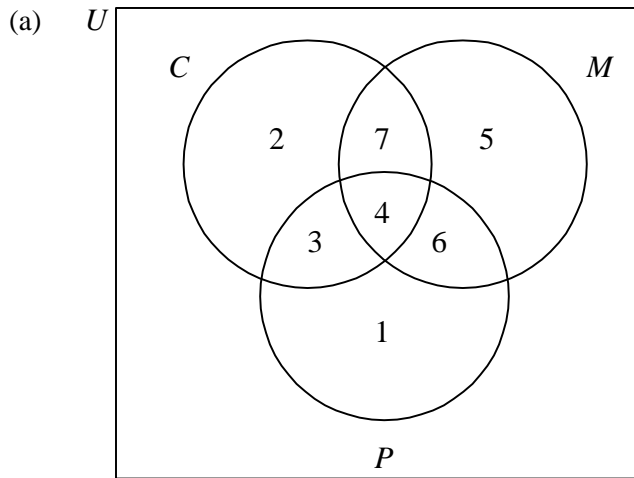
Award *(A1)* for 3 intersecting circles and rectangle, *(A1)* for 1, 3, 4 and 7,
*(A1)* for 2, *(A1)* for 6 and 5.

(b)   (i)   2                                                                     *(A1)***(ft)**

(ii)   6                                                                     *(A1)***(ft)**

(iii)   $40-\left(1+6+2+3+4+7+5\right)$                                *(M1)*

> **Note:** Award *(M1)* for subtracting all their values from 40.

$=12$                                                           *(A1)***(ft)***(G2)*   *[4 marks]*

> **Note:** Follow through from their Venn diagram for parts (i), (ii) and (iii).

(c)   (i)   $\dfrac{16}{40}\left(\dfrac{2}{5},0.4,40\%\right)$                      *(A1)(A1)(G2)*

> **Note:** Award *(A1)* for numerator, *(A1)* for denominator.  Answer must be less than 1 otherwise award *(A0)(A0)*.  Award *(A0)(A0)* if answer is given as incorrect reduced fraction without working.

(ii)   $\dfrac{20}{40}\left(\dfrac{1}{2},0.5,50\%\right)$                       *(A1)***(ft)** *(A1)* *(G2)*

> **Note:** Award *(A1)***(ft)** for numerator, *(A1)* for denominator. Follow through from their Venn diagram.  Answer must be less than 1 otherwise award *(A0)(A0)*. Award *(A0)(A0)* if answer is given as incorrect reduced fraction without working.

*Question 1 continued*

(iii)    $\frac{6}{40}\left(\frac{3}{20}, 0.15, 15\%\right)$                    *(A1)*(ft)*(A1)(G2)*

> **Note:** Award *(A1)*(ft) for numerator, *(A1)* for denominator. Follow through from their Venn diagram.  Answer must be less than 1 otherwise award *(A0)(A0)*.  Award *(A0)(A0)* if answer is given as incorrect reduced fraction without working.

(iv)    $\frac{11}{16}(0.6875, 68.75\%)$                    *(A1)*(ft)*(A1)(G2)*    *[8 marks]*

> **Note:** Award *(A1)*(ft) for numerator, *(A1)* for denominator. Follow through from their Venn diagram. Answer must be less than 1 otherwise award *(A0)(A0)*.  Award *(A0)(A0)* if answer is given as incorrect reduced fraction without working.

(d)    $\frac{16}{40}\times\frac{15}{39}$                    *(A1)(A1)*(ft)

