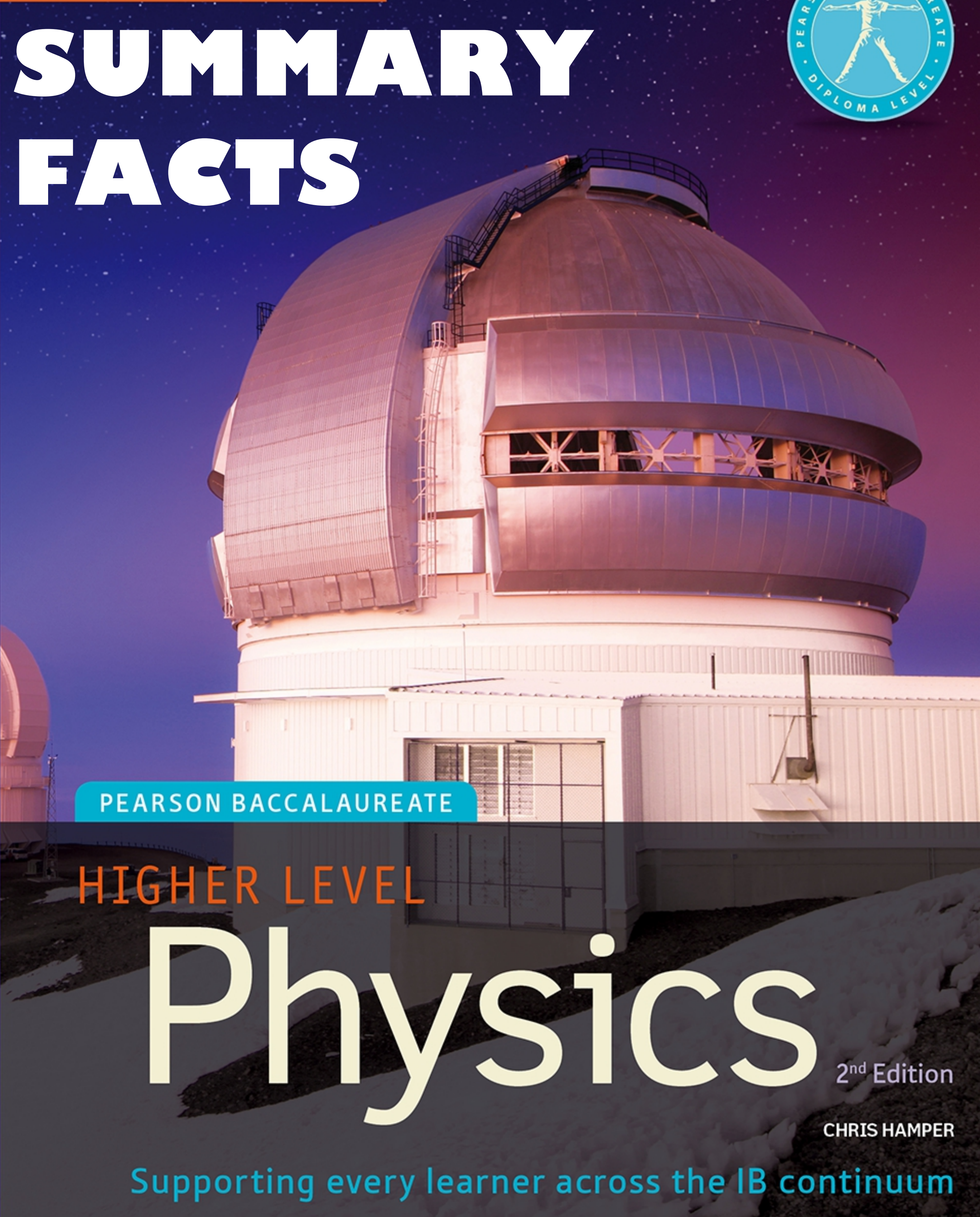
HIGHER LEVEL

SUMMARY FACTS



PEARSON BACCALAUREATE

HIGHER LEVEL

Physics

2nd Edition

CHRIS HAMPER

Supporting every learner across the IB continuum

ALWAYS LEARNING

PEARSON

Summary facts 1

Measurements and uncertainties

Measurements in physics

Standard form and significant figures

3.6×10^8 is the standard form for 360 000 000 and has 2 significant figures.

3.600×10^8 has 4 significant figures.

It is worth remembering the most common prefixes:

n	10^{-9}
µ	10^{-6}
m	10^{-3}
c	10^{-2}
k	10^{3}
M	10^{6}

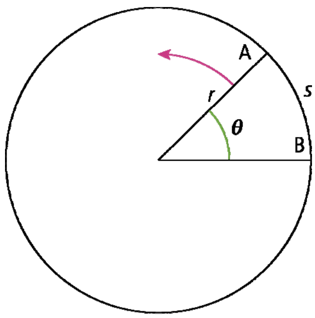
Fundamental quantities and their units

length	m
mass	kg
time	s

Angle – measured in radians or degrees

360° = 2π radians

The radian is defined as $\frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$



Uncertainties and errors

A *random error* can be due to changes in the quantity you are measuring (e.g. the varying size of a plasticine ball), or the way you are measuring it (e.g. the bounce height of a rubber ball). Random errors can be reduced by taking lots of measurements and finding the average.

Random errors will randomly cause a measurement to be sometimes bigger and sometimes smaller. A ruler that is too short will always cause the value to be too big. This is a *systematic error.*

When you measure a quantity you should always estimate the uncertainty in the value. One way of doing this is to repeat the measurement and find the range of values. Uncertainty = $\pm\frac{1}{2}$(range). Alternatively, find the standard deviation.

Uncertainties can be quoted as absolute values (e.g. 12.6 ± 0.1 m) or as percentages (12.6 m ± 0.8%).

Uncertainties are expressed with either 1 or 2 significant figures.

Uncertainties in calculated values can be found by finding the $\frac{(\max - \min)}{2}$ or by using percentage or fractional uncertainties.

(100 ± 5) + (100 ± 5) = 200 ± 10

(100 ± 5%) + (100 ± 5%) = 200 ± 5%

(100 ± 5) × (100 ± 5) = 10 000 ± 1000 $\left[\frac{(\max - \min)}{2} = \frac{(105\times105-95\times95)}{2} = 1000\right]$

(100 ± 5%) × (100 ± 5%) = 10 000 ± 10%

Linearizing

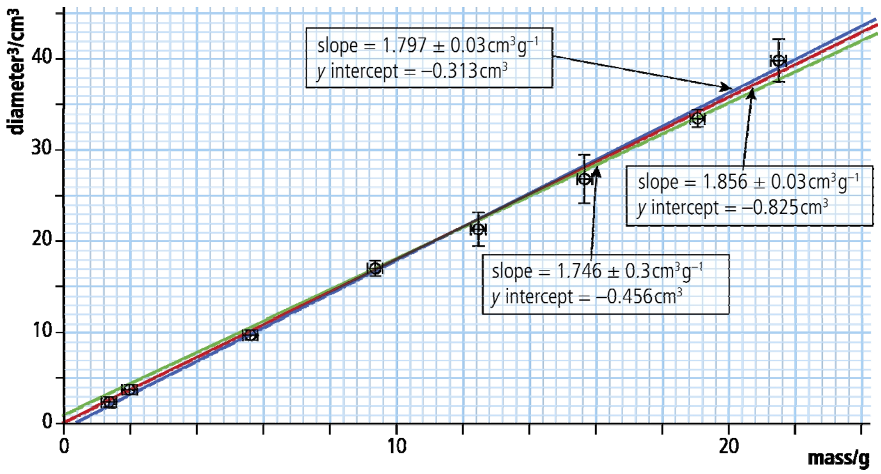
It is much easier to interpret a straight-line graph than a curve so we try to choose *x* and *y* so that they are linearly related. So if $s = \frac{1}{2}at^2$ we would plot *s vs* t^2 as this would give a straight line.

The equation of a straight line is of the form $y = mx + c$.

Error bars

Error bars are lines drawn onto data points to show the extent of the uncertainties. To conclude that a relationship is linear, the line of best fit must touch all the error bars. Note that hitting *either* the *x* or *y* error bar on a point is sufficient.

Uncertainty in a gradient

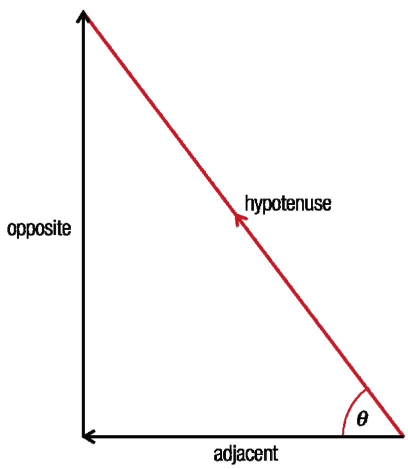


To find the uncertainty in a gradient we can plot the steepest and least steep lines through the error bars.

Vectors

Vectors are quantities with magnitude and direction that can be represented by arrows. The length of the arrow is proportional to the magnitude and the direction is the same as the direction of the quantity (e.g. displacement).

Trigonometry



$$\sin \vartheta = \frac{O}{H}$$

$$\cos \vartheta = \frac{A}{H}$$

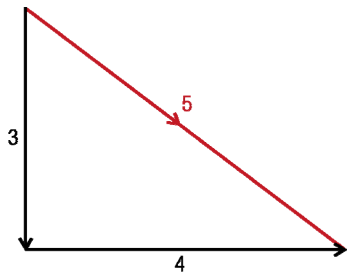
Used to find lengths of triangles and components of vectors.

If ϑ is small and measured in radians, $\sin \vartheta = \vartheta$.

Pythagoras: $H^2 = O^2 + A^2$ – used for adding perpendicular vectors.

Adding vectors

Arrange nose-to-tail and draw a line connecting the free tail to the free nose (works with any number of vectors).

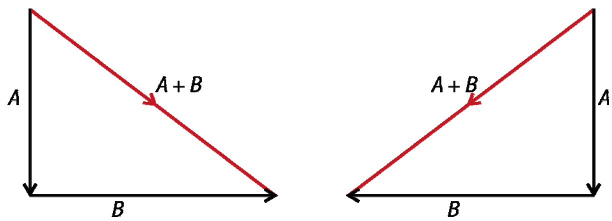


$Resultant^2 = 3^2 + 4^2$ so resultant = 5

This is a 345 triangle (look out for these – they are very easy to solve).

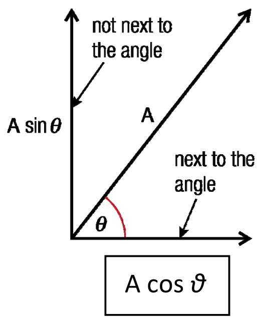
Subtracting vectors

If you change the direction of the arrow then the vector becomes negative so you can subtract a vector by flipping it round and adding it.



Components of vectors

If two vectors can add to give one then one can be split into two. This is done in two perpendicular directions using trigonometry.



Summary facts 2

Mechanics

Motion

Velocity and speed

Velocity is a vector, speed is a scalar. Both have unit m s^{-1}.

$$\text{average velocity} = \frac{\text{displacement}}{\text{time}} = \left(\frac{s}{t}\right)$$

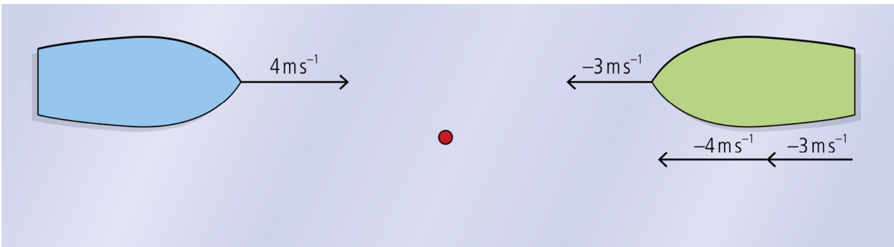
$$\text{speed} = \frac{\text{distance}}{\text{time}} = \left(\frac{d}{t}\right)$$

You can go round a corner at constant speed but *not* constant velocity.

Instantaneous velocity is velocity measured over a small displacement (Δs) for a short time (Δt).

$$v = \frac{\Delta s}{\Delta t}$$

Relative velocity



Subtract the velocity of the blue boat from the velocity of the green boat to find the velocity of the green boat relative to the blue one.

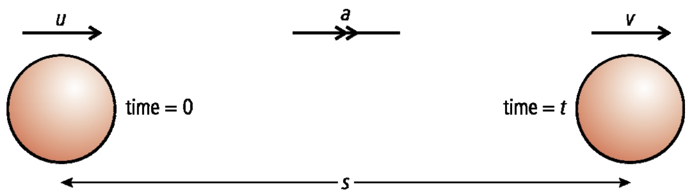
Acceleration

$$\text{acceleration} = \text{rate of change of velocity} \left(\frac{\Delta v}{\Delta t}\right)$$

Unit: m s^{-2}

Vector

Equations for motion with constant acceleration



This example is used many times in different forms.

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{(v+u)t}{2}$$

Remember that *u*, *v*, *s*, and *a* are vectors so the sign gives the direction (in 1-dimension).

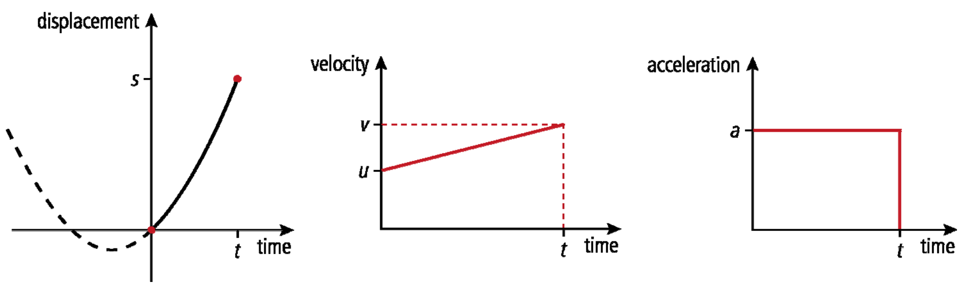
sign of displacement = direction moved

sign of velocity = direction moving

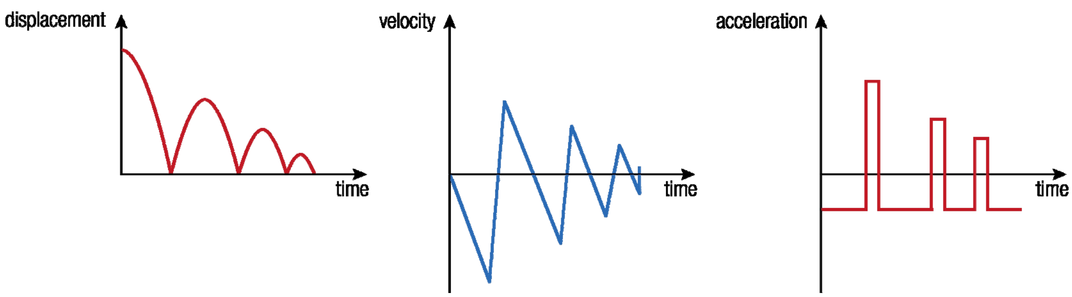
sign of acceleration is + if getting *faster* in a + direction or getting *slower* in a – direction

Graphs of motion

suvat



Bouncing ball



Measuring g

Time how long it takes for a ball to fall from a range of different heights. The time and height are related by the equation $h = \frac{1}{2}gt^2$ so a graph of h *vs* t^2 will have gradient $\frac{1}{2}g$.

Projectile motion

When a body is thrown forwards its horizontal component of motion has constant velocity but its vertical component has uniform acceleration. The resulting motion has a parabolic path. To solve projectile problems apply the *suvat* equations to the separate components remembering:

- the time of flight is the same for both
- at maximum height vertical velocity is zero
- the final vertical displacement is zero
- the vertical initial velocity has the same magnitude but opposite direction to the final velocity.

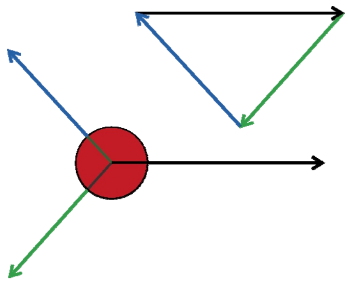
Forces

A force is a push or a pull.
Unit: newton (N)

Force is a vector so forces must be added vectorially and can be split into components.

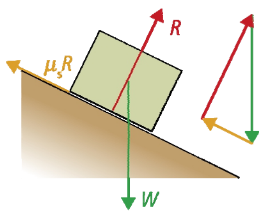
Equilibrium

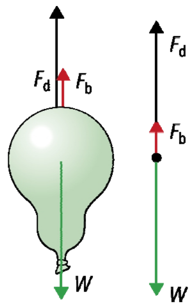
If the resultant force on a body is zero then the forces are said to be *in equilibrium*.



Types of force

Weight – downward (towards the centre of the Earth) acting on the centre of a mass ($W = mg$).
Tension – force exerted by pulling a string.
Normal reaction – force perpendicular to two touching surfaces.
Friction – force that opposes the relative motion of two surfaces ($F = \mu R$).
Buoyancy – upward force experienced by a body immersed in a fluid. Equal to the weight of fluid displaced.
Drag or air resistance – opposes the motion of a body through a fluid.





