

**MATHEMATICAL STUDIES  
STANDARD LEVEL  
PAPER 2**

Wednesday 5 November 2003 (morning)

2 hours

---

**INSTRUCTIONS TO CANDIDATES**

- Do not open this examination paper until instructed to do so.
- Answer all five questions from Section A and one question from Section B.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator in the appropriate box on your cover sheet  
*e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

### SECTION A

Answer all **five** questions from this section.

1. [Maximum mark: 10]

The heights of 200 students are recorded in the following table.

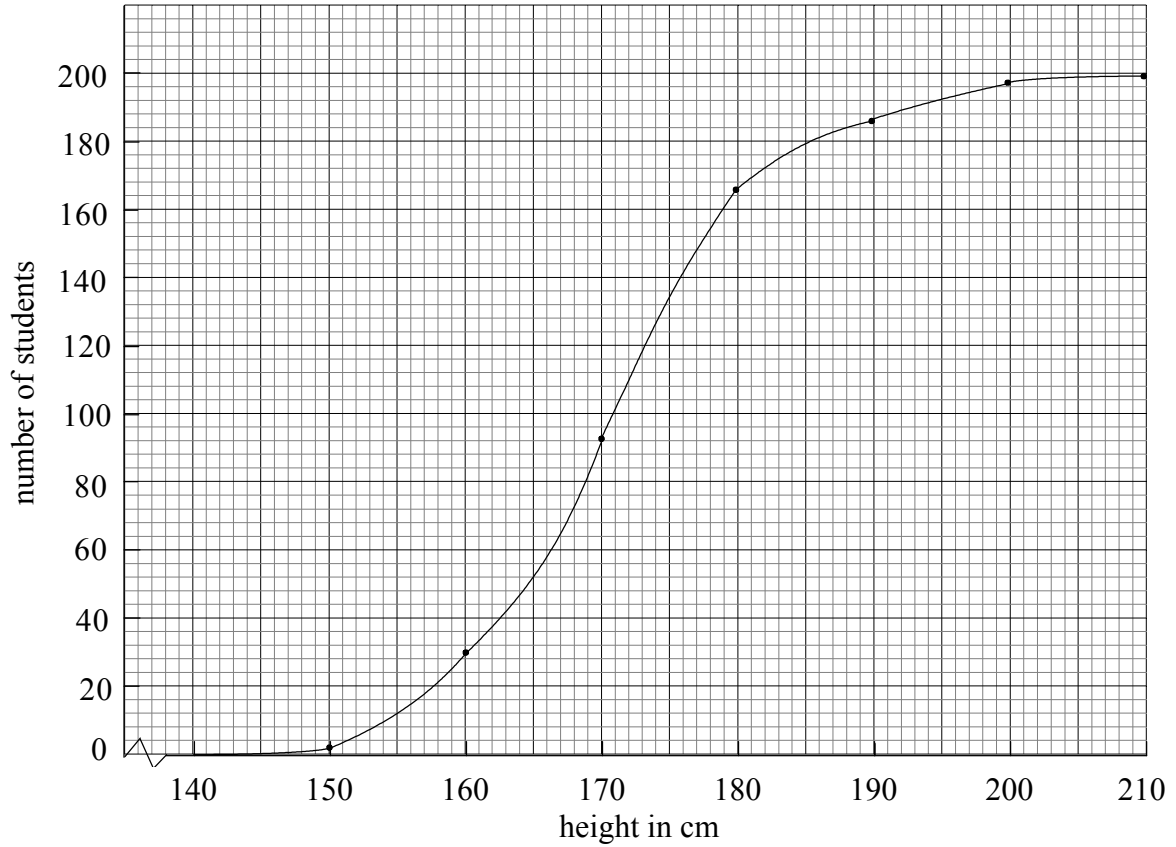
Height ( $h$ ) in cm	Frequency
$140 \leq h < 150$	2
$150 \leq h < 160$	28
$160 \leq h < 170$	63
$170 \leq h < 180$	74
$180 \leq h < 190$	20
$190 \leq h < 200$	11
$200 \leq h < 210$	2

- (a) Write down the modal group. [1 mark]
- (b) Calculate an estimate of the mean and standard deviation of the heights. [4 marks]

(This question continues on the following page)

(Question 1 continued)

The cumulative frequency curve for this data is drawn below.