> **Note:** Award *(A1)* for multiplication of their probabilities, *(A1)*(ft) for their correct probabilities.
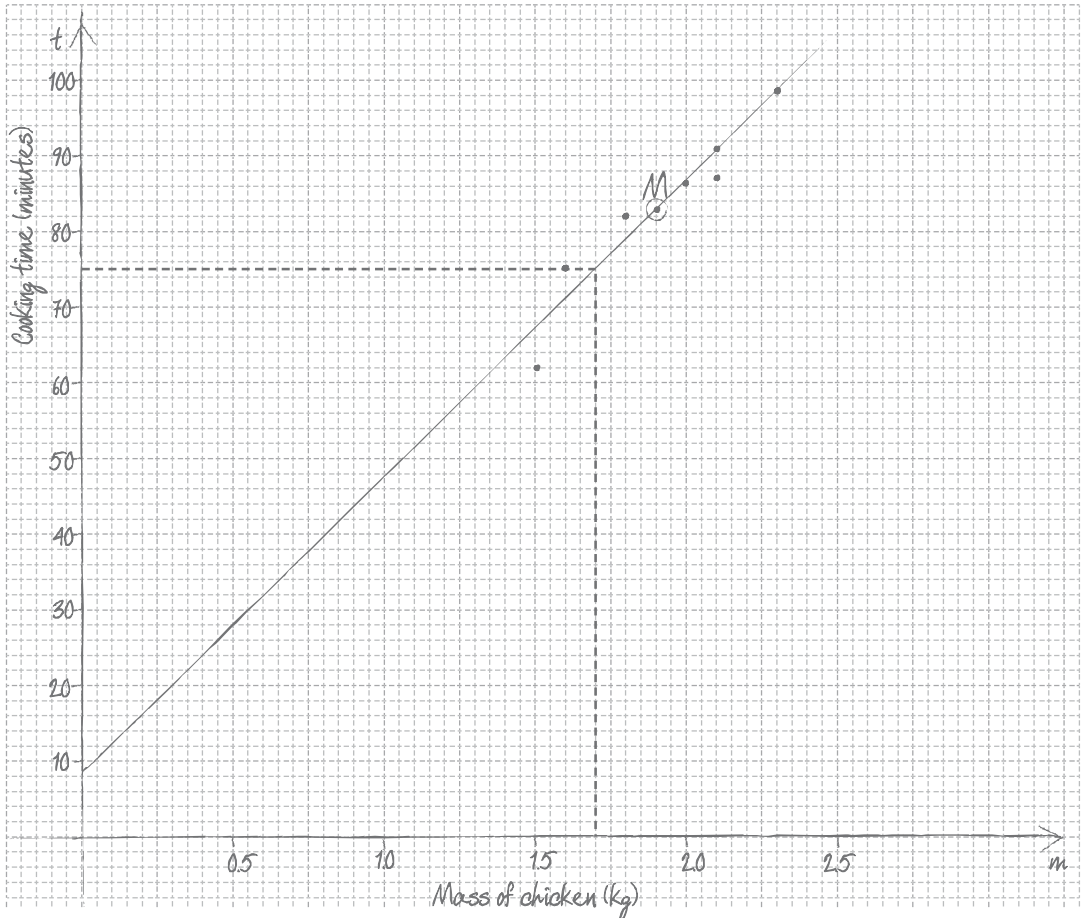
$\frac{240}{1560}\left(\frac{2}{13}, 0.153846\ldots, 15.4\%\right)$                    *(A1)*(ft)*(G2)*    *[3 marks]*

> **Note:** Follow through from their answer to part (c)(i).
> Answer must be less than 1 otherwise award at most *(A1)(A1)(A0)*(ft).

**Total [19 marks]**

**QUESTION 2**

(a)



*(A1) for correct scales and labels (mass or m on the horizontals axis, time or t on the vertical axis)*
*(A3) for 7 or 8 correctly placed data points*
*(A2) for 5 or 6 correctly placed data points*
*(A1) for 3 or 4 correctly placed data points, (A0) otherwise.*          *(A4)*     *[4 marks]*

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**Note:** If axes reversed award at most *(A0)(A3)(ft)*. If graph paper not used, award at most *(A1)(A0)*.

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(b)  (i)   1.91 (kg)   (1.9125 kg)                                           *(G1)*

    (ii)  83 (minutes)                                                   *(G1)*     *[2 marks]*

(c)  Their mean point labelled.                                          *(A1)(ft)*     *[1 mark]*

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**Note:** Follow through from part (b). Accept any clear indication of the mean point. For example: circle around point, (*m*, *t*), M , etc.

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(d)  Line of best fit drawn on scatter diagram.                    *(A1)(ft)(A1)(ft)*     *[2 marks]*

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**Notes:** Award *(A1)(ft)* for straight line through their mean point, *(A1)(ft)* for line of best fit with intercept $9(\pm2)$. The second *(A1)(ft)* can be awarded even if the line does not reach the *t*-axis but, if extended, the *t*-intercept is correct.

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*Question 2 continued*

(e)     75                                                                   *(M1)(A1)(ft)(G2)*      *[2 marks]*

> **Notes:** Accept 74.77 from the regression line equation.
> Award *(M1)* for indication of the use of their graph to get an estimate
> **OR** for correct substitution of 1.7 in the correct regression line
> equation $t = 38.5m + 9.32$.

(f)     0.960 (0.959614…)                                                    *(G2)*      *[2 marks]*

> **Note:** Award *(G0)(G1)(ft)* for 0. 95, 0.959

(g)     Strong and positive                                                 *(A1)(ft)(A1)(ft)*      *[2 marks]*

> **Note:** Follow through from their correlation coefficient in part (f).