Newton's first law

A body will remain at rest or moving with constant velocity unless acted upon by an external unbalanced force.

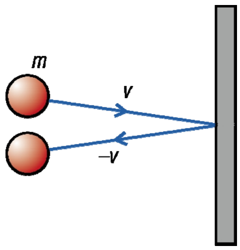
So if a body accelerates then the forces on it are unbalanced.

Momentum

momentum = mass × velocity ($p = mv$)

Unit: N s

Vector (in the same direction as motion)

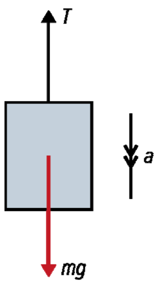


Change of momentum = $-2mv$

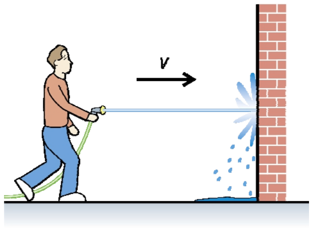
Newton's second law

The rate of change of momentum is directly proportional to the unbalanced force and takes place in the same direction.

For a constant force exerted on a body of constant mass $F = ma$



Unbalanced force = $mg - T = ma$



For examples like water hitting a wall you must use $F = \dfrac{\Delta(mv)}{\Delta t}$

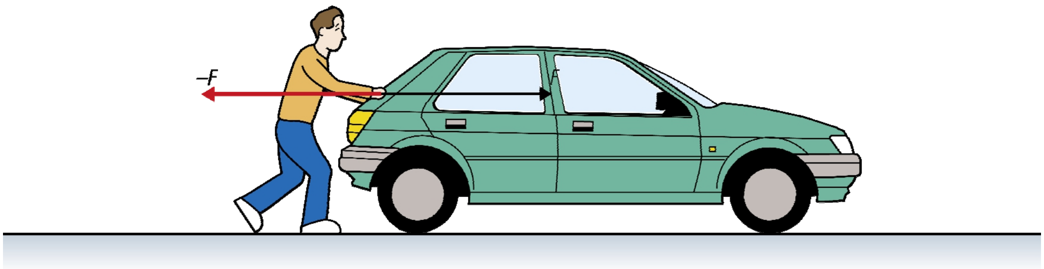
This simplifies to $v \times$ (mass hitting wall per unit time).

Examiners often ask questions like 'what is the resultant force acting on a helicopter travelling with constant velocity?' If the velocity is constant the resultant is zero. This does *not* mean that no forces are acting on the helicopter; just that they are *balanced*.

Similarly, to lift a body at a constant velocity you only need to exert a force equal to its weight. To get it moving the force needs to be a bit bigger.

Newton's third law

If body A exerts a force on body B then body B will exert an equal and opposite force on body A.

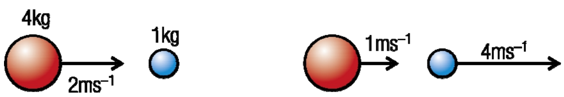


Note that the forces in this diagram act on different bodies; if both forces acted on the car then it wouldn't start moving. Don't be fooled by exam questions.

When a box rests on a table the Earth is pulling it down so Newton's third law implies that the box must pull the Earth up. You might think that the opposite force implied by Newton's third law is the normal force pushing up from the table, but this is wrong. These forces are also equal and opposite but both act on the box. Here you have applied Newton's first law.

Conservation of momentum

A consequence of Newton's three laws is that **the momentum of a group of isolated bodies is always the same**.

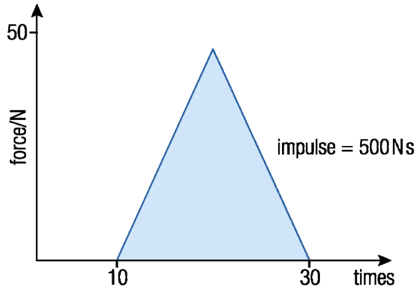


Impulse (Δp)

impulse = change in momentum

impulse = force × time of application of force = $F\Delta t$

If force is not constant, it can be found from the *area* under a force–time graph.



Pressure (P)

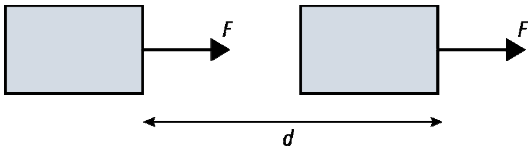
pressure = force per unit area = $\frac{F}{A}$

Unit: N m^{-2} or pascal (Pa)

Most commonly used when dealing with gases but also when a force is exerted over an area. The normal reaction between the ground and your feet.

Work

Work is done when the point of application of a force moves in the direction of the force.

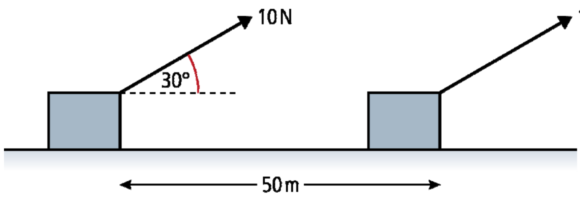


work = force × distance moved in direction of force = $F \times d$

Unit: joule (J)

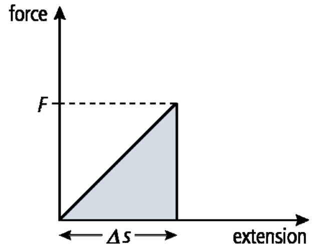
Work is the product of two vectors which in this case gives a scalar.

If there is an angle between F and d then use the component of force in the direction of motion.



Work = $F \times \cos 30°$

If F is *not* constant, work is the area under a force–displacement graph.



$$\text{work} = \frac{1}{2}F \times \Delta s$$

$$\text{work done stretching a spring} = \frac{1}{2}K\Delta x^2$$

Energy

When body A does work on body B, energy is transferred from body A to body B. Energy is the quantity that enables body A to do work.

Unit: joule (J)

Energy, like work, is scalar.

Conservation of energy

Energy cannot be created or destroyed, only changed from one form to another.

Kinetic energy

Energy a body has due to its motion.

$$\text{KE} = \frac{1}{2}mv^2$$

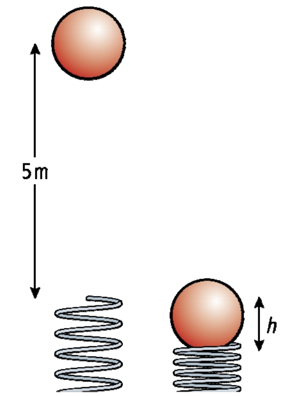
Potential energy

PE = energy a body has due to its position

Gravitational PE = mgh (for bodies close to the Earth)

Elastic PE = $\frac{1}{2}kx^2$ (for springs that obey Hooke's law)

Using conservation of energy is often a simple way to solve problems.



gravitational PE at the top = elastic PE at the bottom

Efficiency

$$\text{efficiency} = \frac{\text{useful work out}}{\text{energy in}} = \frac{\text{power out}}{\text{power in}}$$

Elastic collision

= when KE and momentum are conserved.

Simple case of two balls of equal mass is often used, in this case velocities swap.

Inelastic collision

Only momentum is conserved.

Examples are explosions and bodies sticking together.

Power

power = work done per unit time

Unit: watt (W)

Scalar

power = Fv

If a constant force moves a body at constant velocity (this implies there is another force, e.g. friction, acting against the pushing force).

Summary facts 3

Thermal physics

Atoms and molecules

In this chapter atoms are said to behave like small elastic balls. A molecule is two or more atoms joined together. Most materials are made of molecules but we only consider the simplest situation of matter being made of atoms.

Avogadro's constant (N_A)

$N_A = 6.02 \times 10^{23}$ molecules
This is the number of atoms in 12 g of ^{12}C.

If we take this number of molecules of different materials then the mass is different because the mass of the molecules is different.

Relative atomic mass

The mass of an atom compared to the mass of $\frac{1}{12}$ the mass of a ^{12}C atom.

hydrogen – 1
helium – 4

Mole

One mole of a substance contains 6.02×10^{23} molecules.
One mole of ^{12}C is 12 g.

1 mole of hydrogen – 1 g
1 mole helium – 4 g

States of matter

Solid – fixed shape and volume since atoms are held together by interatomic force.

Liquid – fixed volume but *not* fixed shape since atoms are able to move relative to each other.

Gas – *not* fixed shape or volume (will fill whatever container it is put into); atoms are free to move about with no force between them.

Brownian motion

= the motion of smoke particles viewed under a microscope.

= evidence that gases consist of fast-moving particles.

Thermal concepts

Internal energy

= the sum of the KE and PE of the molecules in a body.

When a force slides a box along the table with constant velocity, work is done but the PE and KE of the block does not increase. Energy is transferred to the molecules of the box.

Temperature

A measure of how hot or cold a body is, related to the average KE of the molecules.

Units: °C (Celsius) or K (kelvin)

To convert from Celsius → kelvin add 273.

Changes in temperature are the same measured in K or °C.

If the temperature of a body is used in calculation, K are always used.

Celsius scale is based on the length of a column of mercury at the boiling and freezing points of pure water at normal atmospheric pressure.

Kelvin scale is based on the pressure of a gas at the triple point of water and absolute zero.

The average KE of a molecule of monatomic gas = $\frac{3}{2}kT$ (T measured in kelvin).

Heat

Heat flows from a body with high temperature to a body with low temperature until they reach thermal equilibrium.

Heat transfer

Conduction – when the molecules at one end of a solid rod are given energy they vibrate more. This disturbs the neighbouring molecules thereby passing the energy along the rod. Solids are generally better conductors than liquids and gases. Metals are good conductors, plastics tend to be poor conductors or *insulators*.

Convection – takes place in fluids. When given heat the fluid expands making it less dense, causing it to rise in the surrounding denser cold fluid. Trapping air in pockets like the gaps between feathers in a down jacket prevents convection.

Radiation – direct transfer from one body to another via infrared radiation. Doesn't require a medium. Black bodies both radiate and absorb best.

Thermal capacity (C)

thermal capacity = the heat required to raise the temperature of a body by 1 K

$$Q = C\Delta T$$

Specific heat capacity (c)

specific heat capacity = the heat required to raise the temperature of 1 kg of a substance by 1 K

$$Q = mc\Delta T$$

Phase change (= change of state)

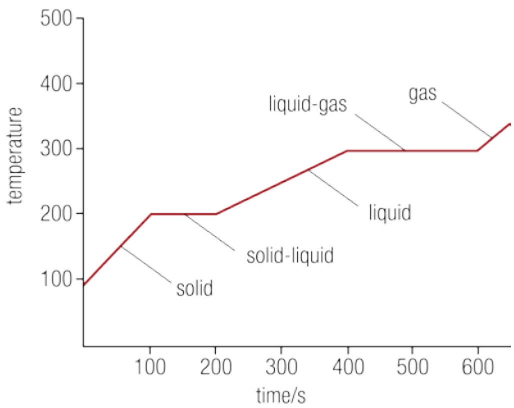
Changing state requires energy to change the position of the molecules but does not result in a change of temperature. The energy increases the PE but *not* the KE.

Specific latent heat (L)

L = the amount of heat required to change the state of 1 kg of a substance without change in temperature

$$Q = mL$$

Vaporization : liquid ↔ gas
Fusion : solid ↔ liquid



The graph shows the changing temperature of 1 kg of a substance heated by a 100 W (100 joules per second) heater so it changes state from solid to liquid to gas.

Specific heat capacity of solid = 100 J kg^{-1} K^{-1}
Specific latent heat of fusion = 10 000 J kg^{-1}
Specific heat capacity of liquid = 200 J kg^{-1} K^{-1}
Specific latent heat of vaporization = 20 000 J kg^{-1}

Modelling an ideal gas

- Volume of atoms is much smaller than the volume of the gas.
- Atoms are perfectly elastic spheres so no energy is lost when they collide with each other.
- No interatomic forces so atoms travel with uniform velocity between collisions.

Volume is the same as container.

$$\text{average KE of molecules} = \frac{3}{2}kT$$

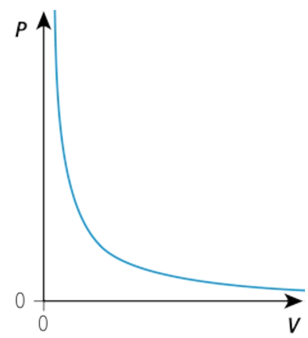
Gas exerts *pressure* on the walls of the container because the gas molecules change momentum when they collide with the walls. Pressure is equal to the rate of change of momentum per unit area.

Boyle's law

The pressure of a fixed mass of gas at constant temperature is inversely proportional to its volume.

$$P \propto \frac{1}{V}$$

If the volume of a gas is increased the molecules will be less dense and therefore hit the walls less often.

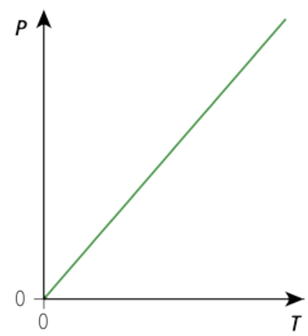


Pressure law

The pressure of a fixed mass of gas with constant volume is directly proportional to its temperature in kelvin.

$$P \propto V$$

Increasing temperature increases the average KE of the molecules which means the change in momentum is greater when they hit the walls and they do it more often.

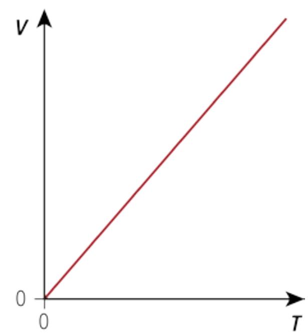


Charles' law

The volume of a fixed mass of gas at a constant pressure is directly proportional to its temperature in kelvin.

$$V \propto T$$

A rise in temperature causes the molecules to move faster, so to keep pressure constant the volume will have to increase proportionally.



Ideal gas equation

$PV = nRT$

n = number of moles

R = molar gas constant = 8.31 J mol^{-1} K^{-1}

To solve problems, use the form $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

Real gases

Real gases behave ideally at high temperature and low pressure but at low temperatures and high pressures the interatomic forces are no longer negligible and the gas may turn into a liquid.

Summary facts 4

Circular motion and gravitation

Circular motion

time period (T) = time for one complete revolution

frequency (f) = number of revolutions per second

$$f = \frac{1}{T}$$

Unit: Hz

angular velocity (ω) = angle in radians swept out by the radius per unit time

$$\omega = \frac{\Delta \vartheta}{\Delta t} = 2\pi f$$

speed (v) = distance travelled per unit time

In time T distance travelled is the circumference, so $v = \frac{2\pi r}{T} = \omega r$

If a body travels in a circle its direction and therefore velocity is changing. This means it must be *accelerating* so the forces on it are not balanced.

Centripetal force

The force towards the centre of a circle causing a body to travel in circular motion. This is not a new type of force but the name given to the resultant force.

Work is done by centripetal force since it is perpendicular to the direction of motion.

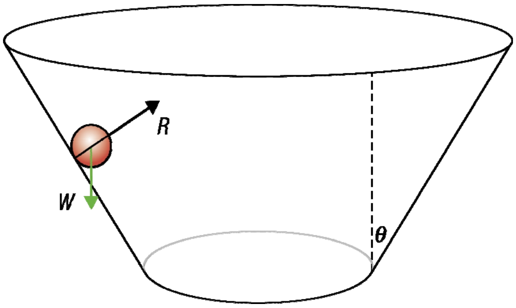
$$F = \frac{mv^2}{r} = m\omega^2 r$$

Centripetal acceleration

The acceleration of the body due to the unbalanced force towards the centre.

$$a = \frac{F}{m} \text{ so } a = \frac{v^2}{r} = \omega^2 r$$

Example of circular motion



First identify which force is acting towards the centre.

This is the horizontal component of *R*.

$$R \cos \vartheta = \frac{mv^2}{r}$$

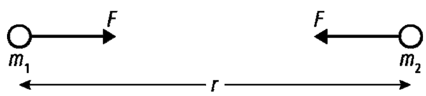
In this case the vertical forces are balanced so $R \sin \vartheta = W$

Gravitational field

gravitational field = the region of space where a mass experiences a force due to its mass

Newton's universal law of gravitation

Every single point mass attracts every other point mass with a force that is proportional to the product of their masses and inversely proportional to their separation.



$$F = \frac{Gm_1m_2}{r^2}$$

Spheres of mass behave as if all their mass was at their centre (centre of gravity).

Gravitational field strength (g)

gravitational field strength = force per unit mass experienced by a small test mass placed in the field

Unit: N kg^{-1}

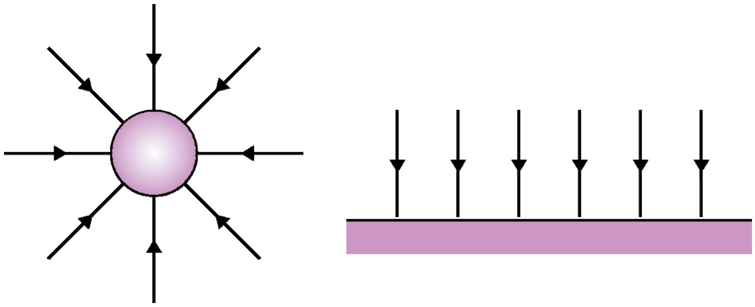
Field strength is a vector so add vectorially.

