International Baccalaureate Extended Essay

Physics

What is the optimum position of street lighting in a linear model?

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Abstract

This essay is an attempt to explore better lighting design for the roads of Hong Kong to minimize the light pollution and energy consumption at night. Hence, this essay, titling **"What is the optimum position of street lighting in a linear model?"**, aims to find the height and spacing of street lights that provide maximum illuminance to the roads with the least amount of electric power through experimentation.

The scope of this essay is to focus on finding the optimum lighting design for the most common, Hong Kong L1 category primary distributor with speed limit under 70km/h, single-sided lighting linear model, by first identifying the different variables that affect the illuminance of a light source. The investigation is further elaborated by creating an experimental model that simulates street lighting for a road, calculating the photometric properties through the application of photometric laws and relationships, and finally scaling up the model according to the criteria based on the *Public Lighting Design Manual* devised by the Hong Kong Highways Department in 2006 (second edition).

The conclusion is drawn from this investigation by comparing the uniformity and average illuminance of the experimental model with the Highways Department's lighting criteria to calculate the optimum height to spacing ratio for this lighting design to be 1 : 2.69. The other photometric properties are then found and experimental model scaled to life-size. The two different heights of street lights the government provides for L1 category, 10m and 12m, are calculated most efficiently placed with spacing 26.9m and 32.3m respectively using the ratio previously found. Finally, a possible continuation of this essay suggested in the conclusion could lead to further investigations of different street lighting designs and the calculations of the costs of street lighting saved per year with the new model.

Abstract Word Count: 295

Contents

Introduction

Scope

Over the past years there have been concerns about the "increasing light pollution problem in Hong Kong becoming more serious"[1]. Not only does the "exposure to nightlights… increases the chance of having cancer" [2], but also creates "energy wastage …from the excessive use of external lighting"[1]

Since Hong Kong is a densely populated area with lots of traffic and street lighting, misplacement of street lights can lead to a huge waste of energy. "Electricity consumption for lighting rose 15.6 per cent between 1997 and 2005 - yet the population grew by just 4.9 per cent."[3] Hence, it proves interesting and necessary to investigate a more effective layout of street lighting to decrease light pollution and energy consumption in Hong Kong. Therefore, this essay focuses on answering the question:

What is the optimum position of street lighting in a linear model?

The "single-sided" lighting system along a linear, one road model is chosen for this investigation, as it proves to be the simplest and most common lighting system in Hong Kong. More complex lighting arrangements such as those used in junctions, expressways and tunnels are also important but will not be discussed in this essay.

The investigation to find the most effective street lighting layout to provide maximum illuminance with least power requires the application of the Inverse-square law and Lambert's cosine law. A scaled model of street lighting is then created for the analysis of the illuminance uniformity ratio of each height. The optimum ratio of post height to spacing is then calculated according to the street lighting criteria stated in the *Public Lighting Design Manual* by Hong Kong Highways Department.

This essay concludes by scaling up the model according to the ratio differences in illumination properties of the experiment bulb and lighting sources used by government. An evaluation is also given to assess the accuracy and reliability of such experiments and conclusion drawn in this essay.

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[1] Hon Ip, K and Cheng, E (2009) *Press Releases - External lighting and light pollution*
[2] Friends of the Earth (2008) *Streetlights by Bed Cancer - Your Bedfellow*
[3] Carvalho, C (2008) *Light Pollution Is Plaguing Hong Kong*

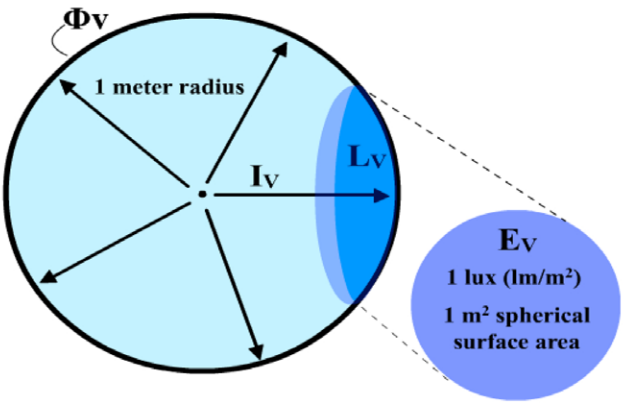
Theoretical hypothesis

Theoretical hypothesis and terminology

This essay mainly focuses on five of the metrics in photometry:
Luminous flux, Luminous intensity, Illuminance, Luminance and Luminous efficacy.

Let us consider an isotropic light point-source[4] as shown below:

Figure 1: Theoretical Isotropic Light source (point source) and terminology



Φ_v - Luminous Flux, lumen (lm), is the measure of 'power' of light

I_v - Luminous Intensity, candela (lm/ sr), is the luminous flux emitted by a light source per unit solid angle (steradian)

E_v - Illuminance, lux (lm / m^2), is the measure of luminous flux per meter squared of spherical surface area

L_v - Luminance, candela per square (cd / m^2), is the measure of luminous intensity per meter squared of spherical surface area

Luminous Efficacy, lumen per watt (lm/W), measures the efficiency the light source (Luminous flux per power)

The relationships between the five metrics of photometry, as shown in figure 1 above, can be found by comparing the definitions and units of the metrics. As the units of luminous flux and luminous intensity are the lumen (lm) and lumen per steradian (lm/sr), a homogenous equation[5] can be established: [6]

Luminous flux (lm) = luminous intensity (lm/sr) × 4π (sr)

The two other units, illuminance and luminance, are both 'per meter squared spherical surface area' measurements of luminous flux and luminous intensity respectively. Since the surface area per steradian[6] covered is the radius squared, the following can be obtained:

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[4] "Isotropic implies a spherical source that radiates the same in all directions" - Palmer, J (1999) *Radiometry and photometry FAQ*

[5] A homologous equation is an equation with balanced units

[6] The steradian (sr) is defined by $\text{sr} = \frac{\text{surface area}}{\text{radius}^2}$. Surface area of a sphere is $4\pi r^2$; thus, it has 4π steradian.

$$\textbf{Illuminance} = \frac{\textbf{Luminous flux}}{\textbf{radius}^2} \qquad \textbf{Luminance} = \frac{\textbf{Luminous intensity}}{\textbf{radius}^2}$$

Since luminous flux = luminous intensity at 1 sr, and is proportional to luminous intensity, the above equation can be combined and simplified into $\mathbf{E_v = \frac{I_v}{d^2}}$ or $\mathbf{E_v = \frac{\Phi_v}{d^2}}$, for an isotropic source. This is also known as the inverse-square law.

Since luminous flux is proportional to power, we can deduce that theoretically, as the distance (spacing and height) of the street lights decreases by a factor of 2, the illuminance can be maintained by only one quarter of the original power per street light.

However, as the spacing between the lights decreases by a factor of 2, twice amount of street lights are required to power the streets. Hence, the total power required when the distance (spacing and height) is halved, is:

$$\frac{1}{4} original\ power\ \times 2\ number\ of\ street\ lights = \frac{1}{2} original\ power$$

Thus, it is impossible to find the optimum position of street lighting only using the above calculations, as a decrease of the height and spacing will always decrease the total power required. Therefore, this investigation employs an important limitation, the uniformity ratio criterion, to help determine the optimum position of street lights.