(h)     (i)     Cooking time is much larger (or smaller) than the other eight       *(A1)*

      (ii)    The gradient of the new line of best fit will be larger (or smaller)    *(A1)*      *[2 marks]*

> **Note:** Some acceptable explanations may include but are not limited to:
>
> *The line of best fit may be further away from the plotted points*
> *It may be steeper than the previous line (as the mean would change)*
> *The t-intercept of the new line is smaller (larger)*
>
> Do not accept vague explanations, like:
>
> *The new line would vary*
> *It would not go through all points*
> *It would not fit the patterns*
> *The line may be slightly tilted*

*Total [17 marks]*

**QUESTION 3** *Units are required in parts (b), (c), (d) and (e). The first time the correct units are not given the final (A1) is not awarded; do not penalize more than once.*

(a)   $ST = \dfrac{1.6}{\cos 35°}$                                                    *(M1)(A1)*

> **Note:** Award *(M1)* for correctly substituted trig equation, *(A1)* for 1.6 seen.

**OR**

$\dfrac{ST}{\sin 35°} = \dfrac{3.2}{\sin 110°}$                                           *(M1)(A1)*

> **Note:** Award *(M1)* for substituted sine rule equation, *(A1)* for correct substitutions.

$ST = 1.95323…$                                                    *(A1)*
$= 1.95\,(m)$                                                          *(AG)*      *[3 marks]*

> **Notes:**  Both unrounded and rounded answer must be seen for final *(A1)* to be awarded.

(b)   $\dfrac{1}{2} \times 3.2 \times 1.95323… \times \sin 35°$ **or** $\dfrac{1}{2} \times 1.95323… \times 1.95323… \times \sin 110°$       *(M1)(A1)*

> **Note:** Award *(M1)* for substituted area formula, *(A1)* for correct substitutions.  Do not award follow through marks.

$= 1.79\,m^2$     $(1.79253…m^2)$                                         *(A1)(G2)*

> **Notes:**  The answer is $1.79\,m^2$, **units are required**. Accept $1.78955…$ from using 1.95.

**OR**

$\dfrac{1}{2} \times 3.2 \times 1.12033...$                                              *(A1) (M1)*

> **Note:** Award *(A1)* for the correct value for TM $(1.12033...)$ OR correct expression for TM (i.e. $1.6\tan 35°$, $\sqrt{(1.95323...)^2 - 1.6^2}$ ), *(M1)* for correctly substituted formula for triangle area.

$= 1.79\,m^2$     $(1.79253…m^2)$                                         *(A1)(G2)*      *[3 marks]*

> **Notes:**  The answer is $1.79\,m^2$, **units are required**.  Accept $1.78\,m^2$ from using 1.95.

*continued…*

*Question 3 continued*

(c)    $9.18\,\text{m}^2$ $(9.18022\ldots\text{m}^2)$    *(A1)(G1)*    *[1 mark]*

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**Notes:** The answer is $9.18\,\text{m}^2$, **units are required**. Do not penalize if lack of units was already penalized in (b). Do not award follow through marks here.
Accept $9.17\,\text{m}^2$ $(9.165\,\text{m}^2)$ from using 1.95.

---

(d)    $2\times1.79253\ldots+2\times9.18022\ldots+4.7\times3.2$    *(M1)(A1)*(ft)

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**Note:** Award *(M1)* for addition of three products, *(A1)*(ft) for three correct products.

---

   $=37.0\,\text{m}^2$ $(36.9855\ldots\text{m}^2)$    *(A1)*(ft)*(G2)*    *[3 marks]*

---

**Notes:** The answer is $37.0\,\text{m}^2$, **units are required**. Accept $36.98\,\text{m}^2$ from using 3sf answers.
Follow through from their answers to (b) and (c).
Do not penalize if lack of units was penalized earlier in the question.

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(e)    $1.79253\ldots\times4.7$    *(M1)*

---

**Note:** Award *(M1)* for their correctly substituted volume formula.

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   $=8.42\,\text{m}^3$ $(8.42489\ldots\text{m}^3)$    *(A1)*(ft)*(G2)*    *[2 marks]*

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**Notes:** The answer is $8.42\,\text{m}^3$, **units are required**. Accept $8.41\,\text{m}^3$ from use of 1.79.
An answer of 8.35, from use of $TM=1.11$, will receive follow-through marks if working is shown. Follow through from their answer to part (b). Do not penalize if lack of units was penalized earlier in the question.

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*continued…*

*Question 3 continued*

(f)   (i)   $\text{TM} = 1.6 \tan 35°$                                                            *(M1)*

> **Notes:** Award *(M1)* for their correct substitution in trig ratio.