On the Earth's surface g = 9.81 m s^{-2} (depends where you live).

Note $\frac{F}{m}$ is gravitational field strength but *also* acceleration.

Field lines

Give direction and strength of field (strength is related to line density).



Gravitational potential energy

gravitational potential energy = the energy a body possesses due to its position in a gravitational field

Equal to the work done putting it there from a place with zero PE
(taken to be the surface of the Earth for 'close to the Earth' examples).

PE = mgh

Gravitational potential (V)

V = work done per unit mass in taking a small point mass from zero potential to the point in question

$V = mgh$

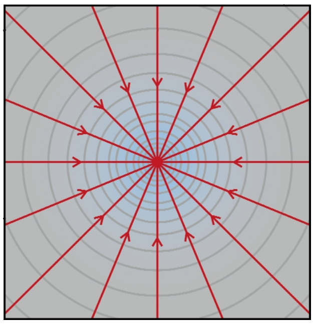
When considering the Earth to be a sphere in space we must take the zero in potential to be at an infinite distance. Potential then equals the work done per unit mass taking a small test mass from infinity to the point.

$$V = \frac{GM}{r}$$

So the PE on a mass m placed at that point would be PE = $\frac{GMm}{r}$

Lines of equipotential

Join points of equal potential. If these were drawn on the Earth they would be the same as contours.



Lines of equipotential are always perpendicular to the field lines. If you think of equipotentials as contours then this would represent a hole in the ground. A ball placed on the side of the hole would roll down the hill in the direction of the field line. In this way we can use equipotentials to visualize the field.

The steeper the hill the greater the acceleration.

field strength = potential gradient

$$g = -\frac{\Delta V}{\Delta x}$$

Escape speed

escape speed = the minimum speed that a body needs to leave the Earth and not come back

Escape speed can be found by considering the change of energy as the body leaves the Earth and travels to an infinite distance from it.

$$v = \sqrt{\frac{2GMm}{R_E}}$$

For the Earth this is about 11 $\mathrm{km s^{-1}}$.

Orbits

Planets orbit the Sun in approximately circular (actually elliptical) orbits. The gravitational force is providing the centripetal force.

$$m\omega^2 r = \frac{GMm}{r^2}$$

This can be rearranged to give Kepler's law: $r^3 \propto T^2$

Geostationary satellites orbit the Earth once a day; they are about $6R_E$ away from the Earth. GPS satellites are much nearer and orbit more often.

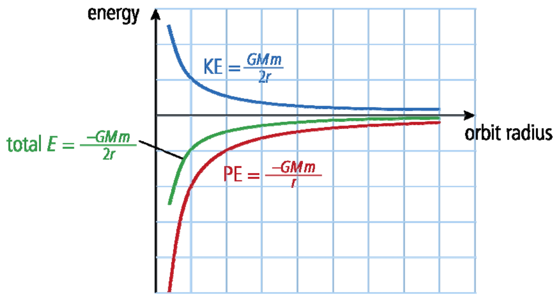
Energy of satellites

total energy = PE + KE

PE = $-\dfrac{GMm}{r}$ and KE = $\dfrac{1}{2}mv^2$

Use centripetal equation to find KE = $\dfrac{GMm}{2r}$

so total $E = -\dfrac{GMm}{2r}$



Summary facts 5

Oscillations and waves

Oscillations

Simple harmonic motion (SHM)

simple harmonic motion = motion where the acceleration is proportional to the displacement from a fixed point and always directed towards that point.

$$a = -\omega^2 x$$

The − sign means that the direction of the acceleration is always in the *opposite* direction to the displacement.

Cycle – one there and back swing.

Amplitude (x_o) – the maximum displacement from the equilibrium position.

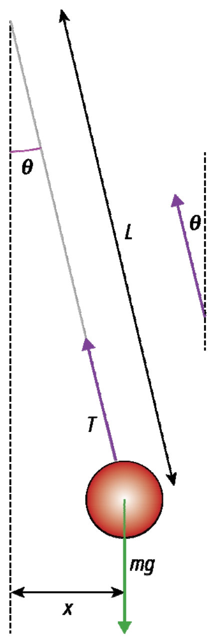
Time period (T) – time for one complete cycle.

Frequency (f) – number of cycles per second $f = \frac{1}{T}$

Angular frequency (ω) – $2\pi f$

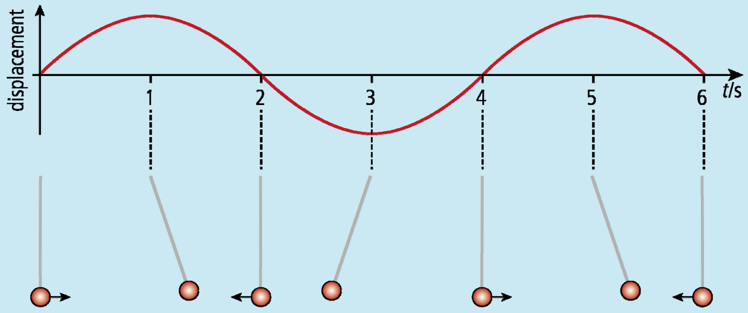
Simple pendulum

A mass hanging on a string oscillates with SHM since the force increases as the mass moves away from the equilibrium position.



restoring force = $T \sin \vartheta$ which increases as the angle increases

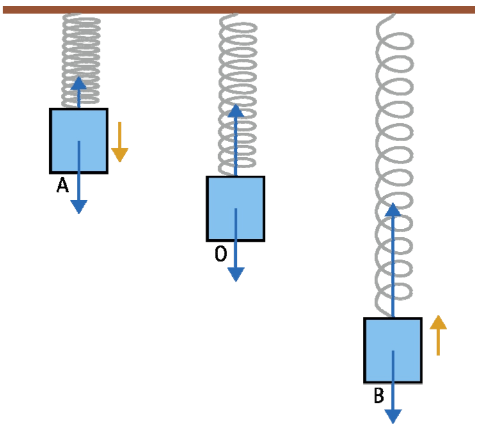
$$\omega = \sqrt{\frac{g}{L}}$$



Displacement is sinusoidal

$$x = x_0 \sin \omega t$$

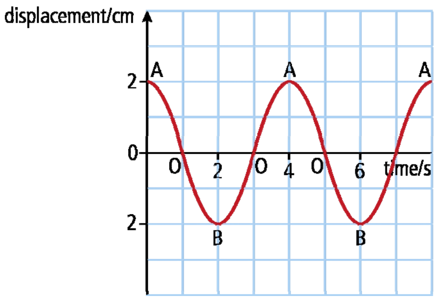
Mass on a spring

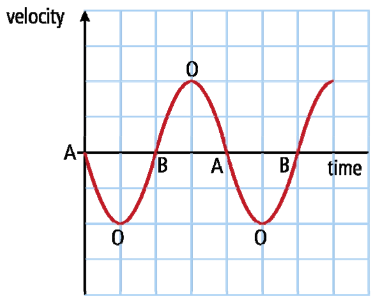


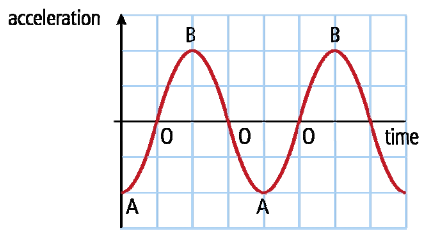
The tension increases as the mass is pulled down so the acceleration is proportional to the displacement but in the opposite direction.

$$\omega = \sqrt{\frac{k}{m}}$$

Graphical representation







Summary of equations

$$x = x_0 \cos \omega t$$

$$v = -\omega x_0 \sin \omega t$$

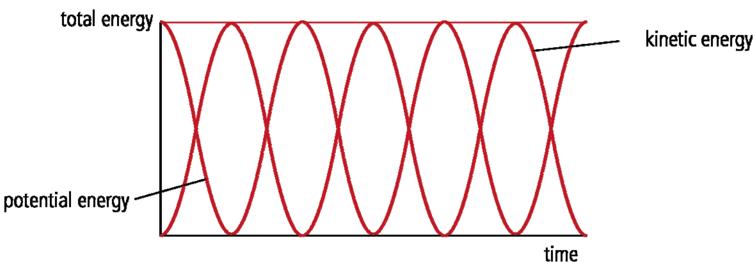
$$a = -\omega^2 x_0 \cos \omega t$$

$$v = \omega\sqrt{x_0^{\ 2} - x^2}$$

$$a = -\omega^2 x$$

Energy changes in SHM

When a pendulum bob is displaced to the left it is given PE. As it swings to the middle the PE is converted to KE which changes back to PE as the bob swings to the high point on the right. Total energy (= KE + PE) is constant.



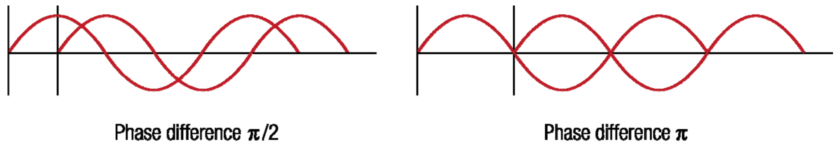
$$\text{PE} = \frac{1}{2} m\omega^2 x_0^{\ 2} \cos^2 \omega t$$

$$\text{KE} = \frac{1}{2} m\omega^2 x_0^{\ 2} \sin^2 \omega t$$

$$\text{total energy} = \text{PE} + \text{KE} = \frac{1}{2} m\omega^2 x_0^{\ 2}$$

Phase

Two bodies oscillating with the same frequency are in phase if they travel through the centre in the same direction at the same time.Phase difference is measured by phase angle. Completely out of phase is equivalent to a phase angle of π.



Waves and wave behaviour

When a stone is dropped into a pool it disturbs the surface. This disturbance propagates across the surface in the form of a *wave*. As this happens energy is transferred.

Wave properties

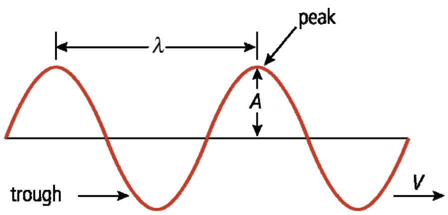
Reflection – when a wave hits the boundary between two media part of the wave comes back at the same angle.

Refraction – when a wave hits the boundary between two media part of the wave passes into the new medium with a change in angle.

Diffraction – when a wave passes through a narrow opening it spreads out.

Superposition – when two waves coincide the displacements at a point add.

Wave quantities



Wavelength (λ) – the distance between two points that are in phase (e.g. two peaks).

Amplitude (A) – the maximum displacement.

frequency (f) – the number of wavelengths passing a point per unit time or the number of cycles or oscillations per unit time.

Wave speed (v) – the distance travelled by the wave profile per unit time.

$$v = f\lambda$$

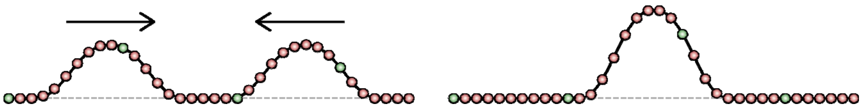
Waves in a string

Transverse wave – disturbance is perpendicular to direction of energy transfer.

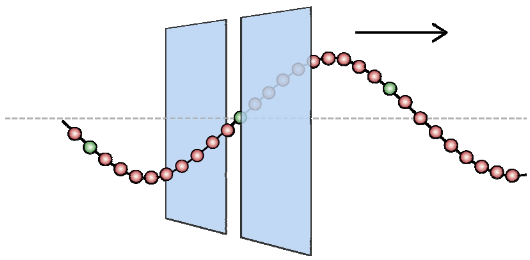
$$v = \sqrt{\frac{T}{\mu}}$$

Reflection – when a wave meets a fixed end the wave is reflected with π phase change. If the end is free the wave reflects without phase change.

Superposition – displacements add vectorially.

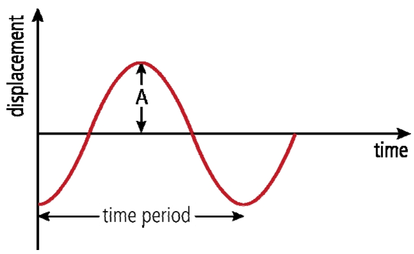


Polarization – a wave in a string can be polarized by passing it through a narrow slit. This only allows the transmission of disturbance in the plane of the slit.

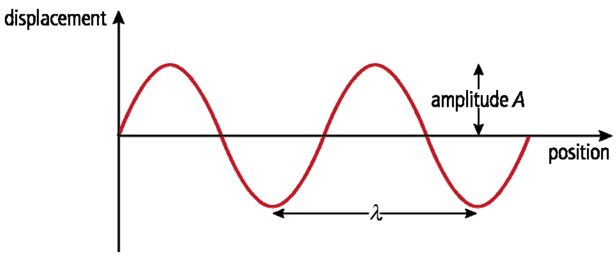


Graphical representation

Displacement–position is like a snapshot of the wave showing the displacement of all points at one time.



Displacement–time shows how the displacement of one point varies with time.



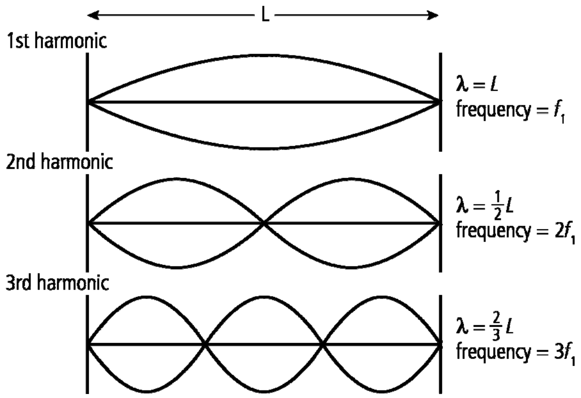
Standing waves in strings

Formed when two waves of the same frequency travel in opposite directions along the same string. This happens when a wave reflects back and forth along a clamped string.

Nodes – positions where the displacement is always zero, separated by $\frac{\lambda}{2}$

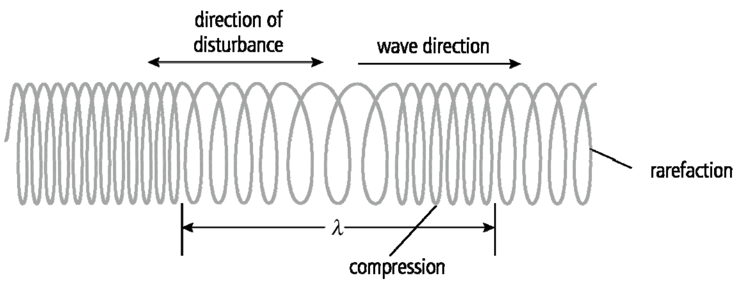
Antinodes – positions of maximum displacement.

There are many possible frequencies of wave that can form standing waves in a string: these are called *harmonics.*

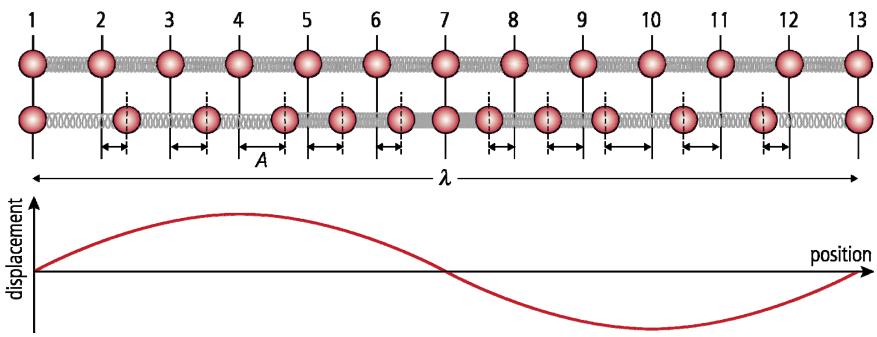


Waves in a slinky spring

Longitudinal wave – displacement is in same plane as the direction of transfer of energy.



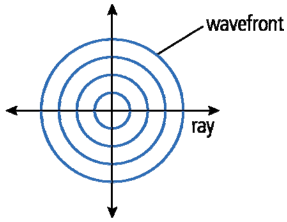
Can also plot displacement–position and displacement–time graphs for this wave but displacement is plotted on the *y*-axis so displacement position no longer looks like a snapshot.



Water waves

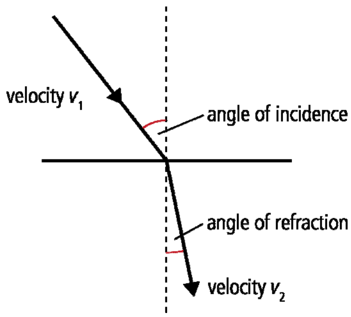
Disturbance on the surface of water. Speed depends on depth (slower when shallow).

2D waves represented by *wavefronts* and *rays.*



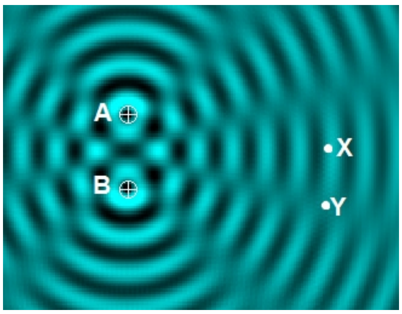
Reflect so the angle of incidence = angle of reflection

Refract according to Snell's law $\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$



Diffraction takes place when the wave passes through a narrow opening.