Uniformity ratio determines the spread of illuminance across the road surface. A good uniformity ratio should be maintained to minimise the imbalance of lighting across the road and ensure visual comfort and performance of the driver. It can be separated into two different measurements: Overall uniformity and Longitudinal uniformity. The overall uniformity ratio is the ratio of the minimum illuminance to the average total illuminance across an area section of the road, while the longitudinal uniformity ratio is the ratio of minimum to average illuminance along a straight line parallel to the road. Thus, this uniformity criterion, used by the Hong Kong government, confines the distribution of lighting and limits how low the height of street lights can be.

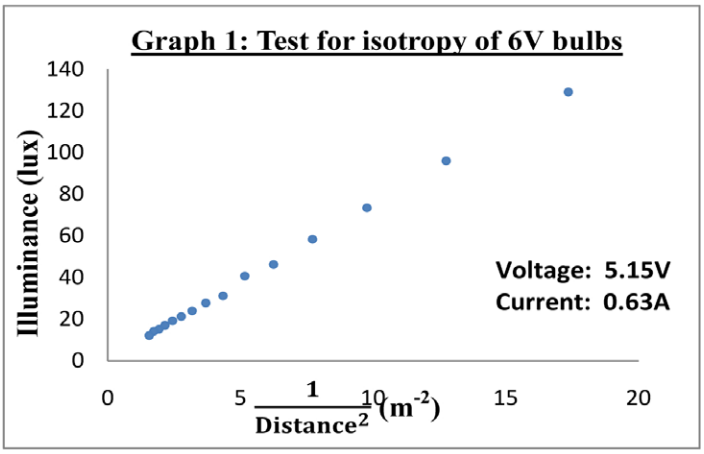
Hence, by investigating uniformity ratio of an experimental model of street lighting, the optimum position that provides maximum illuminance with the least power can be found. However, as the above relationships and laws only apply to isotropic sources, a few simple tests are performed to verify the isotropy of our testing light bulb.

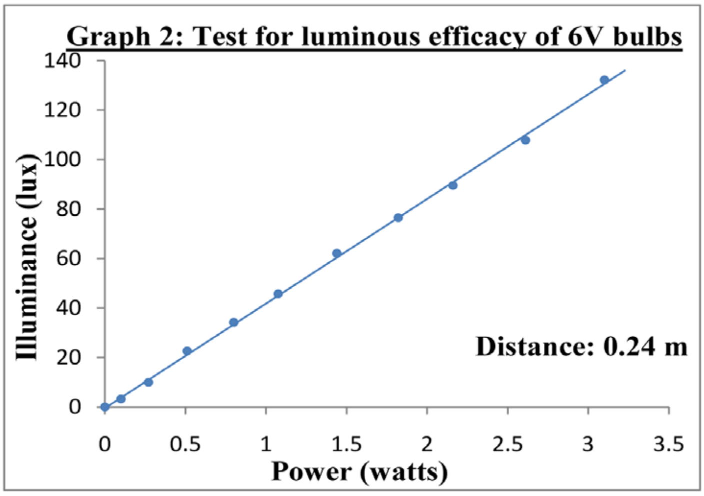
Pre-tests

Testing the nature of the experiment bulbs

For this investigation, two identical 6V incandescent light bulbs will be used to simulate street lighting. As the experiment bulbs differ from the ones used in street lighting, the properties of the bulb, such as the isotropy and luminous efficacy, have to be tested and scaled accordingly for our final model.

Two simple tests are performed to test the nature of the bulb. The first test requires measuring the illuminance of a bulb from varying distances with a light sensor. This tests the isotropy of the bulbs, as isotropic sources follow the inverse-square law. The second test requires measuring the illuminance with varying power to calculate the luminous efficacy of our experiment bulbs. The results are shown below in graph 1 and graph 2:[7]





From the line of best fit in graph 1, we can see that the illuminance provided by the 6V incandescent bulbs is inversely proportional to the distance squared, obeying the inverse-square law. Hence, they are proven isotropic and can then be used for this investigation.

From graph 2 on the left, we can see a direct relationship between the illuminance and power. Since luminous flux is directly proportional to the illuminance, as discussed earlier, luminous flux is also directly proportional to power. Using the line of best fit, the luminous efficacy is calculated to be 2.5 ±0.2 lm/watt.

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[7] Raw sets of results and calculations for the two tests and luminous efficacy can be found in Appendix 1

Experiment setup and description

This part of the essay focuses on creating an experimental model of street lighting to investigate the optimum spacing of the lights. For this investigation, the *Public Lighting Design Manual* by Hong Kong Highways Department is chosen as a guideline for street light placements.

This investigation focuses on the L1 category roads with speed limit under 70km/h, single-sided linear model of street lighting. The criteria are as follow:

Street lighting criteria

Most of Hong Kong's roads belong to Primary Distributors: L1category,[8] with speed limit under 70 km/h.[9]

The Design Standards for L1 are as follow:[10]

Maintained Average Luminance (cd/m^2):	≥ 2.0
Overall Uniformity Ratio:	≥ 0.4
Longitudinal Uniformity Ratio:	≥ 0.5

For the "single-sided" lighting system, the width of road should be less than or equal to the mounting height of luminaries. [11] For this investigation, the "single-sided" lighting system is chosen as the width of the roads in Hong Kong is generally less than the mounting height of luminaries.

Width of roads $\leq$ mounting height of luminaries

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[8] Primary Distributors: L1category contains "roads forming the major network of the urban area. Roads having high capacity junctions, though may be at-grade, segregated pedestrian facilities wherever possible and frontage access limited if not entirely restricted, and 24 hour stopping restrictions."(Hong Kong Lighting Division, 2006, *Public Lighting Design Manual*, p.89)

[9] "In general, 50 km/h is the standard speed limit on roads in the built-up areas" (Legislative Council Panel on Transport (1999-2000) *Speed Limit in Hong Kong*, p.2)

[10] Design Standards (Hong Kong Lighting Division, 2006, *Public Lighting Design Manual*, p.87, Table 13)

[11] Criteria from Hong Kong Lighting Division, 2006, *Public Lighting Design Manual*, p.63, Design Layout

Experiment setup

The method of measuring illuminance and uniformity of the lights in this investigation is adapted from *Illuminance Measurements of Roadways* by XVIII IMEKO World Congress.

