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MATHEMATICS

Higher Level

The portfolio - tasks

For use in 2012 and 2013

For final assessment in 2012 and 2013

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INTRODUCTION

What is the purpose of this document?

This document contains new tasks for the portfolio in mathematics HL. These tasks have been produced by the IB, for teachers to use in the examination sessions in 2012 and 2013. It should be noted that any tasks previously produced and published by the IB will no longer be valid for assessment after the November 2010 examination session. These include all the tasks in any teacher support material (TSM), and the tasks in the document "Portfolio tasks 2009-2010". The tasks in the document "Portfolio tasks 2011-2012" can be used in the 2012 examinations but **NOT** in 2013.

Copies of all TSM tasks published by the IB are available on the Online Curriculum Centre (OCC), under Internal Assessment, in a document called "Old tasks published prior to 2008". These tasks should **not** be used, even in slightly modified form.

What happens if teachers use these old tasks?

The inclusion of these old tasks in the portfolio will make the portfolio non-compliant, and such portfolios will therefore attract a 10-mark penalty. Teachers may continue to use the old tasks as practice tasks, but they should not be included in the portfolio for final assessment.

What other documents should I use?

All teachers should have copies of the mathematics HL subject guide (second edition, September 2006), including the teaching notes appendix, and the TSM (September 2005). Further information, including additional notes on applying the criteria, is available on the Online Curriculum Centre (OCC). Important news items are also available on the OCC, as are the diploma programme coordinator notes, which contain updated information on a variety of issues.

Which tasks can I use in 2012?

The only tasks produced by the IB that may be submitted for assessment in 2012 are the ones contained in this document, and those in the document "Portfolio tasks 2011-2012". There is no requirement to use tasks produced by the IB, and there is no date restriction on tasks written by teachers.

Can I use these tasks before May 2012?

These tasks should only be submitted for final assessment from May 2012 to November 2013. Students should not include them in portfolios before May 2012. If they are included, they will be subject to a 10-mark penalty. Please note that these dates refer to examination sessions, not when the work is completed.

Which tasks can I use in 2013?

The only tasks produced by the IB that may be submitted for assessment in 2013 are the ones contained in this document.

Technology

There is a wide range of technological tools available to support mathematical work. These include graphic display calculators, Excel spreadsheets, Geogebra, Autograph, Geometer sketch pad and Wolframalpha. Many are free downloads from the Internet. Students (and teachers) should be encouraged to explore which ones best support the tasks that are assigned. Teachers are reminded that good technology use should enhance the development of the task.

Extracts from diploma program coordinator notes

Important information is included in the DPCN, available on the OCC. Teachers should ensure they are familiar with these, and in particular with the ones noted below. Please note that the reference to the 2009/2010 document is outdated.

Copies of tasks and marking/solution keys

Teachers are advised to write their own tasks to fit in with their own teaching plans, to select from the 2009/2010 document, or to use tasks written by other teachers. In each case, teachers should work the task themselves to make sure it is suitable, and provide a copy of the task, and an answer, solution or marking key for any task submitted. This will help the moderators confirm the levels awarded by the teacher.

It is particularly important if teachers modify an IB published tasks to include copy of the modified task. While this is permitted, teachers should think carefully about making any changes, as the tasks have been written with all the criteria in mind, to allow students to achieve the higher levels.

Non-compliant portfolios from May 2012

Please note the following information on how to deal with portfolios that do not contain one task of each type. This will be applied in the May 2012 and subsequent examination sessions.

If two pieces of work are submitted, but they do not represent a Type I and a Type II task (for example, they are both Type I or both Type II tasks), mark both tasks, one against each Type.
For example, if a candidate has submitted two Type I tasks, mark one using the Type I criteria, and the other using the Type II Criteria. Do **not** apply any further penalty

This means that the current system of marking both tasks against the same criteria and then applying a penalty of 10 marks will no longer be used.

SHADOW FUNCTIONS **HL TYPE I**

To the student: The work that you produce to address the questions in this task should be a report that can stand on its own. It is best to avoid copying the questions in the task to adopt a "question and answer" format.