Interference of two sources creates the pattern shown. Waves add when path difference = whole numbers of wavelengths.



Sound

Propagation of the variation of pressure in air.

speed about 340 ms^{-1}

range of hearing from 20–20 000 Hz

340 Hz sound has wavelength 1 m

Frequency is related to *pitch*; *amplitude* is related to *loudness*.

Reflection off a solid object (echo).

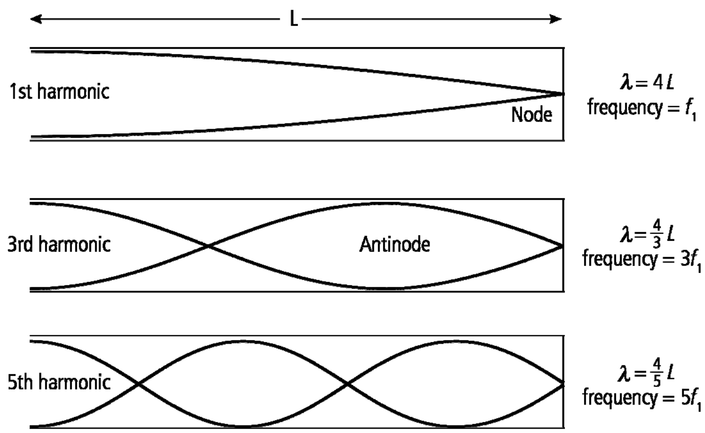
Refraction when travelling through layers of air at different temperatures (since the velocity of sound is higher if temperature is higher).

Diffraction through apertures of around 1 m.

Interference can be experienced if the same note is played on both speakers of your computer.

Standing waves in pipes

Formed when a sound wave reflects back and forth along a pipe. Pipe can be closed which gives only odd-numbered harmonics (1, 3, 5, …) or open which gives all harmonics (1, 2, 3, …).



Doppler effect

The change of frequency due to the relative motion of observer and source: 'eeeeeeoooooww'.

Moving source causes a change in wavelength (squashing ahead and spreading out behind).

$$f' = \frac{cf_0}{c \pm v}$$

Moving observer causes a change in velocity (faster towards slower away).

$$f' = \frac{(c \pm v)f_0}{c}$$

Light waves

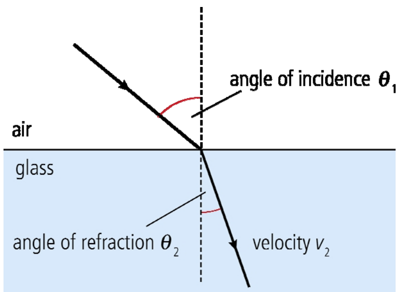
Disturbance of electric and magnetic field *(electromagnetic)*. The fields vary perpendicular both to each other and to the direction of propagation of the wave so this is a *transverse wave*.

Speed in a vacuum = 3×10^8 m s^{-1}. Slower in other media.
Usually represented by rays, *not* wavefronts.
Different wavelengths of em waves have different properties.
Wavelength of visible light related to colour.
intensity $\propto$ amplitude^2

long wavelength: radio (m) → micro (cm) → Infrared → visible(600 nm) → ultraviolet → X → gamma

Reflection – when light meets a boundary between two media part of the light is reflected at the same angle.

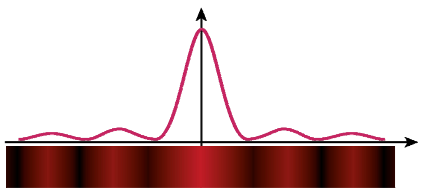
Refraction – when light meets a boundary between two media part of the light is refracted according to Snell's law.



$$\frac{\sin\vartheta_1}{\sin\vartheta_2} = \frac{n_2}{n_1}$$

If the angle of refraction is 90° the angle of incidence is called the *critical angle*; at greater angles total internal reflection takes place.

Diffraction – occurs when light passes through a narrow slit forming the typical single-slit diffraction pattern.

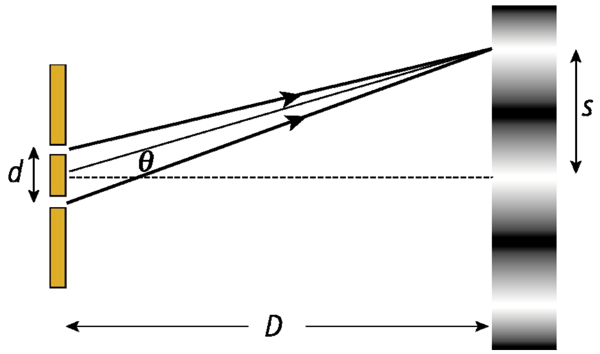


The angle between the central maximum and the first minima is given by $\vartheta = \frac{\lambda}{b}$ (for small angles)

Resolution – **Rayleigh criterion states that two points are just resolved if the principal maxima of one coincides with the first minima of the other.**

For circular apertures $\vartheta = \frac{1.22\lambda}{b}$

Two-slit interference (Young's slits) – Light passing through two narrow slits diffracts causing the light beams to overlap. The light in the overlapping region interferes to give fringes (lines of bright and dark). The separation of the bright lines is given by $s = \frac{\lambda D}{d}$.



Note the diffraction pattern will alter the intensity of the lines, but if the slits are very narrow this won't be noticed.

Diffraction grating – multiple-slit diffraction. Light is diffracted by each slit and then light from each slit interferes with light from all the other slits to give interference fringes. The slits are very narrow so the diffraction angle is big. Interference maxima occur when $d \sin \vartheta = n\lambda$ so different wavelengths will have maxima at different angles. This can therefore be used to create spectra.

Thin film interference – This is what causes the coloured pattern on a soap bubble. Some light reflects off the front and some off the back. The two light waves interfere constructively when $2t = \left(m + \frac{1}{2}\right)\lambda$

So different colours will constructively interfere for different thicknesses.

Doppler effect – This is complicated by relativity but the approximate formula is $\Delta f = \frac{vf_0}{c}$.

This causes the red shift of stars moving away from the Earth.

Polarization – Light is transverse so can be polarized. This is done by some special plastics (e.g. polaroid). Unlike polarization of a string wave, this has nothing to do with slits. When unpolarized light passes through a polarizing filter the intensity will be reduced by $\frac{1}{2}$.

Malus' law – If the angle of a polarizer is ϑ to the direction of polarization of a beam of polarized light of intensity I_0 then the transmitted beam will have intensity $I_0 \cos^2 \vartheta$.

Summary facts 6

Electricity and magnetism

Electric field

A region of space where a charge experiences a force due to its charge.

Charge (Q) can be +, − or neutral.

Charge is conserved.

Unit: coulomb(C)

Charge on an electron = 1.6×10^{-19} C

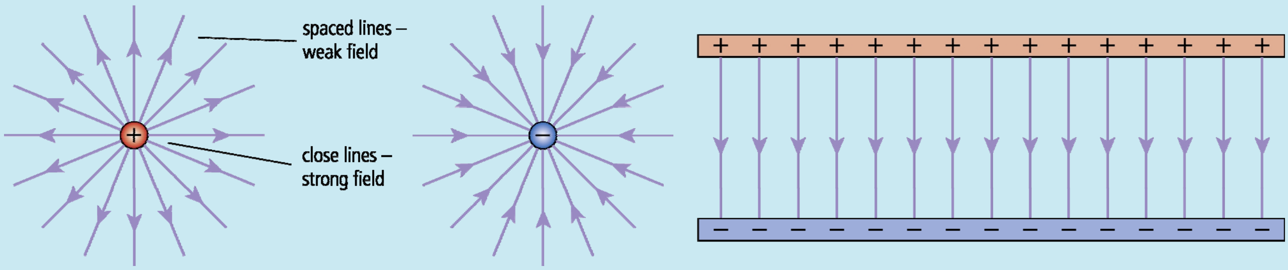
Field strength (E) – the force experienced by a small + charge placed at a point in the field.

Unit: N C^{-1}

Vector

Field lines – show direction and magnitude of field.
Radially outwards from a +point and radially inwards to a −point.

Uniform field – between oppositely charged parallel plates.



Coulomb's law

The force experienced by two point charges is directly proportional to the product of their charge and inversely proportional to the square of their separation. (Rather like Newton's law of gravity.)

$$F = k\frac{Q_1Q_2}{r^2}$$

where $k = 9 \times 10^9$ N m^2 C^{-2}

Electrical PE

The works done in placing a + charge at a point in the field from a place where it has zero potential energy. This can be an arbitrary zero or at an infinite distance.

Electrical potential (V)

The work done per unit charge taking a small + charge from a position of zero potential (infinity) to a point in the field.

Unit: volts ($J\ C^{-1}$)

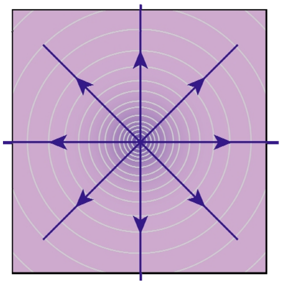
Scalar

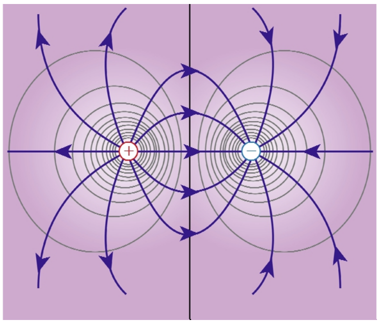
For a sphere of charge Q: $V = \frac{kQ}{r^2}$

potential energy = VQ

Lines of equipotential – join points of equal potential, forming wells and hills in 2D. Always perpendicular to field lines.

Equipotentials and field lines for a positive charge and a dipole:



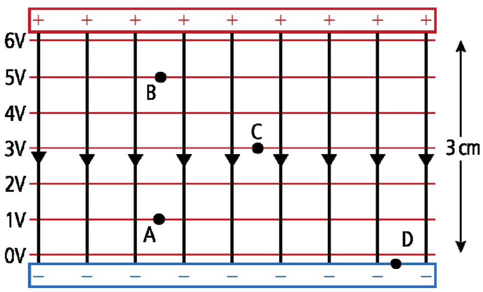


Potential difference – the work done per unit charge taking a small + charge from A to B.

Lines of equipotential in a uniform field are equally spaced parallel lines.

$$E = -\frac{\Delta V}{\Delta X}$$

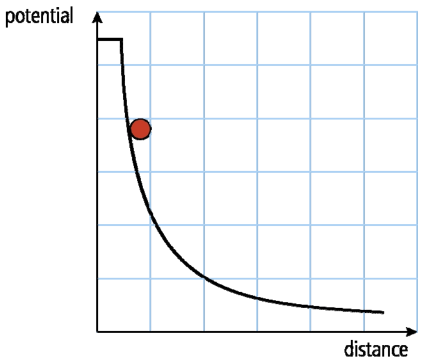
$$E = \frac{6}{0.03} = 200\ V\ m^{-1}$$



Electrical currents

Conductors conduct electricity because they have free electrons, *insulators* don't.

Positive charge will always move from high to low potential (*downhill*).



Current is the rate of flow of charge.

$$I = \frac{\Delta Q}{\Delta t}$$

Conventional current flows from high to low potential (+ to −) but electrons move the other way (*uphill*).

Current is related to the drift velocity of electrons.

$$I = nAve$$

Resistance(R) – the ratio of pd across a conductor to current flowing through it.

Unit: ohm (Ω)

Resistivity (*ρ*) – the resistance of 1 m^3 of substance.

$$R = \frac{\rho l}{A}$$

Unit: Ω m

Ohm's law

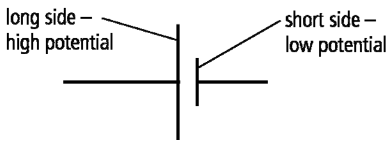
The current through an Ohmic conductor is directly proportional to the pd across it provided the temperature remains constant.

$$V = IR$$

Non-ohmic (e.g. light bulb) = it heats up when current flows which causes its resistance to rise.

Electric cells

Convert chemical energy into electrical potential energy. Tricks of chemistry cause charges to become separated causing one terminal to be more positive than the other.

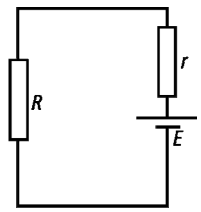


Internal resistance (r) – the resistance of the components inside a battery.

Emf (ε) – the energy converted from chemical to electrical potential when one unit charge flows, *or* the work done taking unit charge from one terminal to the other.

Discharge – when a battery is used, the terminal pd stays the same for a long period of time then suddenly goes down. If it went down constantly then it would be useless after a very short time.

Potential difference (across a resistor) – the amount of electrical energy converted to heat when unit charge flows.

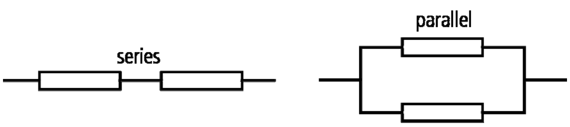


$$\varepsilon = IR + Ir$$

electrical power from battery = εI

electrical power dissipated in a resistor = $VI = I^2R = \dfrac{V^2}{R}$

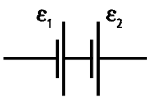
Resistor combinations

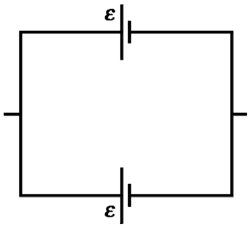


Series – $R = R_1 + R_2$

Parallel – $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

Cell combinations (identical cells)

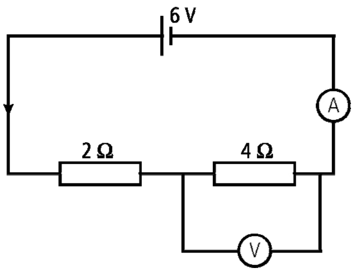




Series – $\varepsilon = \varepsilon_1 + \varepsilon_2$

Parallel – $\frac{1}{\varepsilon} = \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2}$

Measuring voltage and current



Voltmeter in parallel (measures pd across 4 Ω resistor)

Ammeter in series (measures current in circuit)

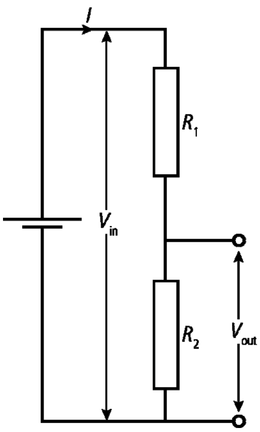
Ideal voltmeter has infinite resistance

Ideal ammeter has zero resistance.

Real meters will slightly change current in circuit.

Potential divider circuit

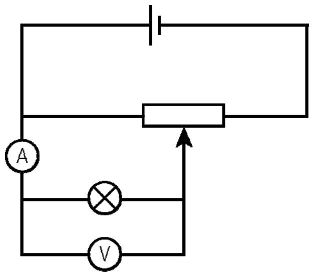
Used to give a variable pd.



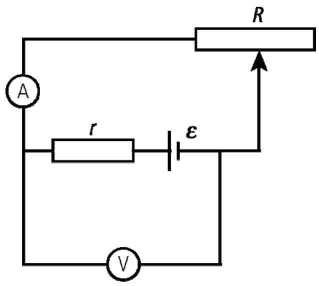
$$V_{out} = V_{in} \frac{R_1}{R_1 + R_2}$$

Variable resistor

Can be used to give varying pd across a light bulb



or varying current through a resistor.

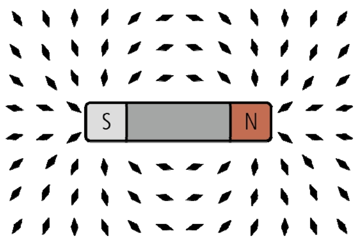


Magnetic fields

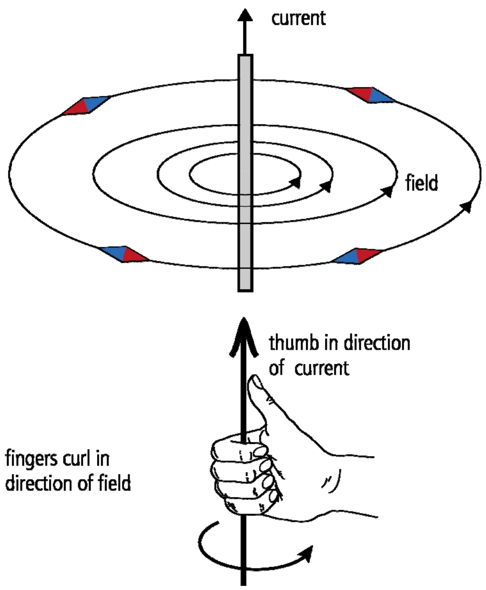
A region of space where a small magnetic dipole experiences a turning force.

Poles – called north-seeking and south-seeking because they point in those directions if suspended. Magnets are always *dipoles.*

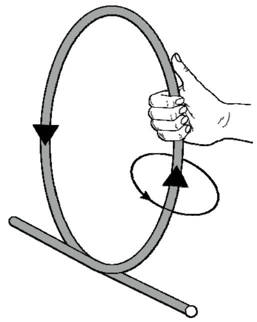
Field lines – point in direction an N pole would point.



Currents cause magnetic field. Find direction of field with the grip rule.