This method involves measuring the illuminance given off by the street lights within a rectangular test-field, formed by the distance between two street lights and the width of the street (measured from curb to curb). The illuminance is then measured from various measuring points within this rectangular field. Although more measuring points within the test-field provide more accurate and precise results, for simplicity and pragmatism, this investigation will use a test-field containing 9 points, in a 3 × 3 format, as shown below in figure 2:

Figure 2: Top view of experiment setup

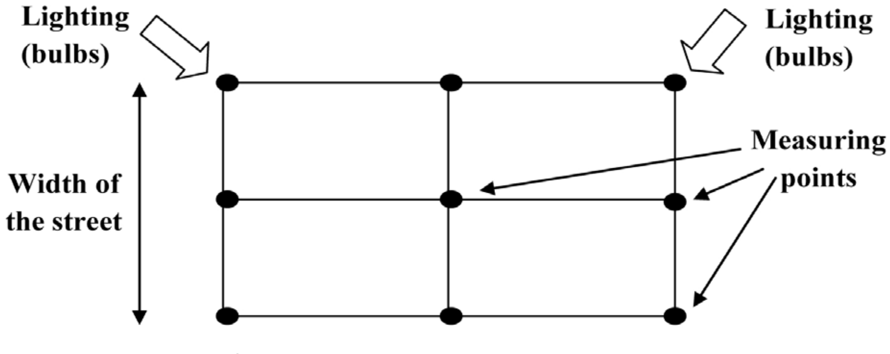
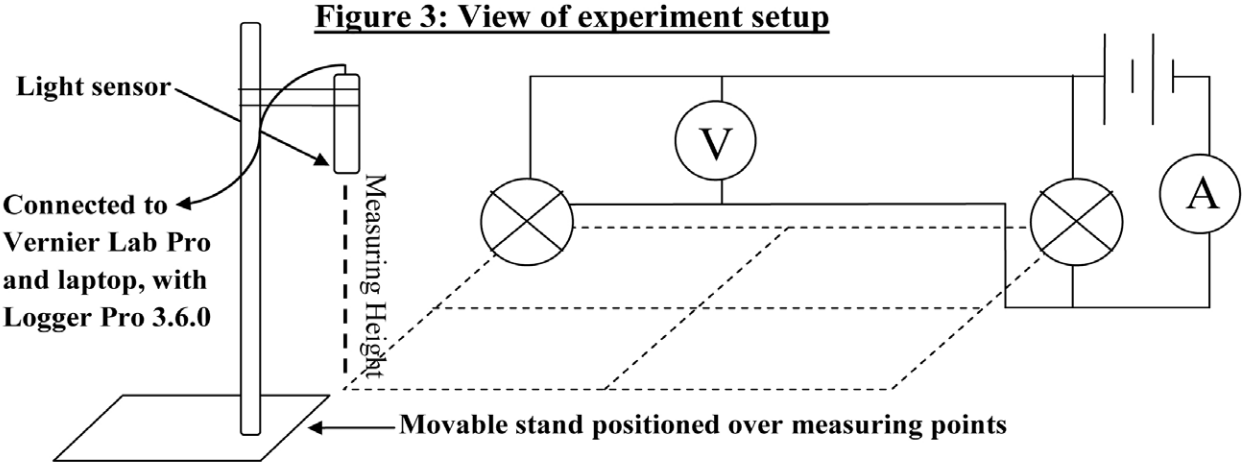


Figure 2 shows a top view of the experiment setup. The two sources of lighting are provided by the two identical 6V incandescent bulbs tested earlier and the measuring points are measured with a light sensor. For practical reasons, the street light model is inverted, as it is easier to measure the illuminance with the light sensor elevated and secured by a stand:

Figure 3: View of experiment setup



Measuring equipments description

The main measuring apparatus in this investigation include: Vernier Light Sensor, Vernier Lab Pro, the computer program “Logger Pro 3.6.0”[12], Voltmeter, Ammeter and meter ruler.

From figure 3 above, we can see the light sensor is mounted and secured by a stand and clamp, which can be adjusted to allow the sensor to measure the illuminance from a certain height in the 9 different measuring-points. The sensor is then connected to a Vernier Lab Pro, which functions as a data-acquisition interface that transfers readings from the sensor onto the computer.

The Voltmeter and Ammeter are used together in this experiment to measure voltage and current of the bulbs, which can then be used to calculate the electrical power. The Voltmeter and Ammeter measures up to 2 decimal places, so they both have an uncertainty value of ± 0.005V and ± 0.005A.

Lastly, the meter ruler is used for measuring the distance between the bulbs, height of the bulbs and light sensor, and position of the measuring points. The meter rule measures up to 0.0010 m accuracy, with an uncertainty value of ± 0.0005m. However, this error is negligible compared to the error in the light sensor detecting surface, which has a thickness of 0.005m. Thus, the distance used to calculate for illuminance has an uncertainty of ± 0.005m.

Calibration and accuracy tests

The Vernier light sensor is calibrated with the calibration stored in logger pro 3.6.0. However, the precision and uncertainty of the light sensor has to be experimentally tested to ensure reliability and reproducibility, and found to be ± 0.5 lux[13].

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[12] Logger Pro 3.6.0 Copyright © 2007 Vernier Software & Technology

[13] Precision tests for light sensor and calculations can be found in Appendix 2

Experiment method

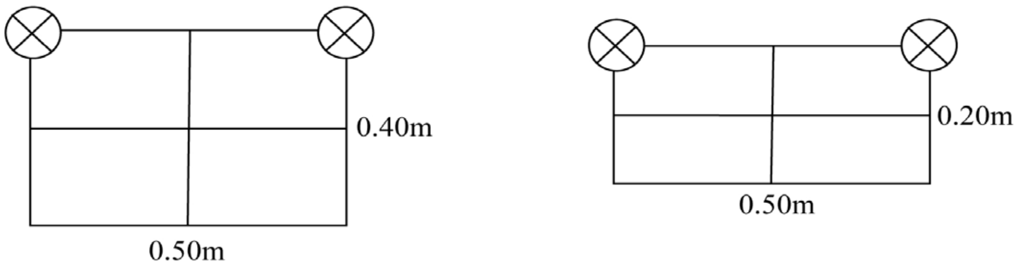
This experiment involves measuring the illuminance of 9 different measuring points at varying heights to find the optimum height to spacing ratio that follows the government's lighting criteria.

One of the lighting criteria previously mentioned, states that width of roads ≤ height of the posts. Theoretically, it is more efficient to have the street lights illuminate the maximum width of the street to minimise light pollution. Hence, it is more effective to have the width of street equal the height of the street posts. The height and width thus becomes the independent variable which we will vary and the spacing between the lights becomes the control variable, as it is impractical to vary the positions of two light bulbs than the height of one light sensor.

The distance between the two bulbs is kept at a constant of 0.500 ± 0.005m, while the height of the light sensor and width of the test-field is varied every 0.050 ± 0.005m. The illuminance is then measured and adjusted for background illuminance for the various heights tested.

Below are two examples showing how the height, width and spacing are varied:

Figure 4: Illustration of how the height, width and spacing vary in the experiment



Test-field for height of 0.40m

Test-field for height of 0.20m

The measuring spots are labeled, as shown in table 1, for the convenience of referencing:

Table 1: Referencing of measuring spots

(1,1)	(1,2)	(1,3)
(2,1)	(2,2)	(2,3)
(3,1)	(3,2)	(3,3)

Measuring spots (1,1) and (1,3) are directly below the light bulbs.