- (c) Write down the median height. [1 mark]
- (d) The upper quartile is 177.3 cm. Calculate the interquartile range. [2 marks]
- (e) Find the percentage of students with heights less than 165 cm. [2 marks]

2. [Maximum mark: 13]

On a particular day 100 children are asked to make a note of what they drank that day.

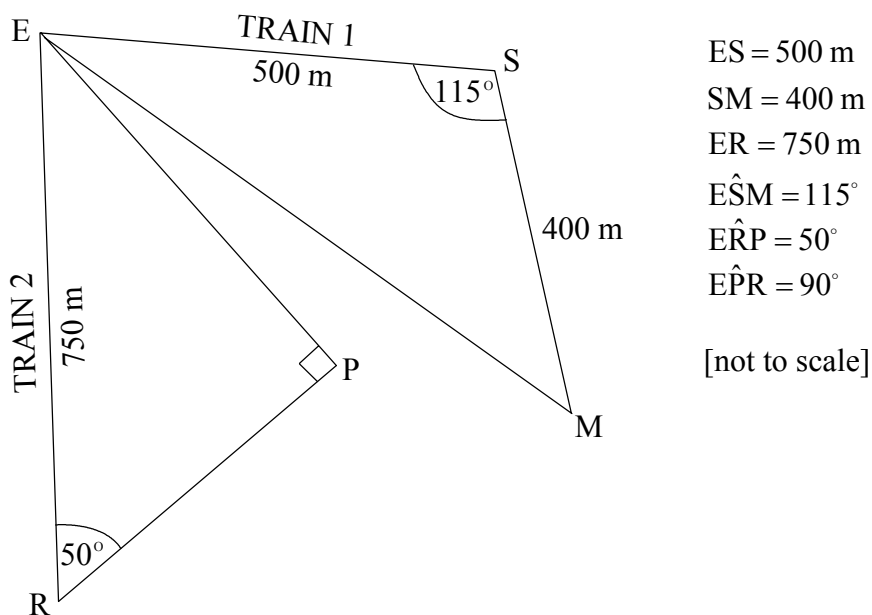
They are given three choices: water (W), coffee (C) or fruit juice (F)

- 1 child drank only water.
- 6 children drank only coffee.
- 8 children drank only fruit juice.
- 5 children drank all three.
- 7 children drank water and coffee only.
- 53 children drank coffee and fruit juice only.
- 18 children drank water and fruit juice only.

- (a) Represent the above information on a Venn Diagram. *[4 marks]*
- (b) How many children drank none of the above? *[2 marks]*
- (c) A child is chosen at random. Find the probability that the child drank
  - (i) coffee;
  - (ii) water or fruit juice but not coffee;
  - (iii) no fruit juice, given that the child did drink water. *[4 marks]*
- (d) Two children are chosen at random. Find the probability that both children drank all three choices. *[3 marks]*

3. [Maximum mark: 17]

- (i) A recreation park has two trains. Train 1 takes visitors from the entrance (E) to the swimming pool (S), to the mini golf (M) and back to the entrance. Train 2 takes visitors from the entrance (E) to the play area (P), to the racing track (R) and back to the entrance. This is shown in the diagram.