> *While real zeros of polynomial functions may be easily read off the graph of the polynomial, the same is not true for complex zeros. In this task, you will investigate the method of shadow functions and their generators, which helps identify the real and imaginary components of complex zeros from key points along the x-axis.*

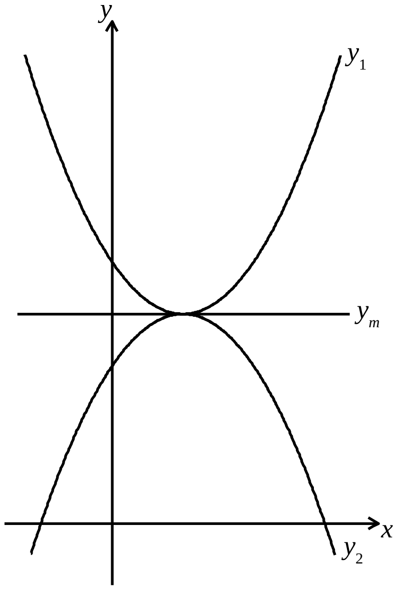
Part A (Quadratic Polynomials)

Consider the quadratic function $y_1 = (x-a)^2 + b^2$, where $a, b \in \mathbb{R}$.

- Write down the coordinates of the vertex.
- Show that y_1 has zeros $a \pm \mathrm{i}b$, where $\mathrm{i} = \sqrt{-1}$.

The "shadow function" to y_1 *is another quadratic* y_2 *which shares the same vertex as* y_1. *However,* y_2 *has opposite concavity to that of* y_1 *and its zeros are in the form* $a \pm b$.

- Use various values for a and b to generate pairs of functions y_1 and y_2.

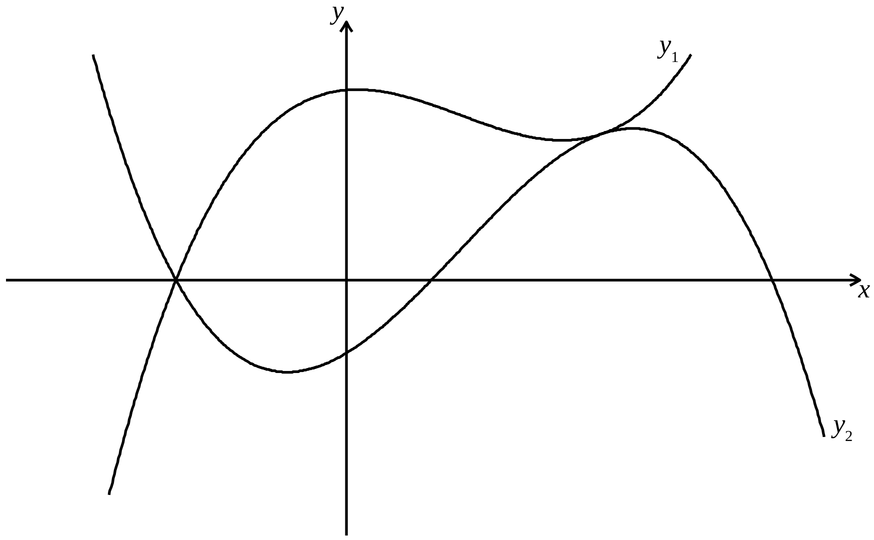


- Hence, or otherwise, express y_2 in terms of y_1 and y_m, where y_m is called the "shadow generating function".
- On a labelled diagram, illustrate how the zeros of y_2 may be helpful in the determination of the real and imaginary components of the complex zeros of y_1.

Part B (Cubic Polynomials)

Now consider the cubic function $y_1 = (x+2)\big(x-(3+2\mathrm{i})\big)\big(x-(3-2\mathrm{i})\big)$.

The shadow function in this case is another cubic function y_2 which shares two points with y_1, has opposite concavity and its zeros are -2 and 3 ± 2 as depicted in the figure below. The shadow generating function in this case passes through the two points of intersection.



- Write down an expression for y_2 and hence find the points of intersection between y_1 and y_2.
- Hence, or otherwise, determine the equation of the shadow generating function y_m in this case.
- Once again, express y_2 in terms of y_1 and y_m. Use technology to investigate similar cubic functions.
- Write a general statement about the functions y_1, y_2 and y_m and prove your statement.
- On a labelled diagram, illustrate how the zeros of y_2 may be helpful in the determination of the real and imaginary components of the complex zeros of y_1.
- How can the findings with quadratics and cubics be applied to quartics?

PATTERNS FROM COMPLEX NUMBERS **HL TYPE I**

To the student: The work that you produce to address the questions in this task should be a report that can stand on its own. It is best to avoid copying the questions in the task to adopt a "question and answer" format.

Part A

- Use de Moivre's theorem to obtain solutions to the equation $z^3 - 1 = 0$.
- Use graphing software to plot these roots on an Argand diagram as well as a unit circle with centre origin.
- Choose a root and draw line segments from this root to the other two roots.
- Measure these line segments and comment on your results.
- Repeat the above for the equations $z^4 - 1 = 0$ and $z^5 - 1 = 0$. Comment on your results and try to formulate a conjecture.
- Factorize $z^n - 1$ for $n = 3,\ 4$ and 5.
- Use graphing software to test your conjecture for some more values of $n \in \mathbb{Z}^+$ and make modifications to your conjecture if necessary.
- Prove your conjecture.