Experiment results

Table 2: Refined results of the illuminance measured across the 9 spots at each height[14]

Height: 0.50m

27.2	28.5	27.9
19.8	22.1	20.3
10.9	12.2	11.6

Height: 0.45m

32.3	33.0	32.8
23.7	25.6	24.0
13.0	14.2	13.5

Height: 0.40m

38.9	38.6	39.4
28.6	30.1	28.9
15.6	16.6	15.4

Height: 0.35m

48.2	44.4	49.1
35.7	35.3	36.8
19.3	20.8	19.7

Height: 0.30m

62.2	50.3	63.1
46.3	39.9	47.6
23.8	24.1	24.2

Height: 0.25m

86.9	55.6	88.3
63.7	47.2	64.9
33.5	29.8	34.0

Height: 0.20m

131.0	60.7	133.6
95.2	53.5	98.7
49.0	36.9	52.3

Height: 0.15m

221.3	59.8	232.8
154.3	56.2	163.7
82.9	40.2	84.0

Height: 0.10m

498.3	49.8	513.2
356.0	48.5	366.8
169.2	42.0	179.8

Height: 0.05m

1724.3	30.2	1895.7
1270.2	27.8	1328.7
706.2	28.4	723.9

(The readings from the voltmeter and ammeter for the experiments above are 4.00V and 1.04A. Since both bulbs are identical, according to Kirchoff's first and second law, the power of each bulb is 4.00V × 0.52A = 2.08W ± 0.02)

The results in table 2 lead to some interesting observations. It can be observed that the trends of illuminance differ across the 9 spots measured. For example, points (3,1) increase with decreasing height while points (1,2) increase first then decrease with decreasing height.

However, as it is desired to study the overall trend instead of those from individual measuring spots, the average illuminance is calculated and plotted in graph 3 below along with the calculated theoretical average illuminance for comparison.

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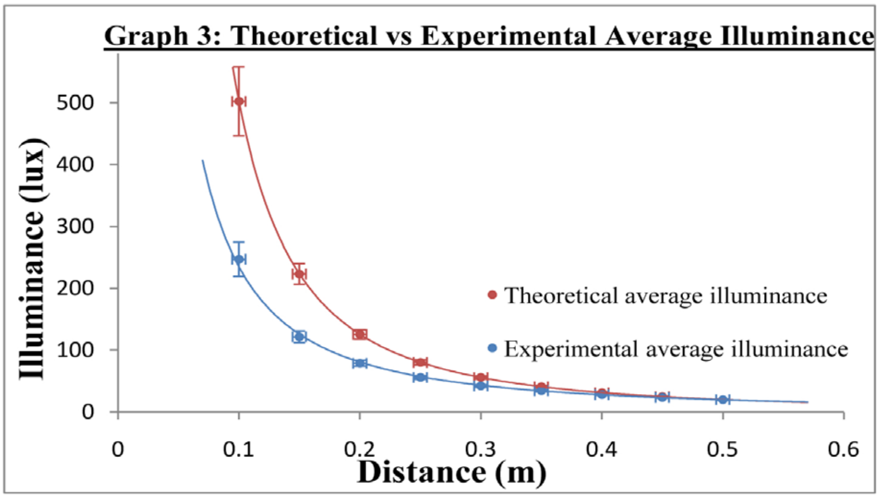
[14] The raw results of the experiment can be found in Appendix 3

Average illuminance analysis



The theoretical illuminance in the above graph is calculated by projecting the illuminance from when height is 0.50 (which has an average illuminance of 20.1 lux) using the inverse-square law: [15] Thus, the errors exist in the theoretical average illuminance can be attributed to the error in distance measured, as illuminance is affected by a change in distance (inverse-square law). [16]

Observations

An immediate observation from graph 3 shows that the trend of experimental average illuminance falls below the theoretical average and that both trends follow a downward curved pattern, showing an inverse relationship between illuminance and distance of light source.

Although this confirms our previous hypothesis that the illuminance increases exponentially as the distance between sensor and light decreases, the values of experimental average illuminance do not lie within the uncertainty bars of the theoretical values, and thus, showing the illuminance of a light source does not solely depend on the distance between the source and sensor (providing power is constant). Hence, our hypothesis and assumption for the theoretical model, $E_v = \frac{\Phi_v}{d^2}$, no longer holds.

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[15] Theoretical average illuminance = $(\frac{0.50}{\text{Vertical distance}})^2 \times 20.1$

[16] Theoretical average illuminance error = $\left(\frac{0.005}{\text{Vertical distance}} \times 2 + \frac{0.5}{20.1}\right) \times 100\%$

This phenomenon is similar to the observation of the trends of measuring points (1,2), (2,2) and (3,2), in which the illuminance of those points increases then decreases with decreasing height of sensor.

A logical explanation considered is that the illuminance of a light source not only depends on the distance of the source to sensor (according to inverse-square law), but also on the angle of incidence to the plane.

Figure 5: Lambert's Cosine Law[17]

Image removed for copyright reasons

According to Lambert's Cosine Law, illustrated by figure 5 on the right, the illuminance is directly proportional to the cosine of the angle of incidence, as follow:

$\mathbf{E_\theta = E\cos\theta}$, illustrated by 'Figure 5' on the right.

Combined with the inverse-square law, $(\mathrm{E} = \frac{\Phi_v}{d^2})$ the following can be obtained:

$$\mathbf{E_\theta = \frac{\Phi_v}{d^2}\cos\theta}$$

Since $\mathbf{\cos\theta} = \frac{\mathbf{Adjacent}}{\mathbf{Hypotenuse}}$, and adjacent = height of light sensor, hypotenuse = distance between light source and sensor, the new illuminance equation for our street light model can be simplified to:

$$\mathbf{E_\theta = \frac{luminous\ flux \times height}{d^3}}$$

This equation can then be used later on for the calculations of luminous flux and the scaling of the final model.

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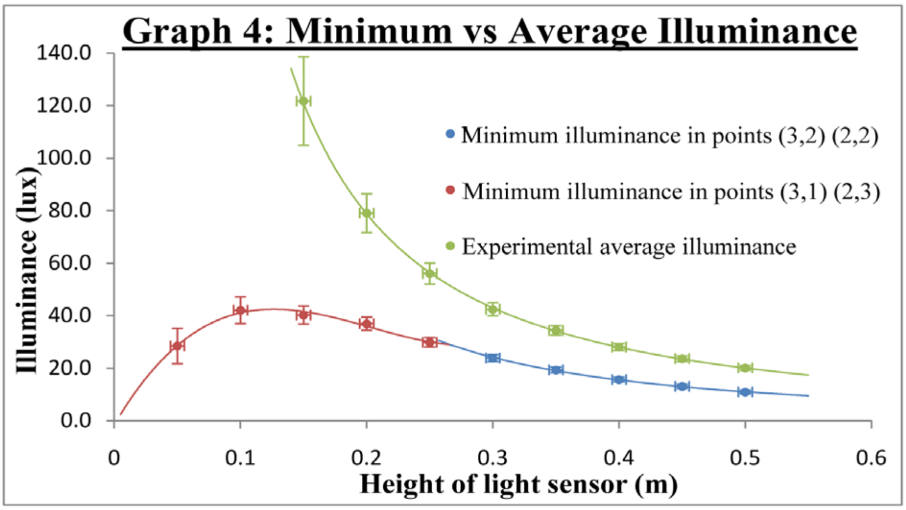
[17] Figure 5 copied from Taylor, A (2000) *Illumination Fundamentals* p.23