- (a) Calculate the total distance **Train 2** travels in one journey from E to P to R to E. [5 marks]
- (b) (i) Show that  $EM = 761 \text{ m}$  correct to 3 s.f.
- (ii) If the trains travel at  $2 \text{ ms}^{-1}$  find the time taken for **Train 1** to complete a journey from E to S to M to E. Give your answer to the nearest second. [6 marks]
- (ii) Ann and John go to a swimming pool. They both swim the first length of the pool in 2 minutes. The time John takes to swim a length is 6 seconds more than he took to swim the previous length. The time Ann takes to swim a length is 1.05 times that she took to swim the previous length.
- (a) (i) Find the time John takes to swim the third length.
- (ii) Show that Ann takes 2.205 minutes to swim the third length. [3 marks]
- (b) Find the time taken for Ann to swim a total of 10 lengths of the pool. [3 marks]

4. [Maximum mark: 18]

The police in Speedytown are considering changing the system for calculating speeding fines. At present, in a 100 km/hr area, the speeding fines are given in the table.

Number of km/hr ( $s$ ) above the limit	Fine (in dollars)
$0 < s \leq 10$	50
$10 < s \leq 20$	100
$20 < s \leq 30$	150
$30 < s \leq 50$	250
$50 < s \leq 70$	500

- (a) On graph paper, draw a graph to represent this information. Use a scale of 2 cm to represent 10km/hr on the  $x$ -axis for  $0 \leq x \leq 70$  and 2 cm to represent 100 dollars on the  $y$ -axis, for  $0 \leq y \leq 700$ .

[5 marks]

The new system proposes the following.

For the first 5 km/hr over the speed limit, there is no fine.

All other fines are calculated using a linear function. This function is represented by the line joining the point (5, 0) to the point (70, 650).

- (b) (i) Draw this line on your graph.  
 (ii) Calculate the equation of this line.

[4 marks]

- (c) The fines are the same for certain speeds.

- (i) How many times are the fines the same?  
 (ii) Calculate one of the speeds where the fine is the same.

[3 marks]

Within a few minutes 5 cars were found to be speeding.

2 were travelling at 104 km/hr,

3 were travelling at 125 km/hr.

- (d) Calculate, for both systems, the amount of money the police would collect.

[6 marks]

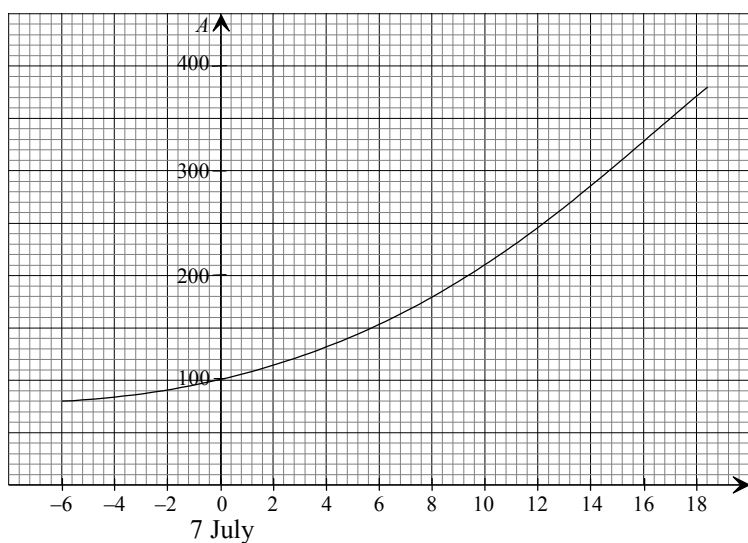
5. [Maximum mark: 12]

- (i) The area,  $A \text{ m}^2$ , of a fast growing plant is measured at noon (12:00) each day. On 7 July the area was  $100 \text{ m}^2$ . Every day the plant grew by 7.5%. The formula for  $A$  is given by

$$A = 100(1.075)^t$$

where  $t$  is the number of days after 7 July. (on 7 July,  $t = 0$ )

The graph of  $A = 100(1.075)^t$  is shown below.