Part B

- Use de Moivre's theorem to obtain solutions to $z^n = \mathrm{i}$ for $n = 3,\ 4$ and 5.
- Use graphing software to represent each of these solutions on an Argand diagram.
- Generalize and prove your results for $z^n = a + b\mathrm{i}$, where $|a + b\mathrm{i}| = 1$.
- What happens when $|a + b\mathrm{i}| \neq 1$?

THE DICE GAME **HL Type II**

To the student: The work that you produce to address the questions in this task should be a report that can stand on its own. It is best to avoid copying the questions in the task to adopt a "question and answer" format.

The aim of this task is to create a dice game in a casino and model this using probability. It is important to examine how best to run the game from both the perspective of a player and the casino. In doing so analyse the game to consider the optimal payments by the player and payouts by the casino.

- Consider a game with two players, Ann and Bob. Ann has a red die and Bob a white die. They roll their dice and note the number on the upper face. Ann wins if her score is higher than Bob's (note that Bob wins if the scores are the same). If both players roll their dice once each what is the probability that Ann will win the game?
- Now consider the same game where Ann can roll her die a second time and will note the higher score of the two rolls but Bob rolls only once. In this case what is the probability that Ann will win?
- Investigate the game when both players can roll their dice twice, and also when both players can roll their dice more than twice, but not necessarily the same number of times.
- Consider the game in a casino where the player has a red die and the bank has a white die. Find a model for a game so that the casino makes a reasonable profit in the case where the player rolls the red die once and the bank rolls the white die once. (When creating your model you will need to consider how much a player must pay to play a game and how much the bank will pay out if the player wins. Do this from the perspective of both the player and the casino and consider carefully the criteria for whether the game can be considered worthwhile for both the player and the casino.)
- Now consider other models for the game including cases such as where the player or the bank rolls their dice multiple times, or where multiple players are involved in the game.

FILLING UP THE PETROL TANK **HL Type II**

To the student: The work that you produce to address the questions in this task should be a report that can stand on its own. It is best to avoid copying the questions in the task to adopt a "question and answer" format.

> *In this task, you will develop a mathematical model that helps motorists decide which of the following two options is more economical.*
> ***Option 1****: to buy petrol from a station on their normal route at a relatively higher price.*
> ***Option 2****: to drive an extra distance out of their normal route to buy cheaper petrol.*

Consider a situation in which two motorists, Arwa and Bao, share the same driving route but own different sized vehicles. Arwa fills up her vehicle's tank at a station along her normal route for US$ p_1 per litre. On the other hand, Bao drives an extra d kilometres out of his normal route to fill up his vehicle's tank for US$ p_2 per litre where $p_2 < p_1$.

- Choose suitable parameters for Arwa's and Bao's vehicles. Justify your choices.
- Suppose $p_1 = \text{US\$1.00}$ per litre, $p_2 = \text{US\$0.98}$ per litre and $d = 10\text{ km}$. Which motorist is getting the better deal? Show all calculations and justify any assumptions you make.
- On a spreadsheet, choose several sets of values for p_1, p_2 and d to investigate further.
- Define a set of variables that would be relevant in the above situation.

"Effective litres" is a method of comparing the cost of petrol bought under the two options described above. Effective litres, for a given vehicle, are those litres used when the vehicle travels its normal route.

- Use your variables and parameters to write algebraic expressions for E_1 and E_2 which represent the cost per effective litre under options 1 and 2 respectively.
- Write a model that helps motorists decide on the more economical option for their vehicles.

In the remainder of this task, you need to consider the two vehicles you have chosen for Arwa and Bao.

- Use your model to find the farthest distance that Bao should drive to obtain a 2 % price saving.
- Investigate the relationship between d and p_2 when E_2 is kept constant (*e.g.* US$0.90, US$1.00, … *etc.*). Use technology to draw a family of curves for Arwa's vehicle. Repeat for Bao's vehicle.
- For $E_2 = \text{US\$0.90}$, provide Arwa with information on three different stations that yield this same cost per effective litre. Discuss how such information may be useful to Arwa.
- For $E_2 = \text{US\$1.00}$ and $p_2 = \text{US\$0.80}$, compare the maximum distance that each motorist should drive and still save money.

Arwa is a busy person and wonders whether the saving in money would be worth the time she would lose in extra driving.

- Modify your model to account for the time taken to drive to and from an off-route station. Clearly justify any assumptions you make.