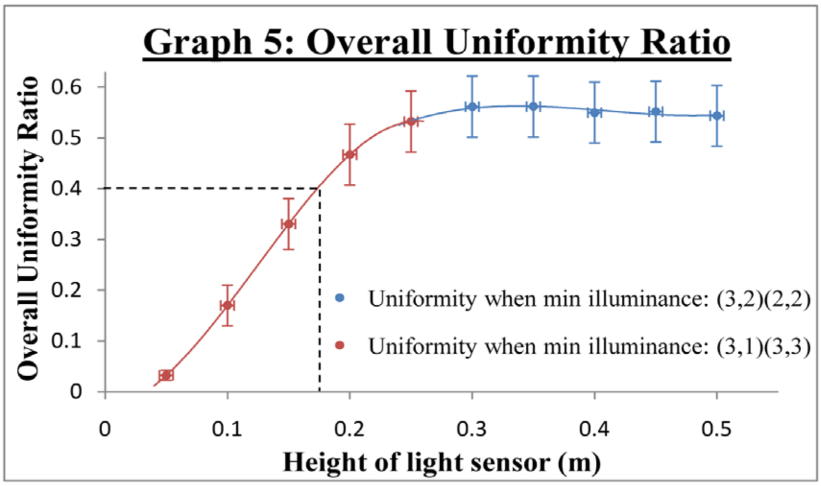
Uniformity ratio analysis

Another important criterion for the street lighting suggested in our hypothesis is the uniformity and longitudinal uniformity of the lighting design. Hence it is essential to compare the minimum illuminance of the 9 measuring spots of each height and compare them to the values of average illuminance (as discussed above) to find the uniformity ratio.

The measuring spots in which the minimum illuminance is found can be separated into two sections. For height of sensor greater than or equal to 0.30m, the minimum illuminance is measured in points (3,1) and (3,3). For height of sensor less than or equal to 0.25m, the minimum illuminance is measured in points (3,2) and (2,2). As shown below in graph 4:

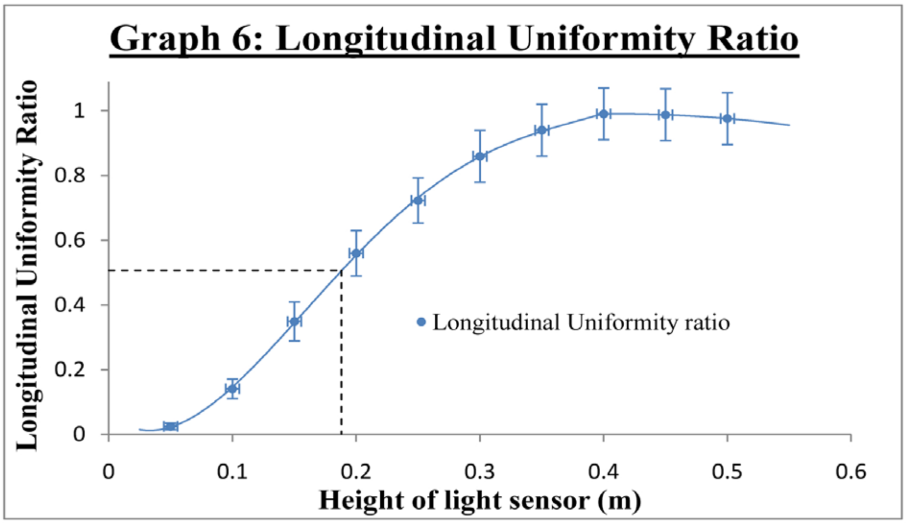


From graph 4 above, we can calculate the overall uniformity ratio with varying height of light sensor, as the uniformity ratio $= \frac{\text{Minimum illuminance}}{\text{Average illuminance}}$. This is represented in graph 5 below.



From graph 5 on the left, the overall uniformity is sloping upwards and stabling at around 0.55 as the height of sensor increases. We are interested in finding the height of light sensor when the overall uniformity is greater than or equal to 0.4 (taken from the lighting criteria, see above). Hence, we can use the line of best fit to calculate the height of light sensor. The height of light sensor that satisfies the overall uniformity criterion is found to be greater than or equal to 0.175 ± 0.005m. (as shown in graph 5 above) [18]

Similarly, the longitudinal uniformity, uniformity of points (1,1)(1,2)(1,3), can be represented by graph 6 below:



From graph 6, we can again use the line of best fit to calculate the height of the light sensor that satisfies the longitudinal uniformity criterion: longitudinal uniformity ratio greater than or equal to 0.5 (according to the street light criteria, see above). The height is found to be greater than or equal to 0.186 ± 0.005m.[18]

Thus, according to both uniformity criteria, the height of light sensor has to be limited to $\geq 0.175\text{m}$ and $\geq 0.186\text{m}$ respectively. Hence, a height that is $\geq 0.186\text{m}$ satisfies both criteria. A previous observation of the average illuminance graph shows that the illuminance increases as height decreases. Therefore, theoretically, the optimum height for this experiment model (that provides most illuminance) is 0.186m.

Since the experiment model has a spacing of 0.500m between the bulbs, the optimum height to spacing ratio for our street lighting model is 0.186 : 0.500, or 1 : 2.69.

After we have calculated the most efficient height to spacing ratio appropriate for Hong Kong's lighting system, a similar experiment is conducted for the optimum height of 0.186m.

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[18] Calculations for height of light sensor can be found in Appendix 4

Experiment results and properties of optimum model

Table 3: Results for height 0.186m

Height: 0.186m

152.6	61.3	154.9
109.7	54.0	112.1
56.2	39.0	57.2

The average illuminance, overall uniformity ratio and longitudinal uniformity ratio of the 0.186m height model is calculated and shown below in table 4:

Table 4: Properties of the optimum experiment model

Height (m)	Width (m)	Spacing between lights (m)	Average illuminance (lux)	Overall Uniformity Ratio	Longitudinal Uniformity Ratio
0.186	0.186	0.500	89	0.440	0.499

Using the two theories of photometry, the Inverse-square law and the Lambert's cosine law, and the relationship between the five metrics of photometry: luminous flux, luminous intensity, illuminance, luminance and luminous efficacy, we can calculate the photometric properties of this optimum experimental model and scale it accordingly.

Using the previously combined equation of illuminance $\mathbf{E_\theta} = \frac{\textbf{luminous flux} \times \textbf{height}}{\textbf{distance}^3}$, the illuminance, vertical height, and direct distance of each measuring point can be substituted into the equation to find the average total luminous flux value for the 0.186m model, which gives an average luminous flux of 5.1 lumens ± 0.3 lumens.