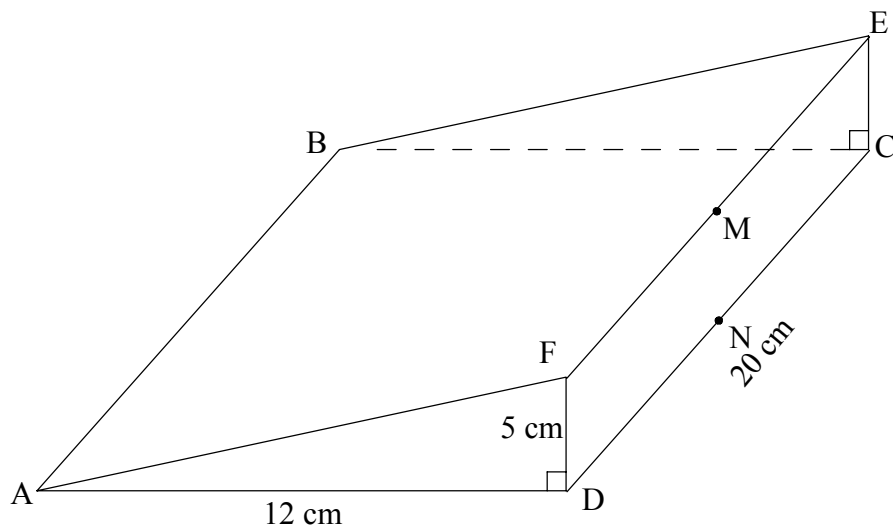


- (a) What does the graph represent when  $t$  is negative? [2 marks]
- (b) Use the graph to find the value of  $t$  when  $A = 178$ . [1 mark]
- (c) Calculate the area covered by the plant at noon on 28 July. [3 marks]

(This question continues on the following page)

(Question 5 continued)

- (ii) In the diagram below  $ABEF$ ,  $ABCD$  and  $CDFE$  are all rectangles.  
 $AD = 12$  cm,  $DC = 20$  cm and  $DF = 5$  cm.  
 $M$  is the midpoint of  $EF$  and  $N$  is the midpoint of  $CD$ .



- (a) Calculate (i) the length of  $AF$ ;  
 (ii) the length of  $AM$ . [3 marks]
- (b) Calculate the angle between  $AM$  and the face  $ABCD$ . [3 marks]



**SECTION B**

Answer **one** question from this section.

**Matrices and Graph Theory**

6. [Maximum mark: 30]

- (i) In a town there are 3 pizza restaurants. The prices of the pizzas vary in each restaurant.

In *Pizza Queen*, a salami pizza costs \$ 6, a ham/cheese pizza costs \$ 5, a tomato pizza costs \$ 4.50 and a spicy pizza costs \$ 7.

In *Prompt Pizza* a salami pizza costs \$ 6, a ham/cheese pizza costs \$ 7, a tomato pizza costs \$ 5 and a spicy pizza costs \$ 5.

In *Pizza Home* a salami pizza costs \$ 5, a ham/cheese pizza costs \$ 6, a tomato pizza costs \$ 6 and a spicy pizza costs \$ 6.

- (a) Write this information in a 3 by 4 matrix, **C**. [3 marks]

The students in Year 1 order 8 salami pizzas, 4 ham/cheese pizzas, 6 tomato pizzas and 12 spicy pizzas. The students in Year 2 order 6 salami pizzas, 12 ham/cheese pizzas, 8 tomato pizzas and 4 spicy pizzas.

- (b) Write this information in a 4 by 2 matrix, **P**. [2 marks]

- (c) Using the matrices **C** and **P**, calculate

(i) the total cost for Year 1 to eat at *Prompt Pizza*.

(ii) the total cost for Year 2 to eat at *Pizza Home*.

(iii) which pizza restaurant is cheapest for Year 1. [7 marks]

- (ii) The adjacency matrix, **M**, for the roads connecting the towns Q, R, S and T is shown below.