     **OR**

     $\text{TM} = \sqrt{(1.95323...)^2 - 1.6^2}$                                              *(M1)*

> **Note:** Award *(M1)* for correct substitution in Pythagoras' theorem.

     **OR**

     $\dfrac{3.2 \times \text{TM}}{2} = 1.79253...$                                              *(M1)*

> **Note:** Award *(M1)* for their correct substitution in area of triangle formula.

     $= 1.12$ (m) (1.12033…)                                              *(A1)(ft)(G2)*

> **Notes:** Follow through from their answer to (b) if area of triangle is used.
> Accept 1.11 (1.11467) from use of $\text{ST} = 1.95$.

   (ii)   $\text{VM} = \sqrt{1.12033...^2 + 4.7^2}$                                              *(M1)*

> **Note:** Award *(M1)* for their correct substitution in Pythagoras' theorem.

     $= 4.83$ (m) (4.83168…)                                    *(A1)(ft)(G2)*     *[4 marks]*

> **Notes:** Follow through from (f)(i).

*Question 3 continued*

(g)    $\sin^{-1}\left(\dfrac{1.12033\ldots}{4.83168\ldots}\right)$                                    *(M1)*

**OR**

$\cos^{-1}\left(\dfrac{4.7}{4.83168\ldots}\right)$                                    *(M1)*

**OR**

$\tan^{-1}\left(\dfrac{1.12033\ldots}{4.7}\right)$                                    *(M1)*

> **Note:** Award *(M1)* for correctly substituted trig equation.

**OR**

$\cos^{-1}\left(\dfrac{4.7^2 + \left(4.83168\ldots\right)^2 - \left(1.12033\ldots\right)^2}{2\times 4.7 \times 4.83168\ldots}\right)$                          *(M1)*

> **Note:** Award *(M1)* for correctly substituted cosine formula.

$= 13.4°$  $(13.4073\ldots)$                              *(A1)***(ft)***(G2)*     *[2 marks]*

> **Notes:** Accept $13.3°$. Follow through from part (f).

*Total [18 marks]*

**QUESTION 4**

(a)    $120 + 10 \times 4$                                                  *(M1)(A1)*

> **Notes:** Award *(M1)* for substituted AP formula, *(A1)* for correct substitutions.
> Accept a list of 4 correct terms.

       $= 160$                                           *(A1)(G3)*    *[3 marks]*

(b)    $120 + (n-1) \times 10 = 260$                          *(M1)(M1)*

> **Notes:** Award *(M1)* for correctly substituted AP formula, *(M1)* for equating
> to 260.
> Accept a list of correct terms showing at least the 14th and 15th terms.

       $= 15$                                             *(A1)(G2)*    *[3 marks]*

(c)    $\dfrac{15}{2}(120 + 260)$ **or** $\dfrac{15}{2}(2 \times 120 + (15-1) \times 10)$           *(M1)(A1)***(ft)**

> **Notes:** Award *(M1)* for substituted AP sum formula, *(A1)***(ft)** for correct
> substitutions.
> Accept a sum of a list of 15 correct terms.
> Follow through from their answer to part (b).

       2850 seconds                                    *(A1)***(ft)***(G2)*

> **Note:** Award *(G2)* for 2850 seen with no working shown.

       47.5 minutes                                  *(A1)***(ft)***(G3)*    *[4 marks]*

> **Notes:** A final *(A1)***(ft)** can be awarded for correct conversion from seconds
> into minutes of their incorrect answer.
> Follow through from their answer to part (b).

(d)    $120 \times 1.06^{3-1}$                                           *(M1)(A1)*

> **Notes:** Award *(M1)* for substituted GP formula, *(A1)* for correct substitutions.
> Accept a list of 3 correct terms.

       $= 135 \;\; (134.832)$                                 *(A1)(G2)*    *[3 marks]*

(e)    $S_4 = \dfrac{120(1.06^4 - 1)}{(1.06 - 1)}$                              *(M1)(A1)*

> **Notes:** Award *(M1)* for substituted GP sum formula, *(A1)* for correct
> substitutions.
> Accept a sum of a list of 4 correct terms.

       $= 525 \;\; (524.953\ldots)$                             *(A1)(G2)*    *[3 marks]*

*Question 4 continued*

(f)     $120 + (n-1) \times 10 < 120 \times 1.06^{n-1}$                                     *(M1)(M1)*

> **Notes:** Award *(M1)* for correct left hand side, *(M1)* for correct right hand side.
>        Accept an equation.
>        Follow through from their expressions given in parts (a) and (d).