Can work out the direction of the field in a coil in the same way.



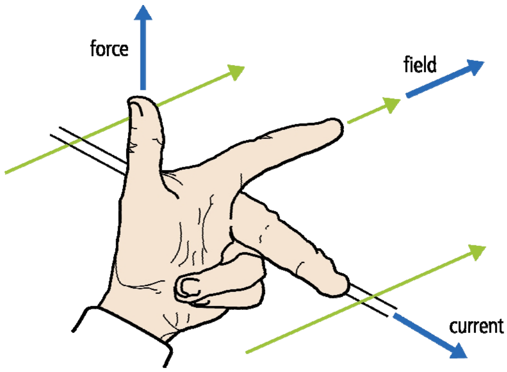
Flux density (*B*) – related to the density of field lines. Gives the size of force experienced by a current-carrying conductor placed in the field.

$F = BIL$
Unit: tesla (T)
Vector

Force on a current-carrying conductor

$F = BIL$

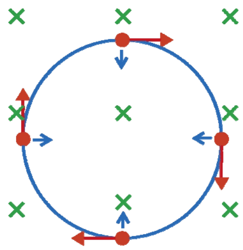
Find direction using Fleming's left hand rule



The ampere –defined in terms of the force between two parallel wires.

One ampere is defined as the current that would cause a force of 2 $\times$ 10^{-7} N per metre between two long parallel conductors separated by 1 m in a vacuum.

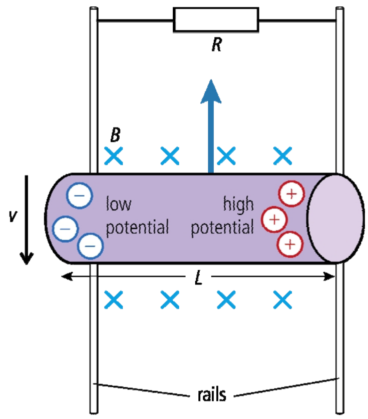
Charges moving in magnetic fields



The force $F = Bqv$ is always perpendicular to the motion so the charge follows a circular path.

Induced emf

When a conductor moves through a magnetic field an emf is induced in it which can cause current to flow in a circuit.



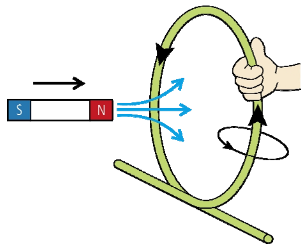
Faraday's law

The size of the induced emf equals the rate at which the conductor cuts the magnetic flux (or the rate of change of magnetic flux enclosed by the circuit)

$$\varepsilon = -\frac{\Delta\phi}{\Delta t}$$

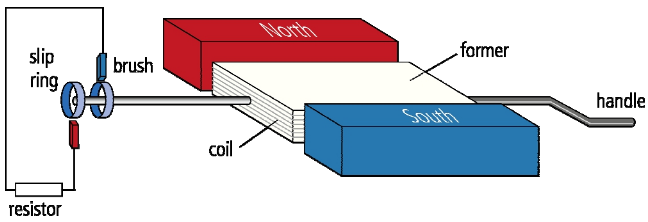
Lenz's law

The direction of induced current is such that it opposes the change inducing it. A consequence of the law of conservation of energy.



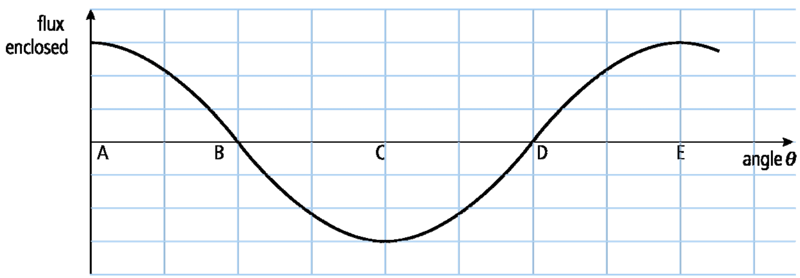
as the magnet approaches the coil the flux enclosed increases → emf induced in coil → current flows (assuming complete circuit) → direction of current must oppose change producing it → current is such that magnet is pushed back

AC generator



as handle is turned the flux enclosed by the coil will increase → emf will be induced in the coil (Faraday) → current will flow in the circuit

as coil moves through vertical position flux enclosed changes very little → no induced emf
as coil continues to turn flux enclosed gets less → induced emf but in opposite direction

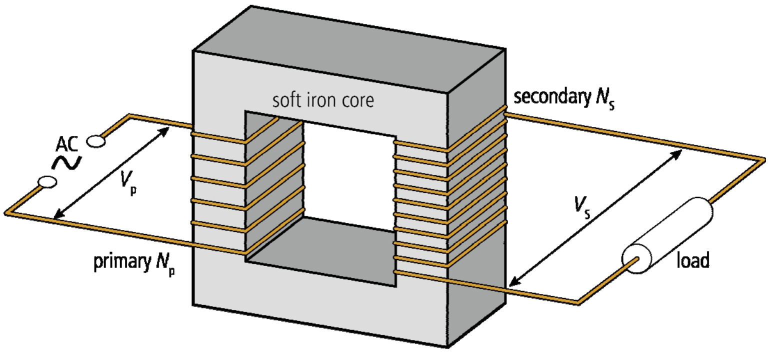


Increasing rate of turning coil will increase both frequency and induced emf.

$$\text{power in AC} = V_{\text{rms}} \times I_{\text{rms}}$$

$$V_{rms} = \frac{V_{peak}}{\sqrt{2}}$$

Transformer



changing current in primary → changing field in core → changing field in secondary → induced emf in secondary → current through load

$$\frac{N_p}{N_s} = \frac{V_p}{V_s}$$

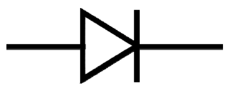
If 100% efficient $V_p I_p = V_s I_s$

Power loss by Joule heating, eddy currents, flux leakage, hysteresis

Transmission of electrical power

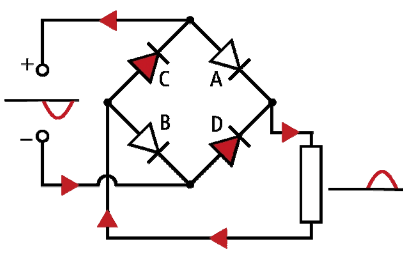
Uses a transformer to step up voltage before transmission so that the current is less and therefore the power loss in the wires (I^2R) is less.

Rectification



Diode only allows passage of current in the direction of the arrow.

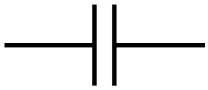
The full-wave rectifier converts AC into DC.



Capacitance

$$C = \frac{Q}{V}$$

parallel plate capacitor



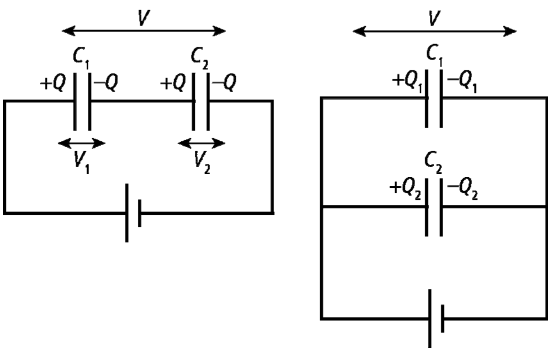
capacitor symbol

$$C = \frac{\varepsilon_o A}{d}$$

Dielectric constant (ε_r) = the ratio of capacitance with dielectric to capacitance without.

$$C = \frac{\varepsilon_0 \varepsilon_1 A}{d}$$

Capacitor combinations



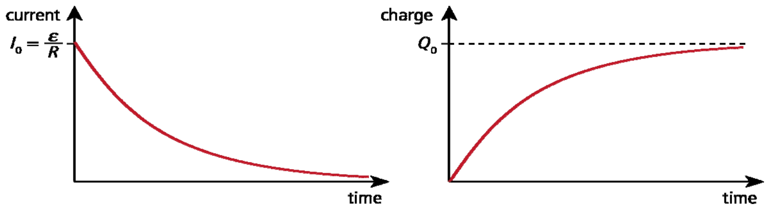
$$Series - \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

Parallel – $C = C_1 + C_2$

Note: this is the *opposite* to resistors.

$$Energy\ stored = \frac{1}{2}CV^2 + \frac{1}{2}QV = \frac{Q^2}{2C}$$

Charging a capacitor

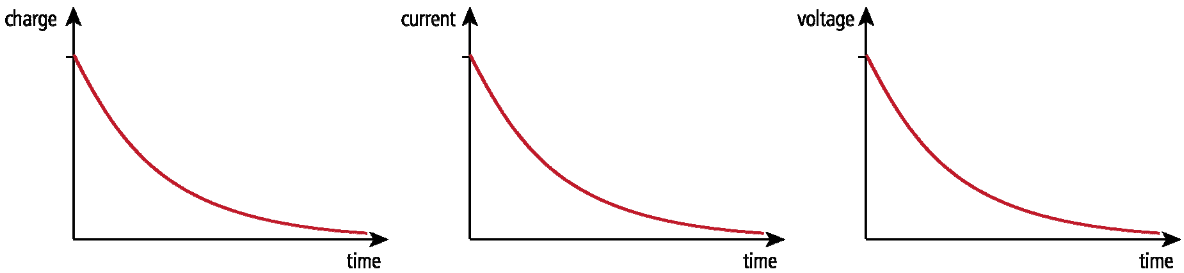


$$\text{Current} - I = I_0 e^{-\frac{t}{RC}}$$

$$\text{Charge} - Q = Q_0\left(1 - e^{-\frac{t}{RC}}\right)$$

Time constant (RC) – time for capacitor to reach approximately $\frac{2}{3}$ charge.

Discharging a capacitor

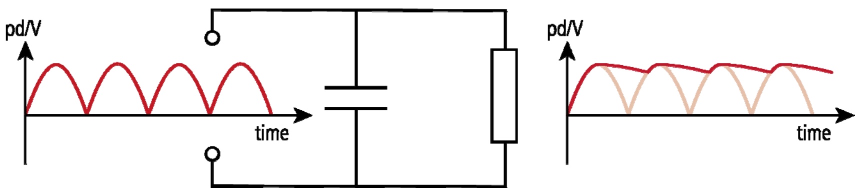


$$Q = Q_0 e^{-\frac{t}{RC}}$$

$$I = I_0 e^{-\frac{t}{RC}}$$

$$V = V_0 e^{-\frac{t}{RC}}$$

Smoothing



Capacitor charges up rapidly since no resistance but discharges slowly through resistance.

Summary facts 7

Atomic, nuclear, and particle physics

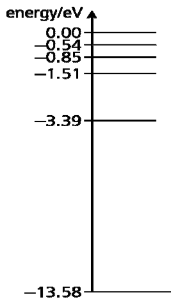
Discrete energy and the interaction of matter with radiation

Rutherford scattering experiment

Alpha particles fired at a gold foil and some bounced back. Showed that an atom has a heavy small, positively charged nucleus.

Atomic spectra

When excited by heating or with an electric field low pressure gases give out light that when split into a spectrum is comprised of discrete lines (unlike excited solid atoms that give continuous spectra). This can be explained if electrons can only exist in discrete energy levels.



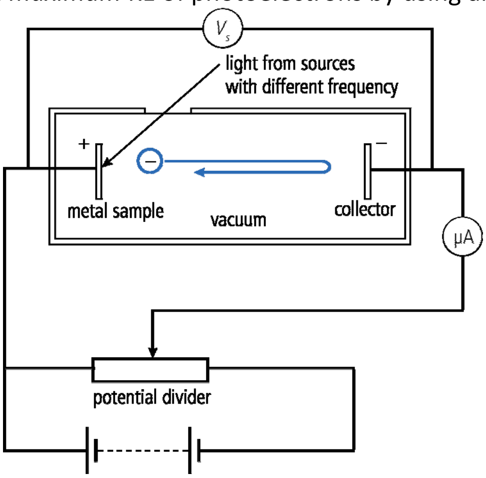
Light is emitted when an electron goes from high to low energy (or absorbed if the reverse).

$$\Delta E = hf$$

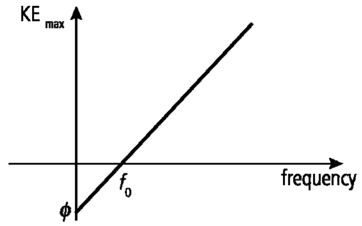
This implies that the light is emitted in packets (*photons*), each with energy = hf

Photoelectric effect

When UV light is shone on a – charged electroscope the leaves fall provided the frequency of light is high enough. It is possible to measure the *maximum* KE of photoelectrons by using an electric field to stop them.



Results show that $KE_{max} = hf - \Phi$ so the max KE equals the energy of the photon (= the energy needed to get the electron out of the metal).



Work function (Φ) – the amount of energy required to liberate an electron from the metal surface.

Threshold frequency (f_0) – the minimum frequency required to liberate electrons.

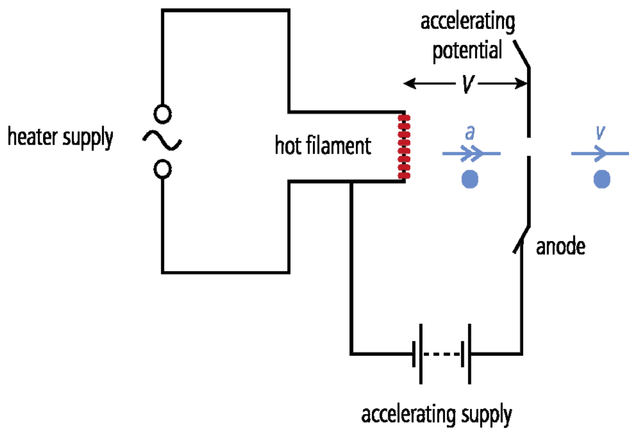
$$\Phi = hf_0$$

electron volt (eV) – a convenient unit of energy.

1 eV is the KE gained by an electron accelerated through 1 V (1.6×10^{-19} J).

Electron gun

Electrons emitted by the hot wire are accelerated by the electric field.



Bohr model

Showed that if the angular momentum was quantized the electrons of hydrogen would have discrete energy levels, giving the observed spectral lines.

$$E = -\frac{13.6}{n^2}$$

Failed to predict the varying intensity or fine detail of hydrogen lines and failed to derive correct energy levels for other atoms.

Wave nature of particles

de Broglie hypothesis – **all matter has a wave-like nature.**

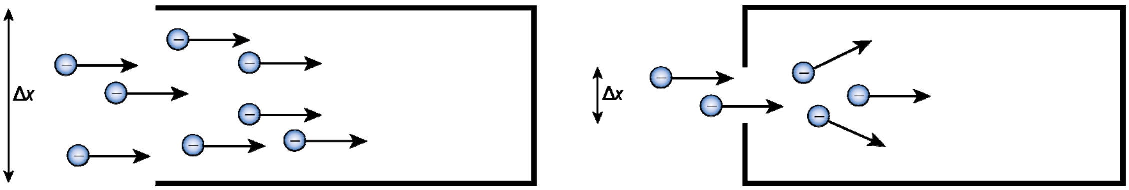
$$\lambda = \frac{h}{p}$$

The de Broglie wave equation gives the *probability* of the electron being at a certain position.

Electron diffraction – electrons are diffracted by crystals in the same way as light is diffracted by a diffraction grating. The de Broglie waves are diffracted by the crystal giving a varying probability for the positions of the electrons.

Heisenberg's uncertainty principle – **it is not possible to determine both position and momentum of a particle at the same time.**

Can be explained in terms of electrons passing into a box. If the box has a wide opening, their position is uncertain but we know their momentum. If the opening is narrow, diffraction causes uncertainty in momentum but we know their position.



$$\Delta p \Delta x \geq \frac{h}{4\pi}$$

Can be used to show that atomic electrons have energy of a few eV.

$$\Delta E \Delta T \geq \frac{h}{4\pi}$$

Schrodinger's equation

Can be solved to find the *wave function* ψ that defines state of an electron. The square of the amplitude of the wave function (ψ^2) gives the probability of finding the electron in a given position. Solving this equation reveals that the energy states of an electron are defined by 3 *quantum numbers* leading to discrete energy levels.

Taking relativity into account predicts a 4th quantum number (*spin*) and the existence of the positive electron (*positron*).

Mass–energy equivalence

As the mass of an electron and positron can be converted into the energy of gamma radiation, mass and energy must be equivalent.

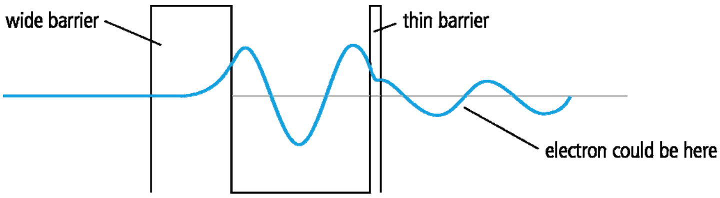
$E = mc^2$

1 u = 931.5 MeV

mass of an electron ≈ 0.5 MeV

Electron tunnelling

If the potential barrier is narrow an electron can exist on the other side without going over the top.



The nucleus

Made of protons and neutrons.