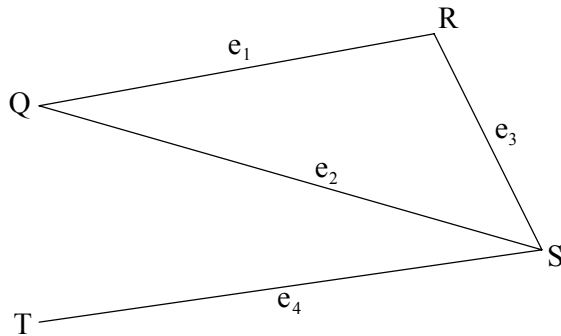
$$M = \begin{matrix} & \begin{matrix} Q & R & S & T \end{matrix} \\ \begin{matrix} Q \\ R \\ S \\ T \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 2 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 2 & 0 \end{pmatrix} \end{matrix}$$

- (a) Draw a directed graph to represent this information. [4 marks]

(This question continues on the following page)

(Question 6 (ii) continued)

The following sub-graph shows four of the roads connecting towns Q, R, S and T.



- (b) Draw an incidence matrix to represent this information. [4 marks]
- (c) Is the sub-graph above connected? Explain your answer. [2 marks]

(iii) Two children  $A$  and  $B$  play a two person zero sum game.  $A$  has two strategies  $a_1$  and  $a_2$ , and  $B$  has two strategies  $b_1$  and  $b_2$ . The entries in the matrix below (pay-off matrix) represent the amount that  $A$  wins or loses from  $B$ .

$$\begin{array}{c}
 \begin{array}{cc}
 & B \\
 & b_1 \quad b_2 \\
 A \quad a_1 & \begin{pmatrix} 3 & -1 \end{pmatrix} \\
 & a_2 \begin{pmatrix} 1 & 6 \end{pmatrix}
 \end{array}
 \end{array}$$

- (a) What is the outcome when  $A$  plays  $a_1$  and  $B$  plays  $b_1$ . [2 marks]
- (b) Let  $P$  be the probability that  $B$  plays  $b_1$ , then  $1 - P$  is the probability that  $B$  plays  $b_2$ . This information is represented in the matrices below.

$$\begin{array}{c}
 \begin{array}{cc}
 & b_1 \quad b_2 \\
 a_1 & \begin{pmatrix} 3 & -1 \end{pmatrix} \\
 a_2 & \begin{pmatrix} 1 & 6 \end{pmatrix}
 \end{array}
 \begin{pmatrix} P \\ 1-P \end{pmatrix}
 \end{array}$$

If  $A$  plays  $a_1$ , then the amount that  $A$  can expect to win is  $4P - 1$ .

- (i) If  $A$  plays  $a_2$ , calculate, in terms of  $P$ , the amount that  $A$  can expect to win.

The amount that  $A$  can expect to win is the same whether he plays  $a_1$  or  $a_2$ .

Calculate (ii) the value for  $P$ .

- (iii) the amount that  $A$  expects to win. [6 marks]

**Further Statistics and Probability**

7. [Maximum mark: 30]

(i) The weights of bags of biscuits are normally distributed with a mean of 123 g and a standard deviation of 2.5 g.

(a) Show that the probability that a bag weighs more than 122 g is 0.6554. [2 marks]

(b) Calculate the probability that a bag weighs less than 120 g. [3 marks]

The manufacturer's aim is that only 7.5 % of the bags weigh more than 126 g.

(c) If the standard deviation remains 2.5 g, calculate the value of the new mean in order to achieve this aim. [5 marks]

(ii) A bag containing 60 sweets is opened. The bag contains sweets of the following colours.

Colour	Frequency
Red	18
Orange	17
Green	10
Purple	9
Blue	6

According to the manufacturer, the various colours should have the following percentages.

Colour	Percentage
Red	35 %
Orange	25 %
Green	20 %
Purple	15 %
Blue	5 %

(a) Calculate the expected number of sweets of each colour in a bag containing exactly 60 sweets. [3 marks]

Before you can perform the chi-squared test on this data, it is necessary to combine the data for one of the colours with that of another colour.