**OR**

List of at least 2 terms for both sequences (120, 130, … and 120, 127.2, …)     *(M1)*

List of correct 12$^{\text{th}}$ and 13$^{\text{th}}$ terms for both sequences (..., 230, 240 and …, 227.8, 241.5)                                                                 *(M1)*

**OR**

A sketch with a line and an exponential curve,                                  *(M1)*
An indication of the correct intersection point                                 *(M1)*

13$^{\text{th}}$ lap                                                    *(A1)*(**ft**)*(G2)*     *[3 marks]*

> **Note:** Do not award the final *(A1)*(**ft**) if final answer is not a positive integer.

*Total [19 marks]*

**QUESTION 5**

(a)     $y = -\dfrac{75^2}{10} + \dfrac{27}{2} \times 75$                                                                                    *(M1)*

> **Note:** Award *(M1)* for substitution of 75 in the formula of the function.

     $= 450$                                                                                                                                        *(A1)*

     Yes, point A is on the bike track.                                                                      *(A1)*     *[3 marks]*

> **Note:** Do not award the final *(A1)* if correct working is not seen.

(b)     $\dfrac{dy}{dx} = -\dfrac{2x}{10} + \dfrac{27}{2} \quad \left( \dfrac{dy}{dx} = -0.2x + 13.5 \right)$                               *(A1)(A1)*     *[2 marks]*

> **Notes:** Award *(A1)* for each correct term.  If extra terms are seen award at most *(A1)(A0)*.  Accept equivalent forms.

(c)     $-\dfrac{2x}{10} + \dfrac{27}{2} = 0$                                                                                                *(M1)*

> **Note:** Award *(M1)* for equating their derivative from part (b) to zero.

     $x = 67.5$                                                                                                                              *(A1)(ft)*

> **Note:** Follow through from their derivative from part (b).

     (Their) $67.5 \neq 75$                                                                                                            *(R1)*

> **Note:** Award *(R1)* for a comparison of their 67.5 with 75.  Comparison may be implied (*eg* 67.5 is the *x*-coordinate of the furthest north point).

*continued…*

*Question 5 continued*

**OR**

$$\frac{dy}{dx} = -\frac{2 \times (75)}{10} + \frac{27}{2}$$　　　　　　　*(M1)*

**Note:** Award *(M1)* for substitution of 75 into their derivative from part (b).

$$= -1.5$$　　　　　　　*(A1)*(ft)

**Note:** Follow through from their derivative from part (b).

(Their) $-1.5 \neq 0$　　　　　　　*(R1)*

**Note:** Award *(R1)* for a comparison of their –1.5 with 0. Comparison may be implied (*eg* The gradient of the parabola at the furthest north point (vertex) is 0).

Hence A is not the furthest north point.　　　　　*(A1)*(ft)　　*[4 marks]*

**Note:** Do not award *(R0)(A1)(ft)*. Follow through from their derivative from part (b).

(d)　(i)　　M(50, 175)　　　　　　　*(A1)*

**Note:** If parentheses are omitted award *(A0)*.
　　Accept $x = 50, y = 175$.

　　(ii)　$\dfrac{350 - 0}{100 - 0}$　　　　　　　*(M1)*

**Note:** Award *(M1)* for correct substitution in gradient formula.

$$= 3.5 \left( \frac{350}{100}, \frac{7}{2} \right)$$　　　　　*(A1)*(ft)(G2)　　*[3 marks]*

**Note:** Follow through from (d)(i) if midpoint is used to calculate gradient. Award *(G1)(G0)* for answer $3.5x$ without working.

*continued…*

*Question 5 continued*

(e)     $y = 3.5x + 150$     *(A1)*(ft)*(A1)*(ft)

> **Note:** Award *(A1)*(ft) for using their gradient from part (d), *(A1)*(ft) for correct equation of line.

  $3.5x - y = -150$ **or** $7x - 2y = -300$ (or equivalent)     *(A1)*(ft)     *[3 marks]*

> **Note:** Award *(A1)*(ft) for expressing their equation in the form $ax + by = c$.

(f)     (18.4, 214) (18.3772…, 214.320…)     *(A1)*(ft)*(A1)*(ft)*(G2)*(ft)     *[2 marks]*

> **Notes:** Follow through from their equation in (e). Coordinates must be positive for follow through marks to be awarded.
> If parentheses are omitted and not already penalized in (d)(i) award at most *(A0)(A1)*(ft).
> If coordinates of the two intersection points are given award *(A0)(A1)*(ft).
> Accept $x = 18.4$, $y = 214$.

*Total [17 marks]*