	Mass/kg	Mass/u	Charge
Proton	1.673×10^{-27}	1.007 28	$+1.6 \times 10^{-19}$
Neutron	1.675×10^{-27}	1.008 66	0

Nucleon number (*A*) – protons + neutrons

Proton number (*Z*) – as it says

Neutron number (*N*) – = *A* – *Z*

Isotopes – nuclei with the same *Z* but different *A.*

Nuclear symbol – $^{A}_{Z}X$

Diameter – approximately 10^{-15} m. Can be determined by alpha scattering/closest approach.

Density – all nuclei have the same density so *A* is proportional to radius^3.

Nuclear force

Strong: much stronger than electric force.

Very short range, does not affect particles outside the nucleus.

Particle independent: same for protons and neutrons.

Exchange force – due to exchange of virtual meson. For such a strong force these mesons must have a large mass so can only exist for a short time (Heisenberg), hence short range.

Binding energy

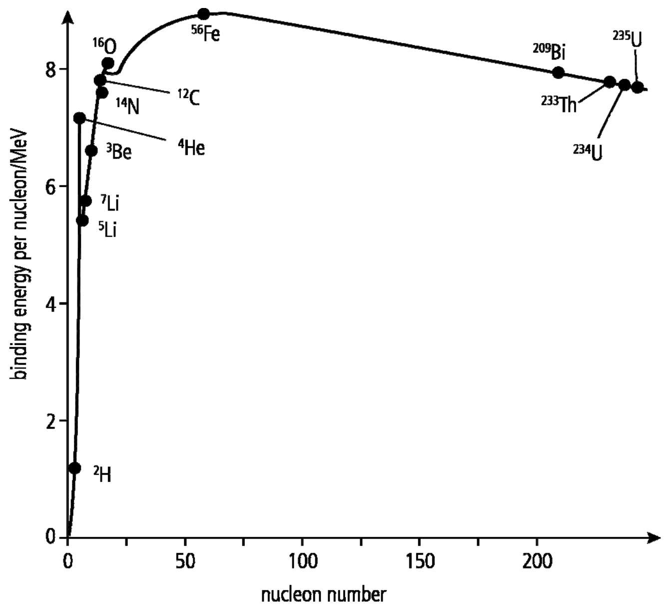
The energy required to pull a nucleus apart or the energy released when put together.

Mass of parts is more than the mass of the nucleus, so some mass is converted into energy. When a nucleus is formed, this energy is released. If a nucleus is pulled apart work must be done, this energy is converted into the mass of the particles.

BE = [mass of parts(kg) – mass of nucleus(kg)]c2
or [mass of parts(u) – mass of nucleus(u)]931.5 MeV

BE per nucleon curve

Used to explain why large nuclei decay and how small nuclei are formed.



Radioactive decay

By changing the constituent nucleons a nucleus can change to one with higher BE, thereby releasing energy. There are 3 common forms of decay (gamma is just the release of energy).

Particle	Mass/u	Charge/*e*	Stopped by
alpha (α)	4	+2	paper
beta (β)	0.0005	–1	aluminium
gamma (γ)	0	0	lead

Alpha (α)

= helium nucleus.

$$^{226}_{88}\text{Ra} \rightarrow {}^{226}_{88}\text{Rn} + {}^{4}_{2}\text{He}$$

($Z = 2$, $A = 4$)
energy released = [mass parent atom(u) – mass daughter atom(u) – mass He(u)] × 931.5 MeV
Use atomic masses and all the electrons cancel out.

Alpha is much smaller than the daughter so takes almost all of the energy. However, sometimes there is more than one discrete alpha energy, suggesting that the *nucleus has energy levels*.

Tunnelling – alphas can get out of the nucleus without going over the potential barrier that holds the nucleons in place. This is called tunnelling and can be explained by considering the wave-like properties of the particles.

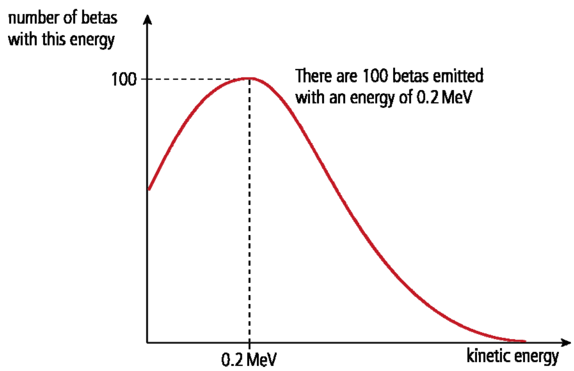
Beta minus (β^-)

Electrons formed when a neutron changes to a proton plus an electron (and antineutrino).

$${}^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + e^{-} + \overline{\nu}$$

(A stays the same, Z increases to $Z + 1$)
energy released =[mass parent atom (u) – mass daughter atom (u)] × 931.5 MeV
Use atomic masses and all the electrons cancel out.

Energy spectrum – betas have a range of energies since the energy is shared between beta and antineutrino.



Beta plus (β^+)

A positive electron (antielectron) is formed when a proton changes into a neutron (and a neutrino).

$${}^{22}_{11}\text{Na} \rightarrow {}^{22}_{10}\text{Ne} + e^{+} + \nu$$

(A stays the same, Z decreases to $Z – 1$)

Energy released is complicated by electron mass so will not be asked for.

Gamma (γ)

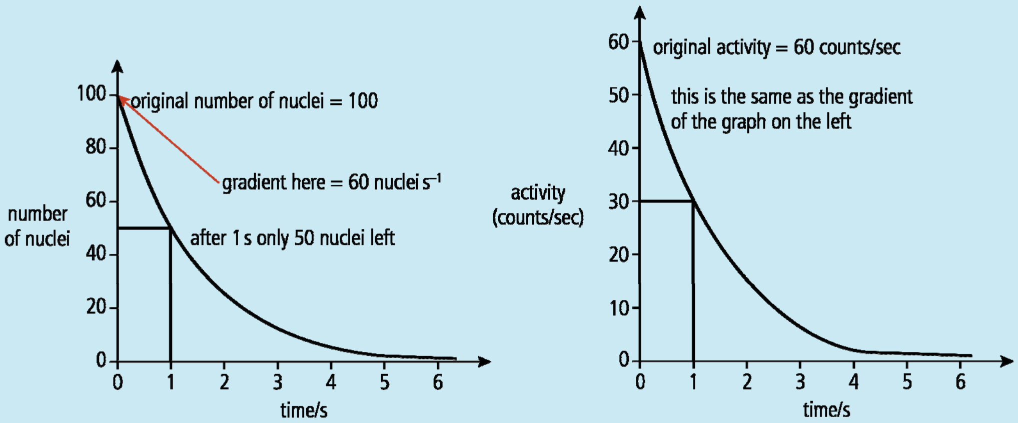
High frequency EM radiation energy loss = hf, no change in nucleons.

Exponential decay

Since radioactive decay is random the number of decays is proportional to the number of nuclei which leads to an exponential decay.

$$\frac{dN}{dt} = -\lambda N$$

$$N = N_0 e^{-\lambda t}$$



Decay constant (λ) – the probability of decay in one second. Gives the rate of decay for a given number of nuclei.

Half- life ($t_{1/2}$) – the time taken for half the nuclei to decay (or activity to halve).

If time for decay is a whole number of half-lives then can solve problems without using exponential equation.

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$$

Activity (A) – the number of decay per second $\left(\frac{dN}{dt}\right)$

Background radiation – radiation of environment, rocks, air, and from the Sun.

Fusion

The joining of small nuclei to make bigger ones with the release of energy. This is the way all nuclei up to iron were made in the cores of stars. To achieve fusion, must have high temperature and density.

$${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n$$

Energy released = [change of mass(u)] × 931.5 MeV

This energy is given to the KE of the products.

Fission

The splitting of large nuclei into smaller ones with the release of energy.

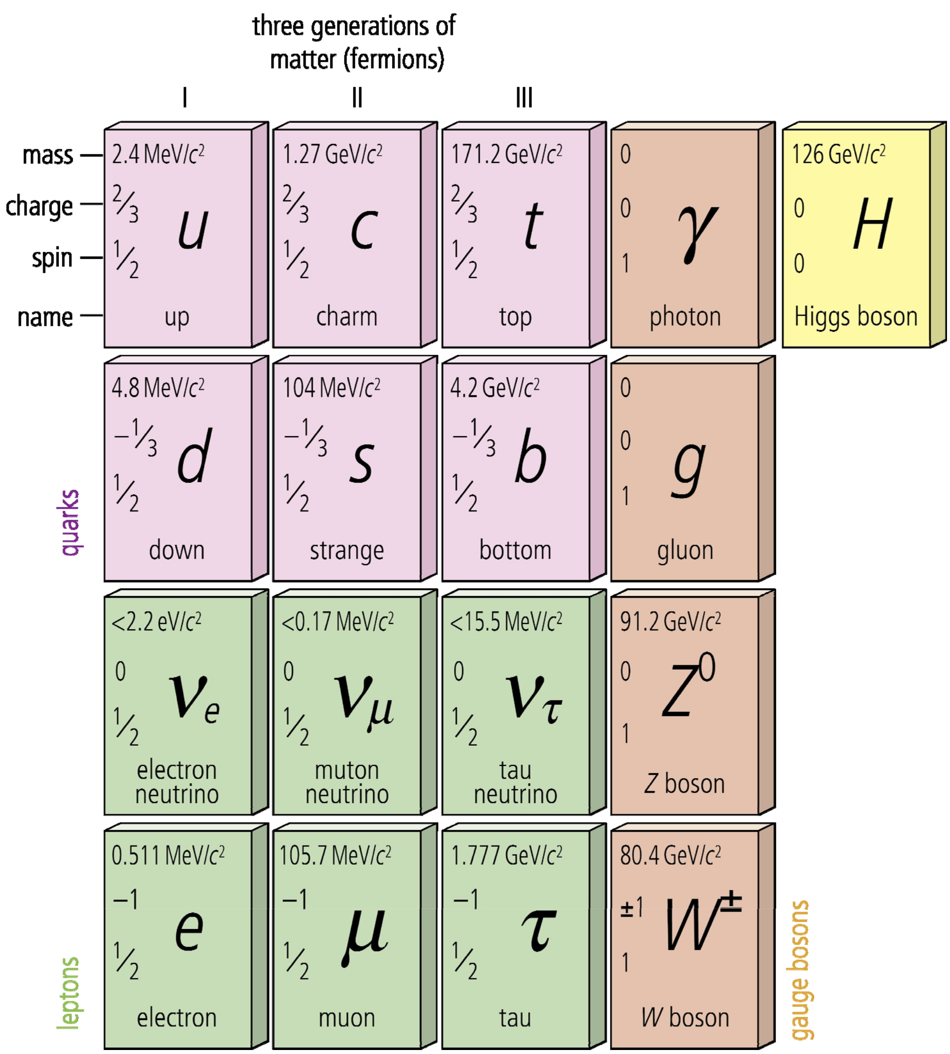
$$^{236}_{92}\text{U} \rightarrow {}^{92}_{36}\text{Kr} + {}^{142}_{56}\text{Ba} + 2n$$

Energy released = [change of mass(u)] × 931.5 MeV

This energy is given to the KE of the products.

The structure of matter

Represented by the standard model.



Quarks

These are the building blocks of all *hadrons* which are split into 2 subgroups: *baryons and mesons*. Quarks have charge and colour so take part in strong and electromagnetic interactions.

6 flavours

Spin $\frac{1}{2}$

charge $\pm\frac{1}{3}$ or $\pm\frac{2}{3}$

3 colours (red, green, and blue + anticolours)

all combinations must be colourless

There are also antiquarks, which have opposite charge.

Baryon –made of 3 quarks (e.g. proton *uud*, neutron *ddu*)

baryon number = 1

charge = multiples of *e*

spin $\frac{1}{2}$ or $1\frac{1}{2}$

Meson – made of a quark + antiquark pair (e.g. pi meson)

baryon number = 0

charge = multiples of *e*

spin = 1 or 0

Confinement – the force required to pull quarks apart is so big that enough energy is transferred to the quarks to produce more quarks so single quarks cannot be observed.

Pauli exclusion principle – particles with spin $\frac{1}{2}$ cannot occupy the same energy state.

Leptons

= electron, muon, tau, and their neutrinos.

Take part in weak interactions and the charged ones also take part in electromagnetic interactions.

baryon number = 0

lepton number = 1 (−1 for antiparticles, e.g. positron)

charge = 0 or ±*e*

spin = $\frac{1}{2}$

Exchange bosons

Responsible for the interactions between particles.

Photon – electromagnetic. According to Heisenberg their energy will be less the longer they live, hence the $\frac{1}{r^2}$ reduction in force.

Gluon – strong (nuclear force is a residual force due to exchange of mesons. However, the mesons are attracted to the nucleons by the strong force). Gluons have colour/anticolour, causing quarks to change colour when exchanged.

Z^0 – weak interactions where there is *no* exchange of charge.

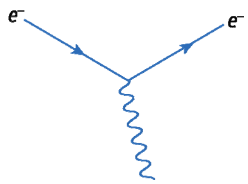
W^+ and W^- – weak interactions with exchange of charge.

Higgs – particle associated with the Higgs field that is responsible for mass.

Force	Strength	Range
strong	1	10^{-15} m
electromagnetic	10^{-2}	infinite
weak	10^{6}	10^{-18} m
gravity	10^{-38}	infinite

Feynman diagrams

- particles straight, exchange particles wavy
- each vertex has 2 particles and 1 exchange particle
- time progresses left to right
- particles are forward, antiparticles backwards
- always one arrow in and one arrow out.



All possible rotations are possible interactions, also can be used to predict the probability of an interaction which is related to the number of ways it can be represented.

Summary facts 8

Energy production

Energy sources

Wood –plants turn CO_2 and water into solid matter that can be burnt to release heat through the process of photosynthesis. Produces CO_2. Renewable.

Coal – dead plants compressed over millions of years. Burnt to release energy. Produces CO_2. Non-renewable.

Oil and gas – microscopic dead organisms collecting on the sea floor covered by sediment and compressed. Can be burnt to produce heat or can be processed into petrol and diesel for use in transport. Produces CO_2. Non-renewable.

Energy density – energy per unit volume (MJ m^{-3})

Specific energy – energy per unit mass (MJ kg^{-1})

Converting heat to work

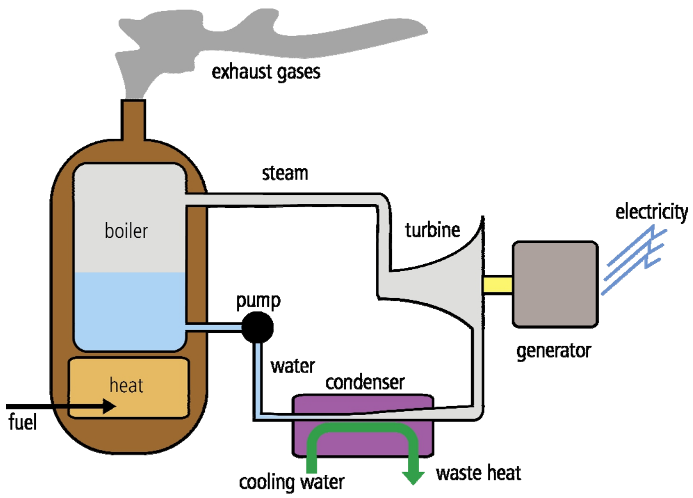
A *heat engine* converts heat to work. A cylinder of gas is heated causing it to expand and do work. The gas is then cooled and compressed back to its original volume ready for the cycle to repeat. To operate, some heat must be transferred from a hot body to a cold one. This principle is used in car engines.

In a *turbine* heat is given to water to turn it into steam, resulting in an increase in pressure. The steam is released onto the blades of the turbine causing it to rotate. This principle is used in most power stations.

Converting mechanical to electrical energy

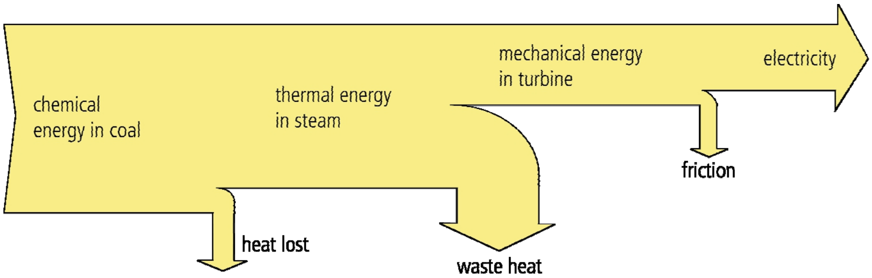
A generator consists of a coil rotating in a magnetic field (or a magnet rotating in a coil). According to Faraday's law an emf is induced when the magnetic flux enclosed by the coil changes.