Since the unit of luminous intensity is the candela (cd), defined by lumen per steradian, the luminous intensity for the model is calculated as $\frac{\textbf{5.1 lumen}}{\textbf{4}\boldsymbol{\pi}\textbf{ steradian}} = 0.41 \text{ cd}$

Lastly, for the calculations on luminous efficiency, since the maximum lumens per watt is 683,[19] the luminous efficiency, in percentage, can be expressed as follow:

$$\textbf{Luminous efficacy } (\mathbf{2.5}) = \textbf{Luminous efficiency } \times \mathbf{683}$$

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[19] For an ideal monochromatic 555nm source

Therefore, from the previously developed relationships, the photometric properties of the optimum model can be found:

Table 5: Photometric properties of the optimum model

Power per bulb (watt) ± 0.02	Luminous Efficacy per bulb ±0.2lm/W	Luminous Efficiency (%) ± 0.02	Luminous flux (lumen) ± 0.3	Luminous intensity (cd) ± 0.03	Average illuminance (lux) ± 6	Average luminance (cd/m^2) ± 0.5	Vertical Height (m) ± 0.005
2.08	2.5	0.37	5.1	0.41	89	7.1	0.186

The phenomenon of how $\frac{\text{Average luminous flux}}{\text{height}^2}\left(\frac{5.1}{0.186^2}\right) \neq$ average illuminance (89) for this experiment model can be explained by Lambert's cosine law and the difference in distances across the 9 measuring points. However, these two factors do not affect the scaling of the model. When the model is scaled, the angle of incidence of the lighting remains constant, and the ratio of the distances for different measuring points also remains constant. Therefore, although $\frac{\text{average luminous flux}}{\text{height}^2} \neq$ average illuminance, they are directly proportional. Since Luminous flux = Power × Lumious efficacy, the relationship between the average illuminance and the distance for this model can be reestablished:

$$\textbf{Averaged Illuminance} = \mathbf{k}\,\frac{\textbf{Power } \times \textbf{Luminous efficacy}}{\textbf{Distance(height of stret light)}^2}$$

(in which k is a constant, calculated to be 0.6)[20]

Hence, the average luminance (average illuminance per steradian) can be represented by:

$$\textbf{Averaged Luminance} = \mathbf{k}\,\frac{\textbf{Power } \times \textbf{Luminous efficacy}}{\mathbf{4\pi} \times \textbf{Distance(height of stret light)}^2}$$

The final criterion for street lighting in this investigation, which states the average luminance should be maintained at least 2.0 cd, is crucial to the scaling of the optimum model to life size for practical use. By substituting the known values, we can establish the final formula for the scaling of the model:

$$\mathbf{2.0\ cd} = \mathbf{0.6} \times \frac{\textbf{Power } \times \textbf{Luminous efficacy}}{\mathbf{4\pi} \times \textbf{Distance(height of stret light)}^2}$$

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[20] Calculations of the constant k can be found in Appendix 5

According to the *Public Lighting Design Manual* by the Hong Kong government, "it is desirable to install tubular high xenon pressure high pressure sodium (SON-T Plus) lamps of rating 50W, 70W, 100W, 150W, 250W, 400W and 600W for road lighting application."[21]

Hence, SON-T Plus lamps are chosen for the light source for this investigation. The luminous flux and luminous efficacy are shown below in table 6 for different powers of SON-T Plus lamps.[22]

Table 6: Properties of different powers of SON-T Plus lamps

Table removed for copyright reasons

By substituting the above information into our equation, the optimum height of street lighting for each power can be found:

Table 7: Optimum height and spacing for different powers of SON-T Plus lamps

	SON-T Plus						
Power (watt)	50	70	100	150	250	400	600
Height of street lights (m)	10.2	12.5	15.5	20.1	28.1	36.2	46.4
Spacing between street lights (m)	27.6	33.5	41.5	54.2	75.5	97.4	124.6

(the spacing is scaled by the height to spacing ratio previously calculated, 1 : 2.69)

For L1road categories, which this investigation focuses on, the Hong Kong lighting department offers two different types of mounting height: 10m and 12m.[23]

Therefore, the electric power of 50W and 70W SON-T Plus is suitable for this L1 primary distributor, single-sided linear road model. For street lights with 10.0m mounting height, the optimum spacing for the posts is 26.9m, while for the street lights with 12.0m mounting height, the optimum spacing for the posts is 32.3m.[24]

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[21] (Hong Kong Lighting Division, 2006, *Public Lighting Design Manual*, p.70, Choice of equipment)

[22] Copied from (Hong Kong Lighting Division, 2006, *Public Lighting Design Manual*, p.86, Table 12)

[23] Mounting heights copied from (Hong Kong Lighting Division, 2006, *Public Lighting Design Manual*, p.64)

[24] The spacing is scaled by the height to spacing ratio previously calculated, 1 : 2.69

Although both heights are lower than the theoretical optimum height, the design still follows the lighting criteria (as it is scaled) and provides a little more illuminance for the road (as the lights are closer to the ground).

Conclusion and Evaluation

For this investigation to find an optimum position of street lighting in a linear model that fulfils the Hong Kong lighting design criteria, an experimental scaled model of lighting design was created and the optimum height and spacing for the SON-T plus street lights were calculated. Hence the answer to the proposed question:

What is the optimum position of street lighting in a linear model?

is found to be 10.0m height, 26.9m spacing for 50W source and 12.0m height, 32.3m spacing for 70W source.

The investigation also led to several interesting observations. Firstly, the average illuminance values were lower than the expected values from the calculations in the hypothesis. This interesting phenomenon led to further research and the discovery of another factor governing the illuminance of a light source, the angle of incidence of the light, described in Lambert's cosine law. It was later noticed that this is the factor that allowed the existence of an "optimum" lighting design. Without this factor, the illuminance will only be inversely proportional to distance squared, and thus, the maximum illuminance is produced when the height of the street light is 0 meters, which is impractical.

Secondly, the minimum illuminance of the model shifts between different measuring spots. This provides an interesting shape to the uniformity graphs and allows us to find the height that satisfies the lighting criteria. It is also noted that the shape of the uniformity graphs might not be as accurate. A repeated set of results with a larger range of heights tested will ideally improve the experiment as it minimizes the computer estimation between points. Nevertheless, the estimated line of best fit of the uniformity graphs led us to calculate the optimum height to spacing ratio to be 1 : 2.69.

For simplicity, this investigation chooses 9 measuring spots within the testing field, which does not give a very precise and accurate measure of average illuminance in the area. An increase in measuring spots within the field will allow more reliable results if time permits.

However, the overall observations from the 9 measuring spots still provide sufficient information which allows us to draw a conclusion and calculate the optimum height and spacing for the SON-T plus street lights.

A few interesting questions and ideas (and also limitations) emerged from this investigation, which include the integration of road safety in the calculation of the optimum position of the lights. It is known that the higher illuminance in the roads, the better visibility and the less accidents. The trade off for visibility requires more power, which increases the cost of lighting. However, without any reports on how road accidents vary with illuminance, and how much each accident costs in respect to the cost of the extra power, we have to assume that the lowest possible illuminance suitable for the lighting criteria is the cheapest layout.

Another major area for further research is the investigation of different lighting layouts, such as double-sided non-linear models, parking lots, junctions and tunnels, which together with the single-sided L1 category model investigated in this essay, can allow us to construct a more holistic lighting design layout for the city of Hong Kong to decrease energy consumption and light pollution.