(b) Which colour is this and why is this necessary? [2 marks]

(c) Using the chi-squared test at the 5 % significance level, investigate the hypothesis that the sweets in the packet may be regarded as a random sample. Remember to state the null hypothesis, the number of degrees of freedom and the critical value of chi-squared. [7 marks]

*(This question continues on the following page)*

(Question 7 continued)

- (iii) Several candy bars were purchased and the following table shows the weight and the cost of each bar.

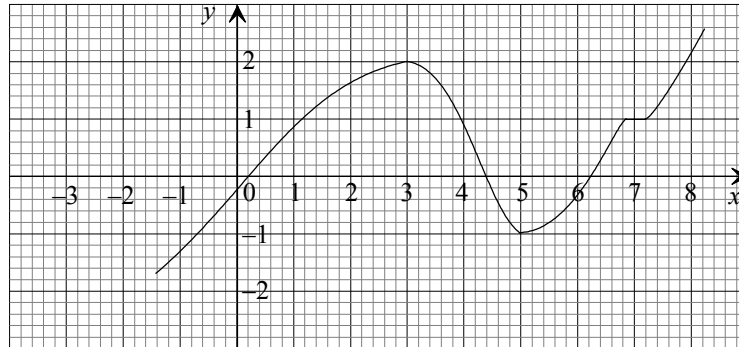
	Yummy	Chox	Marz	Twin	Chunx	Lite	BigC	Bite
Weight (g)	60	85	80	65	95	50	100	45
Cost (Euros)	1.10	1.50	1.40	1.20	1.80	1.00	1.70	0.90

- (a) Given that  $s_x = 19.2$ ,  $s_y = 0.307$  and  $s_{xy} = 5.81$ , find the correlation coefficient,  $r$ , giving your answer correct to 3 decimal places. *[2 marks]*
- (b) Describe the correlation between the weight of a candy bar and its cost. *[1 mark]*
- (c) Calculate the equation of the regression line for  $y$  on  $x$ . *[3 marks]*
- (d) Use your equation to estimate the cost of a candy bar weighing 109 g. *[2 marks]*

**Introductory Differential Calculus**

8. [Maximum mark: 30]

(i) The diagram shows a part of the curve  $y = f(x)$ .



(a) For what values of  $x$  is  $f'(x) = 0$ ? [3 marks]

(b) For what range of values of  $x$  is  $f'(x) < 0$ ? [2 marks]

(ii) A function  $g(x) = x^3 + 6x^2 + 12x + 18$ .

(a) Find  $g'(x)$ . [3 marks]

(b) Solve  $g'(x) = 0$ . [2 marks]

(c) (i) Calculate the values of  $g'(x)$  when

(a)  $x = -3$ ;

(b)  $x = 0$ .

(ii) Hence state whether the function is increasing or decreasing at

(a)  $x = -3$ ;

(b)  $x = 0$ . [4 marks]

(d) Is the value of  $x$  found in part (b) a local maximum, a local minimum or a point of inflexion? Give a reason for your answer. [2 marks]

*(This question continues on the following page)*

*(Question 8 continued)*

(iii) The velocity ( $v \text{ cm s}^{-1}$ ) of a moving object at time  $t$  is given by

$$v = 3t^2 - 16t + 16.$$

- (a) Given that the ball starts from zero, find an expression for the displacement,  $s$ , in terms of  $t$ . *[5 marks]*
  - (b) Calculate the displacement when  $t = 4$ . Comment on your result. *[3 marks]*
  - (c) When the velocity is zero, show that the values of  $t$  are 4 and  $\frac{4}{3}$ . *[1 mark]*
  - (d) Calculate the acceleration of the ball at these times. *[4 marks]*
  - (e) When is the acceleration equal to zero? *[1 mark]*
-