Coal-fired power station



coal burnt → heat → turns water into steam → turns turbine → turns generator

chemical energy in coal → heat → internal energy of water → mechanical energy of turbine → electrical energy by generator



Nuclear power

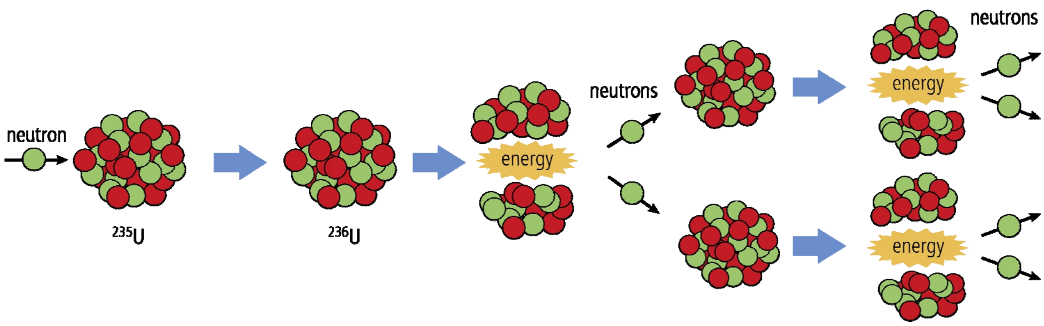
Uses the energy released when large nuclei such as uranium are split into smaller nuclei with higher binding energy.

$$^{236}_{92}\text{U} \rightarrow {}^{92}_{36}\text{Kr} + {}^{142}_{56}\text{Ba} + 2n$$

energy released = [mass of parent(u) – mass of daughters and neutrons(u)] × 932.5 MeV

About 10^8 times more energy released than from burning coal.

Chain reaction is needed to give continuous production of energy.

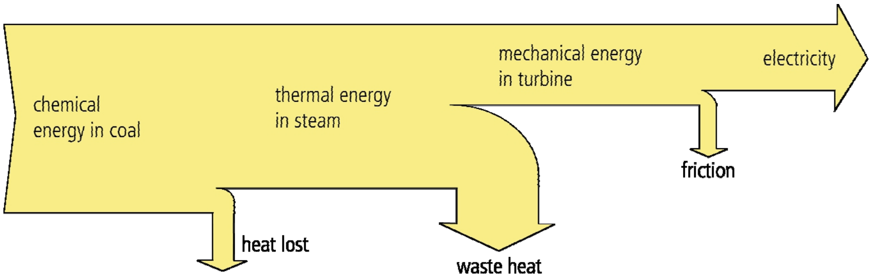


Enrichment – processing of fuel to increase the amount of fissile nuclei (^{235}U).

Moderation – graphite used to slow down neutrons so they are absorbed (this does *not* slow the chain reaction but makes the reaction possible).

Critical mass – the minimum amount of nuclear fuel needed for a chain reaction.

Control rods – boron used to absorb neutrons to slow down the reaction.



nuclear fuel → heat → turns water into steam → steam turns turbine → turbine turns generator

Meltdown – when the core of a reactor gets too hot and melts.

Low level waste – radioactive waste produced during the processing of fuel and parts of the reactor that have become radioactive after exposure to radiation emitted by reactor.

High level waste – the spent fuel contains highly radioactive fission fragments. Needs to be kept away from humans for 100 000 years.

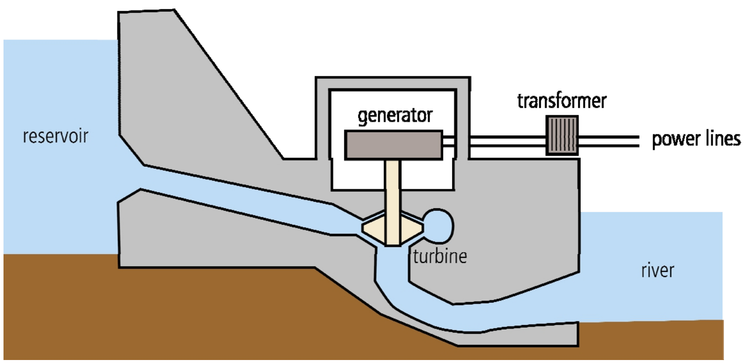
Solar power

Solar heating panel – absorbs radiation from the Sun and converts it into thermal energy.

Photovoltaic cell – absorbs radiation and converts it into electrical energy.

Solar power station – uses mirrors to reflect radiation to a central tower where water is boiled and used to turn a generator to generate electricity.

Hydroelectric power



potential energy in water → mechanical energy in turbine → electrical energy in generator

Can also be used as *pump storage system.*

electrical energy turns motor (generator in reverse) → pump (turbine) moves water into reservoir

Wind power

Turbines placed in windy places.

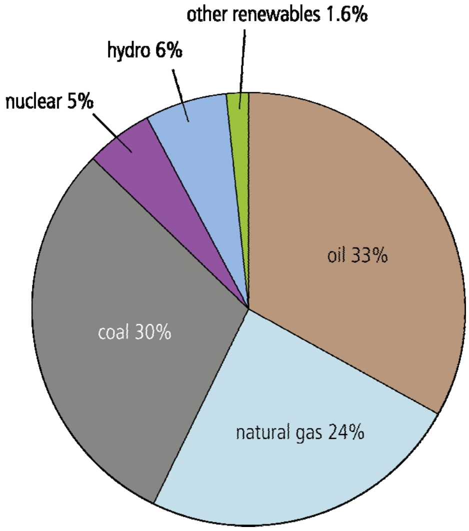
KE in wind → mechanical energy of turbine → electrical energy in generator

Wave power

Oscillating water column – waves cause water to move up and down a column pushing air through a turbine which is connected to a generator.

Pelamis – cylinders connected by joints that bend when waves pass. Movement is used to pump a fluid through a turbine.

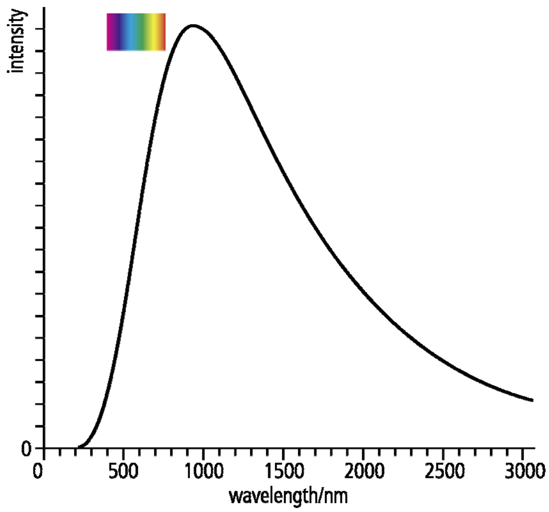
World energy use



Global thermal energy transfer

Energy from the Sun

Sun can be considered to be a black body radiator.



Stefan – Boltzmann law – $P = A\sigma T^4$ can be used to calculate the power radiated per unit area given the temperature.

Emissivity (e) – ratio of energy radiated by body/energy radiated by perfect back body at same temperature. $P = eA\sigma T^4$

Wien's displacement law – $\lambda_{peak} = \frac{0.002\,89}{T}$ can be used to determine the temperature of the Sun from the peak in the spectrum.

Inverse square law – $I = \frac{P}{4\pi r^2}$ used to calculate the power per unit area at a distance from the Sun.

Solar constant – The power per unit area at the Earth (1400 W m^{-2}).

Interaction between radiation and the atmosphere

Ultraviolet – absorbed by ozone layer.

Infrared – excites molecules of water, methane, and carbon dioxide (greenhouse gases). After absorption energy is re-emitted in random directions.

Visible light – mostly passes through.

Interaction between radiation and the ground

The ground reflects about 30% of the incident radiation.

$$\text{albedo} = \frac{\text{total scattered power}}{\text{total incident power}}$$

Absorbed radiation causes the temperature of the surface to rise.

$$\text{surface heat capacity } (C_s) = \frac{Q}{A\Delta T}$$

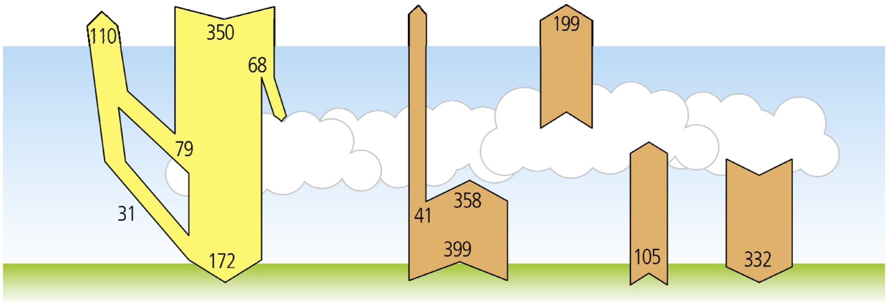
The Earth also radiates but not as a black body.

$$P = eA\sigma T^4$$

Thermal equilibrium is reached when the power absorbed = power radiated.

The greenhouse effect

The Earth radiates EM radiation in the IR region of the spectrum. This is absorbed by the greenhouse gases in the atmosphere and re-radiated in all directions. Some radiation goes back to the Earth. Since less energy is leaving, the temperature of the Earth increases. This increases the amount of energy radiated until thermal equilibrium is re-established.



Enhanced greenhouse effect

The use of fossil fuels increases the concentration of carbon dioxide in the atmosphere so more IR radiation is absorbed leading to an increase in the surface temperature. Can be reduced by using less fossil fuel or replacing with alternative forms of energy.

Summary facts 9

Relativity

Galilean relativity

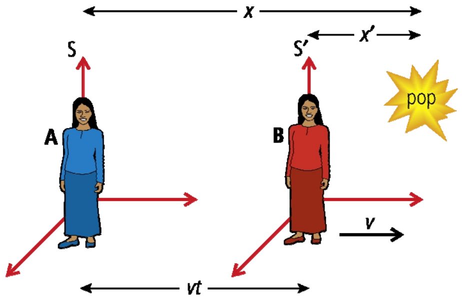
Reference frame – a system of coordinates covered in clocks.

Observer – an experimenter committed to make measurements in their frame of reference.

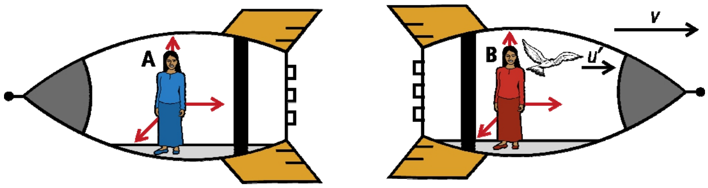
Event – some observable change that takes place at a point in space at a moment in time.

Inertial frame of reference – a frame of reference within which Newton's laws of motion apply.

Coordinate transform – used to calculate the position and time an event would be in someone else's frame of reference.



A (S)	B (S′)	Transformation
x	x'	$x = x' + vt$
y	y'	$y = y'$
z	z'	$z = z'$
t	t'	$t = t'$



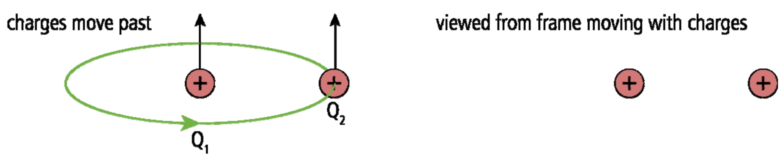
A (S)	B (S′)	Transformation
u	u'	$u = u' + v$

Maxwell discovered that light was an electromagnetic wave with a velocity that is always the same as measured by any inertial observer. This is *not* in agreement with the Galilean transformations.

Although the mass of light is zero, it has momentum.

$$p = \frac{E}{c}$$

When viewed classically, the forces between two moving charges seem to have different origins.



Special relativity and the Lorentz transformations

First postulate – the laws of physics are the same in all inertial frames of reference.

Second postulate – the speed of light in a vacuum is the same as measured by all inertial observers.

Lorentz transforms

= an adaption of the Galilean transforms to make the velocity of light the same for all inertial observers.

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$ (= the *Lorentz factor*).

These can also be written in terms of time interval Δt and distance between two events, Δx .

$$\Delta x' = \gamma(\Delta x - v\Delta t)$$
$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$$

Time dilation – (moving clocks tick more slowly) $T = \gamma T_0$

Proper time – the time interval between two events measured by the same clock (the time of an event has to be measured by a clock at the position of the event).

Length contraction – (moving objects are shorter) $L = \frac{L_0}{\gamma}$

Proper length – the length measured by an observer at rest relative to the object.

Invariance – measurements that all observers will agree on (e.g. proper length and time).

Simultaneity – events that are simultaneous in one frame of reference are not simultaneous in other frames of reference (unless they occur at the same position).

Addition of velocity –

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

With this transformation the velocity of light is the same as measured by all inertial observers. No matter how fast bodies approach each other they will never have a relative velocity greater than *c*.

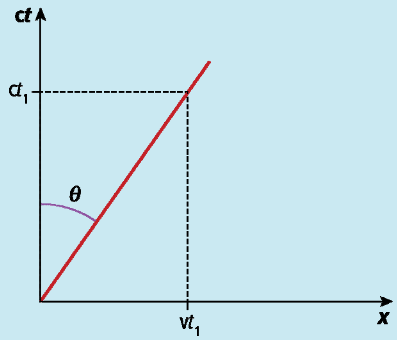
The muon experiment – classical approach predicts that very few muons should reach the Earth but the relativistic approach can explain that due to time dilation the muons have a longer half-life than expected.

Space–time interval $(ct')^2-(x')^2=(ct)^2-(x)^2$ is invariant.

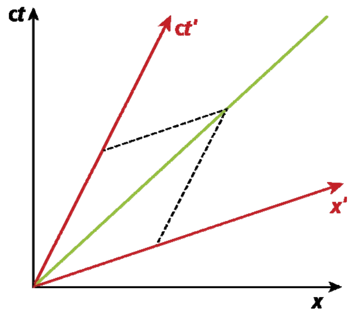
Space–time diagrams

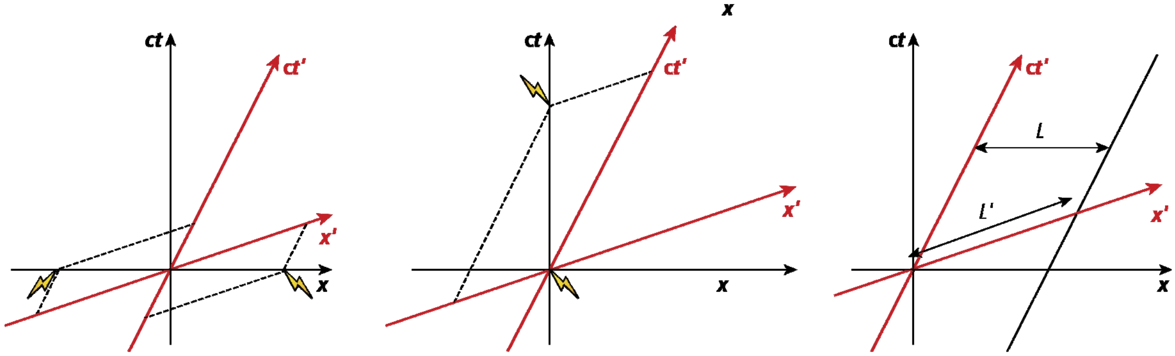
= a way of representing events in space and time. Vertical axis (ct) represents time converted to the distance light would travel. Horizontal axis (x) is space.

$$\tan\vartheta=\frac{c}{v}$$

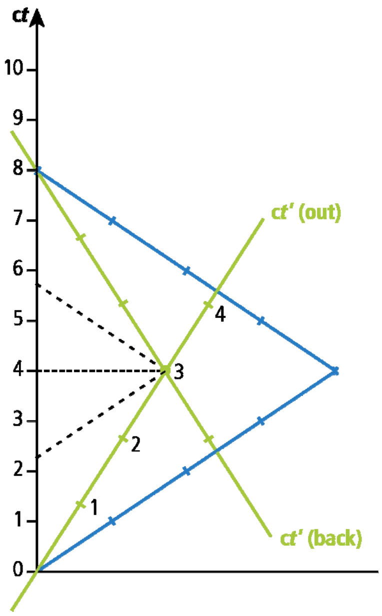


Different frames of reference can be represented on the same graph by drawing two sets of axes. Time and position are found by drawing lines parallel to the axes.





The twin paradox – apparent paradox that if one twin goes on a long trip and comes back, each twin would be older than the other. Can be resolved by looking at the space–time diagram.



Relativistic mechanics

Energy and mass are equivalent.

momentum (p) = $\gamma m_0 v$

kinetic energy = $(\gamma - 1)m_0c^2$

rest energy = m_0c^2

total energy = $\gamma m_0 c^2$

$$E^2 = p^2c^2 + m_0^2c^4$$

Calculations are easy if you use *MeV*

energy	MeV
mass	MeVc^{-2}
momentum	MeVc^{-1}

Calculate the momentum of an electron accelerated to a total energy of 2 MeV.

Rest mass of electron = 0.5 MeV c^{-2}

$E^2 = p^2c^2 + m_0{}^2c^4$

$(2 \text{ MeV})^2 = p^2c^2 + (0.5 \text{ MeV } c^{-2})^2c^4$

$p^2c^2 = 4 - 0.25 = 3.75$ MeV

$p = 1.9$ MeV c^{-1}

Neutral pion decay – KE and rest energy converted into the energy of two photons. To conserve energy and momentum the two photons cannot travel in the same direction.