It is also desirable to collect the data for the current average height and average spacing of street lights in Hong Kong roads, to then compare with our conclusion, and thus, to allow us to calculate the cost of street lighting saved per year. However, as the height and spacing of Hong Kong street lights are quiet uneven, it is extremely difficult to obtain an accurate figure for such calculations. Nevertheless, the optimum positions of street lighting in a linear, single-sided design found in this investigation should theoretically provide lighting to the roads of Hong Kong in the most efficient, energy saving manner.

Appendix 1

Raw sets of results for pretests and calculations of luminous efficacy

Test for isotropy of 6V bulbs

5.15V 0.63A

Distance from sensor to light source (m) ±0.005	$\frac{1}{\text{Distance}^2}$ (m^{-2})	Measured illuminance (lux) ±0.5	Background illuminance (lux) ±0.5	Measured minus background illuminance (lux) ±0.5
0.240	17.4	135.9	6.8	129.1
0.280	12.8	103.1	7.0	96.1
0.320	9.8	81.3	7.8	73.5
0.360	7.7	65.7	7.4	58.3
0.400	6.3	53.9	7.6	46.3
0.440	5.2	48.3	7.6	40.7
0.480	4.3	38.4	7.2	31.2
0.520	3.7	34.7	7.0	27.7
0.560	3.2	31.2	7.2	24.0
0.600	2.8	28.5	7.3	21.2
0.640	2.4	26.7	7.5	19.2
0.680	2.2	24.4	7.4	17.0
0.720	1.9	22.4	7.3	15.1
0.760	1.7	21.0	6.9	14.1
0.800	1.6	19.1	7.0	12.1

Test for luminous efficacy of 6V bulbs

Distance from sensor to light source 0.24m

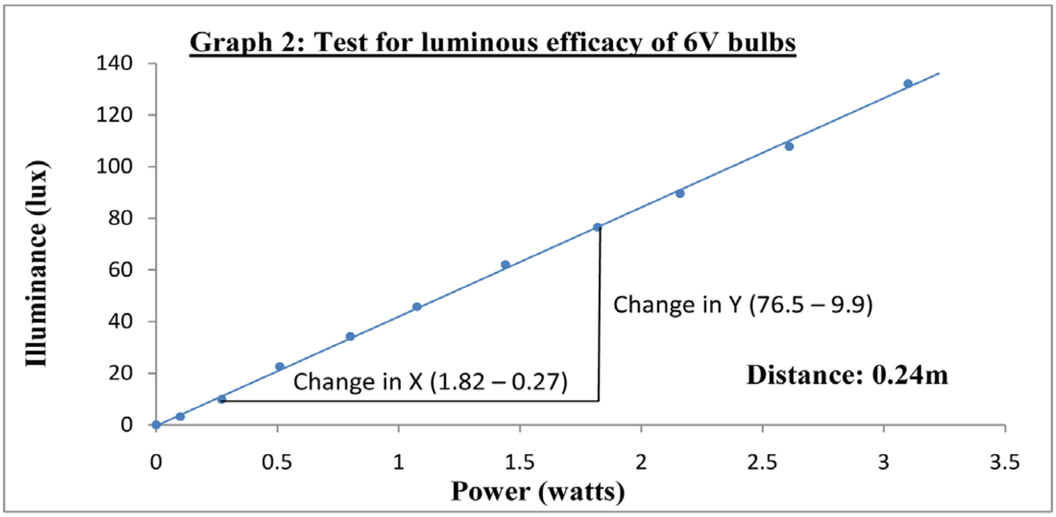
Voltage V	Current A	Power W	Measured illuminance (lux) ±0.5	Background illuminance (lux) ±0.5	Measured minus background illuminance (lux) ±0.5
5.00	0.62	3.10	139.5	7.3	132.2
4.50	0.58	2.61	114.7	6.9	107.8
4.00	0.54	2.16	96.4	6.8	89.6
3.50	0.52	1.82	84.0	7.5	76.5
3.00	0.48	1.44	69.4	7.3	62.1
2.50	0.43	1.08	53.1	7.4	45.7
2.00	0.40	0.80	41.2	7.0	34.2
1.50	0.34	0.51	30.3	7.7	22.6
1.00	0.27	0.27	17.1	7.2	9.9
0.50	0.20	0.10	9.9	6.7	3.2
0.00	0.00	0.00	3.3	3.1	0.2

(The error for voltage and current is ±0.005, which is negligible to the calculations of power)

Appendix 1 continued

Calculations for luminous efficacy

$$Gradient = \frac{Change\ in\ Y}{Change\ in\ X}$$



$$Gradient = \frac{76.5 - 9.9}{1.82 - 0.27}$$

$$Gradient = 42.96774 = 42.97\ (2.d.p)$$

Luminous efficacy is defined by the lumen per watt (luminous flux per power), and since

$$\text{Illuminance} = \frac{\text{Luminous flux}}{\text{distance}^2} \quad => \quad \text{Luminous flux} = \text{Illumiance} \times \text{distance}^2$$

We can deduce:

$$\text{Luminous efficacy} = \frac{\text{Illuminance} \times \text{distance}^2}{\text{power}}$$

$$\text{We previously calculated Gradient of the graph} = \frac{\text{Illuminance}}{\text{power}} = 42.97$$

Luminous efficacy = 42.97 × distance^2

Luminous efficacy = 42.96774 × 0.24^2

Luminous efficacy = 2.4749 = 2.5 lm/watt (1.d.p)

Uncertainty:

% uncertainty of distance^2 + % uncertainty of illuminance + % uncertainty of power

$$\frac{0.005}{0.24} \times 2 + \frac{0.5}{76.5-9.9} + \frac{0.02}{1.82-0.27} = 6.21\% = \pm\ 0.2\ \text{(1.d.p.)}$$

Appendix 2

<u>Precision tests for light sensor</u>

Distance: 0.24m

Power: 5.15V × 0.63A = 3.2445W = 3.24W

Twelve readings are taken from the same light source, distance and power. Already adjusted for background illuminance.

Readings – Illuminance measured (lux)				Average (lux)	Widest increment away from average (lux)
128.7	128.6	129.1	128.7	128.9	± 0.5
129.0	129.2	129.4	129.0		
128.9	129.2	128.9	128.5		

Appendix 3

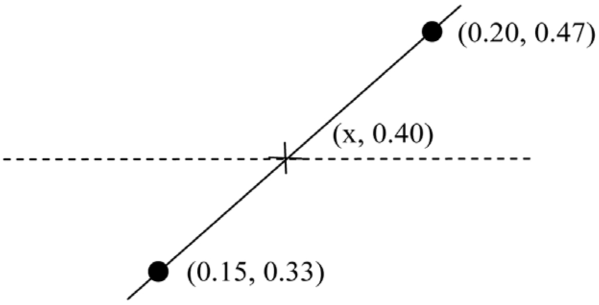
The raw results of the experiment

Vertical Height ± 0.005m	Measured illuminance ±0.5 lux			Background illuminance ±0.5 lux			Measured – background illuminance ±0.5 lux		
0.500	30.0	31.1	31.2	2.8	2.6	3.3	27.2	28.5	27.9
	22.6	25.2	22.8	2.8	3.1	2.5	19.8	22.1	20.3
	14.2	14.3	15.1	3.3	2.1	3.5	10.9	12.2	11.6
0.450	35.6	36.6	36.3	3.3	3.6	3.5	32.3	33.0	32.8
	25.9	28.9	26.3	2.2	3.3	2.3	23.7	25.6	24.0
	15.0	17.2	17.0	2.0	3.0	3.5	13.0	14.2	13.5
0.400	42.1	42.5	42.1	3.2	3.9	2.7	38.9	38.6	39.4
	32.0	33.2	33.5	3.4	3.1	4.6	28.6	30.1	28.9
	18.2	20.6	18.1	2.6	4.0	2.7	15.6	16.6	15.4
0.350	50.7	48.4	52.3	2.5	4.0	3.2	48.2	44.4	49.1
	38.5	38.4	40.6	2.8	3.1	3.8	35.7	35.3	36.8
	21.9	24.2	23.9	2.6	3.4	4.2	19.3	20.8	19.7
0.300	65.1	52.8	65.9	2.9	2.5	2.8	62.2	50.3	63.1
	49.1	42.5	50.1	2.8	2.6	2.5	46.3	39.9	47.6
	26.0	26.9	27.0	2.2	2.8	2.8	23.8	24.1	24.2
0.250	89.7	58.1	91.0	2.8	2.5	2.7	86.9	55.6	88.3
	67.4	49.7	67.4	3.7	2.5	2.5	63.7	47.2	64.9
	37.2	32.5	36.7	3.7	2.7	2.7	33.5	29.8	34.0
0.200	137.3	65.7	138.1	6.3	5.0	4.5	131.0	60.7	133.6
	101.4	58.2	104.0	6.2	4.7	5.3	95.2	53.5	98.7
	54.3	42.6	57.5	5.3	5.7	5.2	49.0	36.9	52.3
0.150	226.7	65.5	239.3	5.4	5.7	6.5	221.3	59.8	232.8
	160.9	62.4	169.7	6.6	6.2	6.0	154.3	56.2	163.7
	88.6	46.8	90.1	5.7	6.6	6.1	82.9	40.2	84.0
0.100	503.8	56.3	520.6	5.5	6.5	7.4	498.3	49.8	513.2
	361.5	54.1	373.6	5.5	5.6	6.8	356.0	48.5	366.8
	174.2	47.6	187.0	5.0	5.6	7.2	169.2	42.0	179.8
0.050	1731.2	37.6	1900.9	6.9	7.4	5.2	1724.3	30.2	1895.7
	1277.3	35.0	1335.8	7.1	7.2	7.1	1270.2	27.8	1328.7
	710.8	35.8	729.5	4.6	7.4	5.6	706.2	28.4	723.9

Appendix 4

Calculations for height of light sensor (Overall Uniformity Ratio)

From the uniformity graph (refer to page 14), then point when overall uniformity is 0.4, lies in between two experiment results,



Since all three of the points lie on the line of best fit, the gradient between x and 0.20 should equal the gradient between 0.15 and x.

$$\frac{0.47-0.40}{0.20-x}=\frac{0.40-0.33}{\mathrm{x}-0.15}$$

$$0.07\ \times(x-0.15)=0.07\ \times(0.20-\mathrm{x})$$

$$x=0.175\pm0.005$$

Calculations for height of light sensor (Longitudinal Uniformity Ratio)

$$\frac{0.56-0.50}{0.20-x}=\frac{0.50-0.35}{\mathrm{x}-0.15}$$

$$0.06\ \times(x-0.15)=0.15\ \times(0.20-\mathrm{x})$$

$$0.21\mathrm{x}\ =0.039$$

$$x=0.186\pm0.005$$

Appendix 5

Calculations of the constant k

From the equations:

$$\mathbf{Averaged\ Illuminance} = \mathbf{k}\ \frac{\mathbf{Power\ \times Luminous\ efficacy}}{\mathbf{Distance(height\ of\ stret\ light)^2}}$$

the known values can be substituted,

$$\mathbf{89 = k \times \frac{2.08\ \times 2.5}{0.186^2}}$$

$$\mathbf{k = 0.6}$$

Thus, the value of constant k is 0.6

Appendix 6

Figures, tables, graphs, photos

Figure 1	Figure 1 is a diagram illustrating the different terminologies relating to a theoretical isotropic light source
Figure 2	Figure 2 shows a detailed labeled top view of our experiment setup, showing the layout of the measuring points, and the position of the lighting. Figure 2 is created in Microsoft Office Word 2007.
Figure 3	Figure 3 shows another view of our experiment setup and apparatus used, showing the electrical circuit of the two bulbs, ammeter and voltmeter. It also shows how the light sensor is positioned. Figure 3 is created in Microsoft Office Word 2007.
Figure 4	Figure 4 is a diagram representation of how the height, width and spacing are varied in our experiment. Figure 4 is created in Microsoft Office Word 2007.
Figure 5	Figure 5 is a diagram representation of Lambert's cosine law. The diagram is copied directly from Taylor, A (2000) *Illumination Fundamentals* p.23
Table 1	Table 1 shows the referencing system of measuring spots used in this essay.
Table 2	Table 2 shows refined results from the raw results in Appendix 6.
Table 3	Table 3 shows the refined results for height 0.186m.
Table 4	Table 4 shows the properties of the experiment model for the optimum height of 0.186m.
Table 5	Table 5 is an extended version of table 4, covering the calculations of more photometric properties of the optimum model.

Table 6	Table 6 shows the properties of different powers of SON-T Plus lamps. It is copied from Hong Kong Lighting Division, 2006, *Public Lighting Design Manual*, p.86, Table 12
Table 7	Table 7 shows the optimum height and spacing for different powers of SON-T Plus lamps. It is obtained by substituting the results in table 6 into the equation mentioned in the essay.
Graph 1	Graph 1 shows a graphical representation of the first pre-test: test for isotropy of the 6V bulbs. The line of best fit is manually drawn, and the graph is created in Microsoft Office Excel 2007.
Graph 2	Graph 2 shows a graphical representation of the second pre-test: test for luminous efficacy of the 6V bulbs. The line of best fit is again manually drawn, and the graph is created in Microsoft Office Excel 2007.
Graph 3	Graph 3 shows a graphical representation of the theoretical average illuminance and the experimental average illuminance of the results in table 2 in the experiment. The graph is created in Microsoft Office Excel 2007. Both lines of best fit are also drawn by the program Microsoft Office Excel 2007.
Graph 4	Graph 4 is a graphical representation of minimum illuminance and average illuminance of the results shown in table 2. The graph and lines of best fit are created by Microsoft Office Excel 2007.
Graph 5	Graph 5 is a refined version of Graph 4, showing the overall uniformity of the results in table 2. The graph and line of best fit are created by Microsoft Office Excel 2007.
Graph 6	Graph 6 is a graphical representation of the longitudinal uniformity ratio of the results in table 2. The graph and line of best fit are created by Microsoft Office Excel 2007.

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