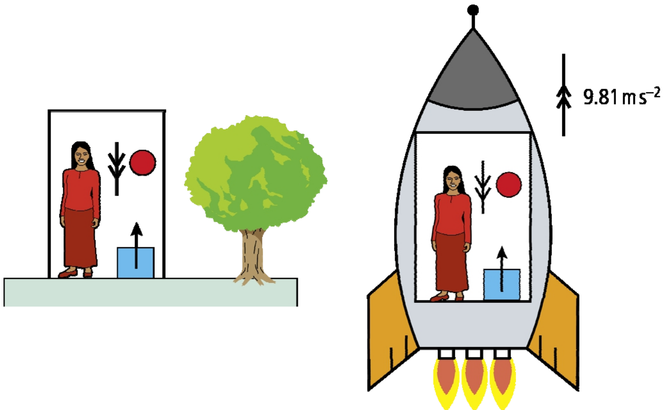
Pair production – particle and antiparticle produced from high energy gamma photon. To conserve energy and momentum there must be a recoiling nucleus.

General relativity

The equivalence principle

No observer can determine by experiment whether they are in an accelerating frame of reference of a gravitational field.

Also cannot distinguish whether in free fall or floating in space.

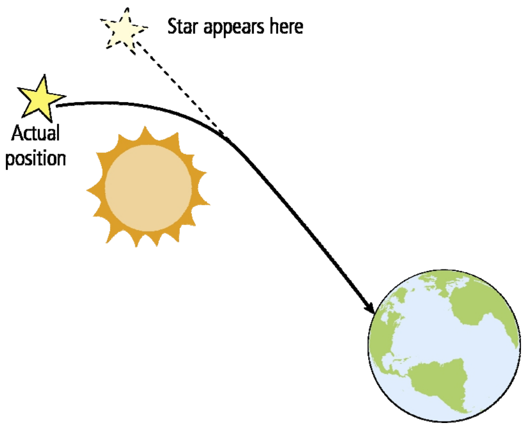


gravitational mass = *mg*

inertial mass = *ma*

These quantities are the same.

Bending of light by gravity – if light were sent across an accelerating spaceship it would have a curved path so should also be curved by a gravitational field. This leads to the idea that large masses curve space–time. Light can be seen to be bent by the Sun.

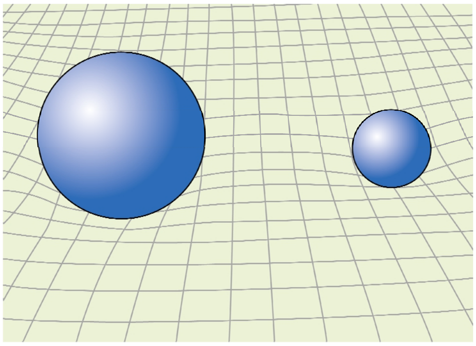


Gravitational lensing – bending of light around distant galaxies causes an Einstein ring.

Gravitational red shift – in the Pound–Rebka experiment the frequency of radiation travelling towards the top of a tower was red-shifted. This can be explained in terms of its loss of energy as it moves upwards.

$$\frac{\Delta f}{f} = \frac{g\Delta h}{c^2}$$

Curvature of space–time – can be used to explain gravitational attraction without a force being involved.



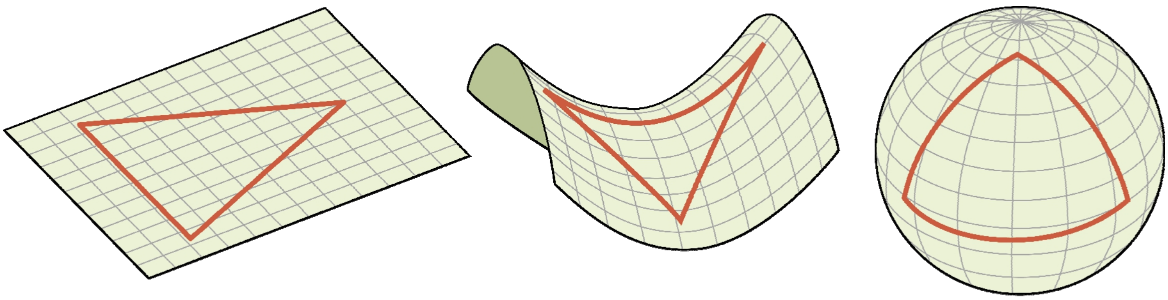
Black hole – a collapsed star that is so dense that the escape velocity is greater than the speed of light.

Schwarzschild radius – if within this distance from a black hole, light cannot escape. At the Schwarzschild radius the escape velocity is *c*.

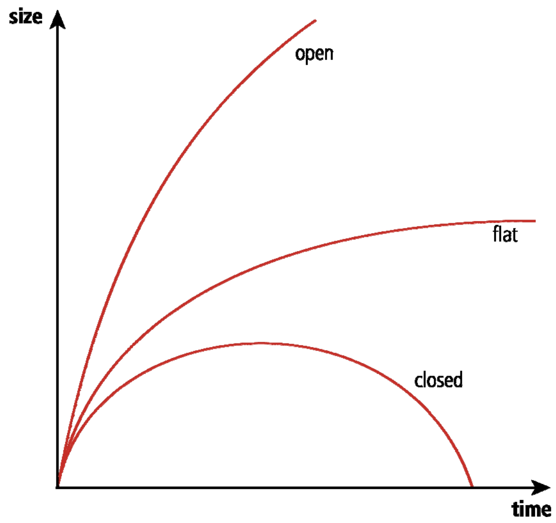
$$R_s = \frac{2GM}{c^2}$$

$$\text{time dilation near a black hole } (\Delta t) = \frac{\Delta t_0}{\sqrt{1 - \frac{R_s}{r}}}$$

Curvature of the Universe – the theory of general relativity predicts that the Universe could have different curvatures depending on several parameters such as the density of matter.



The curvature determines the fate of the Universe.



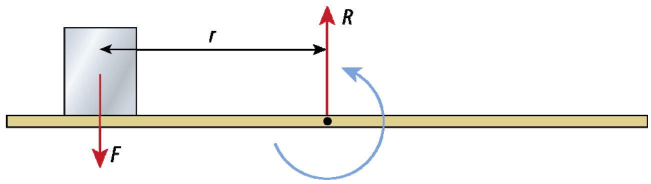
Summary facts 10

Engineering physics

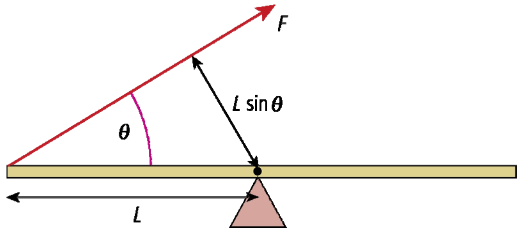
Rigid bodies and rotational dynamics

torque (Γ) = force $\times$ perpendicular distance to pivot

$\Gamma = F \times r$



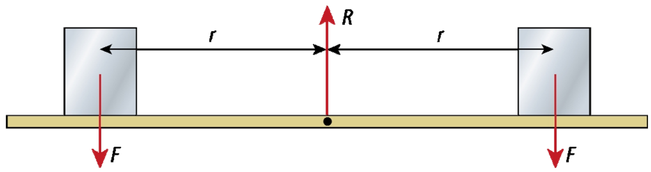
If force not perpendicular then use component: $F \sin\vartheta \times L$



Equilibrium – if torques balanced about any point (clockwise torques = anticlockwise torques)

and forces also balanced.

$Fr = Fr$
$2F = R$



angular displacement (ϑ) = angle swept out by radius

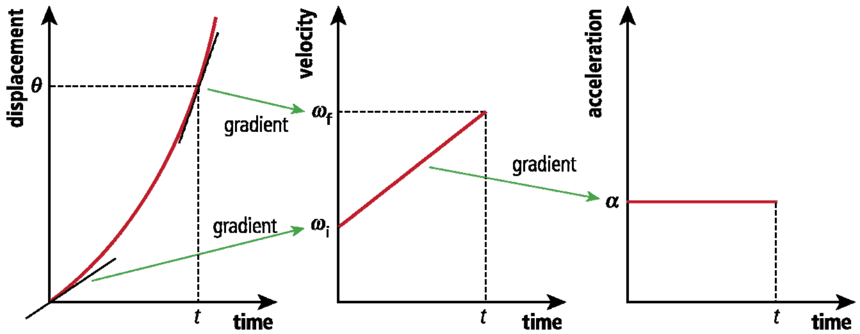
angular velocity (ω) = angle swept out per unit time

angular acceleration (α) = rate of change of angular velocity

Equations for constant angular acceleration – same as *suvat* but linear terms swapped for angular ones.

Angular	Linear
$\omega_f = \omega_i + \alpha t$	$v = u + at$
$\omega_f^2 = \omega_i^2 + 2\alpha\theta$	$v^2 = u^2 + 2as$
$\theta = \omega_i t + \frac{1}{2}\alpha t^2$	$s = ut + \frac{1}{2}at^2$

Graphical representation



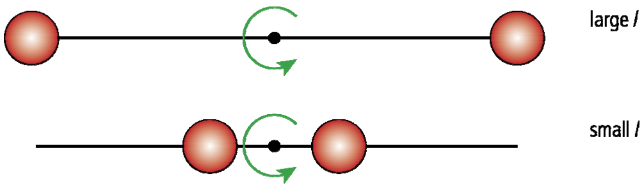
Relationship between angular motion and linear motion

tangential velocity $v = \omega r$

tangential acceleration $a = \alpha r$

Newton's second law for angular motion ($\Gamma = I\alpha$)

Where *rotational inertia* $I = \Sigma mr^2$ a measure of how spread out the mass is from the centre.



rotational KE = $\frac{1}{2}I\omega^2$

work done = $\Gamma\vartheta$

As a ball rolls down a hill the PE is converted into translational KE + rotational KE.

angular momentum (L) = $I\omega$

Conservation of angular momentum

If no external unbalanced torques act then the angular momentum of a system of bodies is conserved.

$I_i\omega_i = I_f\omega_f$

Thermodynamics

Consider a piston containing a fixed mass of monatomic gas.

Internal energy (U) = the KE of all the molecules = number of molecules × average KE

$N \times \frac{3}{2}kt$

For n moles of gas, $U = \frac{3}{2}nRT$

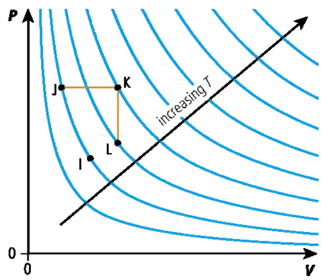
Work done (W) if pressure is constant $W = P\Delta V$

Heat (Q) – heat can be added by placing the gas in thermal contact with something hot.

First law of thermodynamics $Q = \Delta U + W$

If you add heat to a gas it will get hot and do work.

Use P–V diagram to see if work is done and temperature changed, then use 1st law to decide if heat is added or removed.



L→K Isochoric temperature rise (constant volume) – no work done so heat added increases the temperature.

$Q = \Delta U$

J→K Isobaric expansion(constant pressure) – gas does work and gets hot so heat must be added.

$Q = \Delta U + W$

J→I Isothermal expansion (constant temperature) – gas does work but no change in temperature heat must be added.

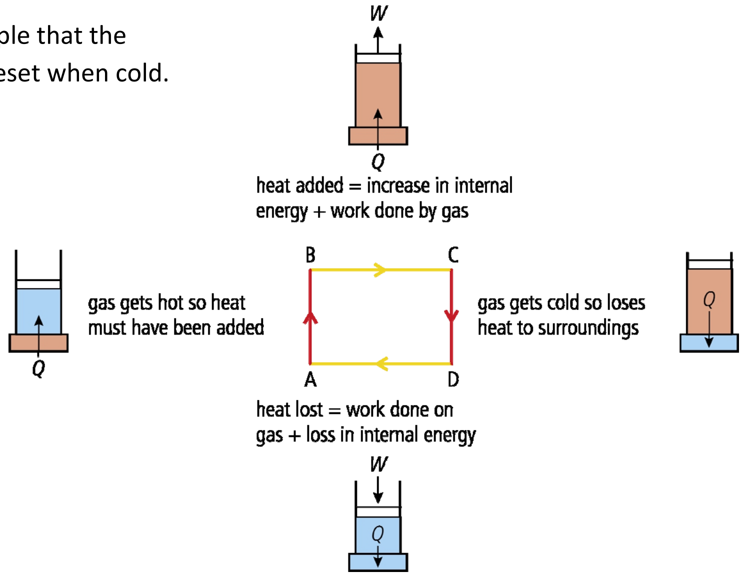
$Q = W$

M→N Adiabatic Compression (no heat exchanged) – work done on gas temperature rises.

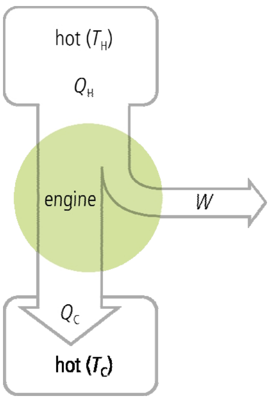
$\Delta U = W$

Cyclic processes

A heat engine operates on the principle that the gas does work when it is hot and is reset when cold.

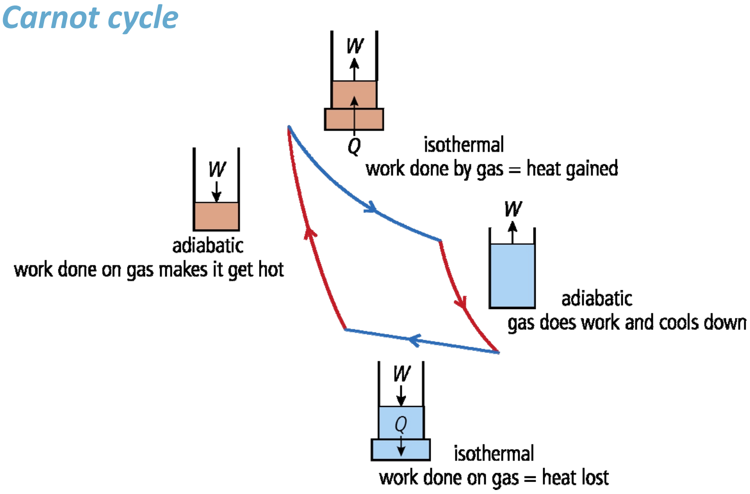


Energy flow



$$\text{thermal efficiency } \eta = 1 - \frac{Q_C}{Q_H}$$

Carnot cycle



$$\text{thermal efficiency } \eta = 1 - \frac{T_C}{T_H}$$

Second law of thermodynamics

It is not possible for a heat engine working in a cycle to absorb thermal energy and convert it *all* to work (Kelvin–Planck).

In a cyclic process the entropy will either stay the same or increase.

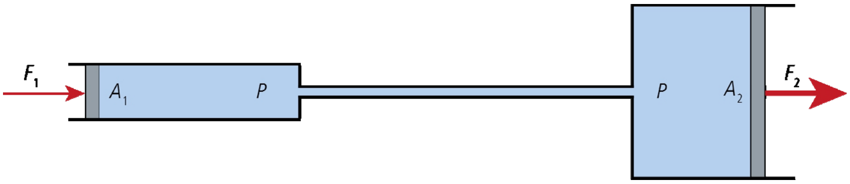
Entropy (S) – a measure of the disorder in a system (the number of ways that energy can be shared).

$$\Delta S = \frac{Q}{T}$$

It is not possible for heat to be transferred from a cold to a warmer body without work being done. (Clausius)

Fluid statics

Pascal principle – pressure applied to a confined fluid increases the pressure throughout the fluid.

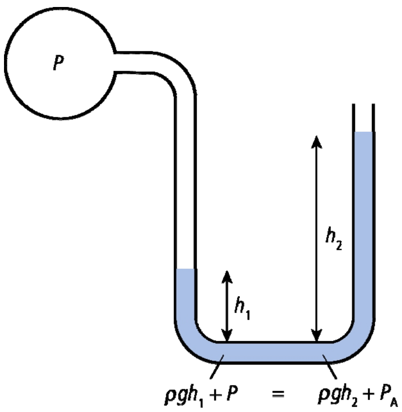


Hydrostatic equilibrium – parts of a fluid don't move about when the forces on them are balanced (e.g. buoyancy = weight).

Pressure at depth – $P = \rho g h$

pressure at depth with atmosphere = $\rho g h + P_A$

U-tube manometer



$$P = \rho g(h_2 - h_1) + P_A$$

Archimedes principle – the **buoyant force on a body immersed in a fluid is equal to the weight of fluid displaced.**

Fluid dynamics

Ideal fluid – incompressible and zero viscosity.

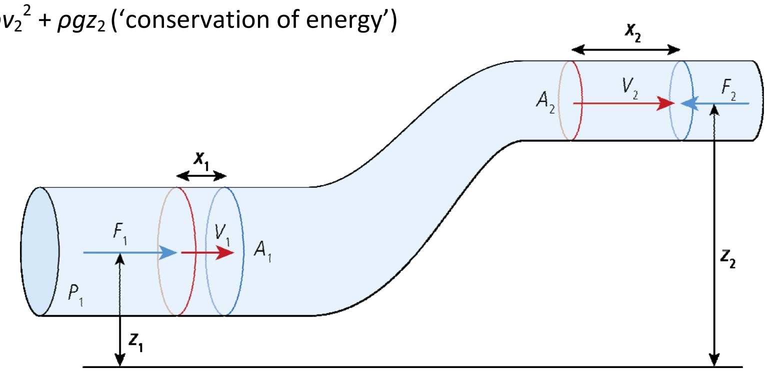
Flowline – the path of a particle of fluid.

Streamline – line whose tangent gives the velocity of a fluid at any point.

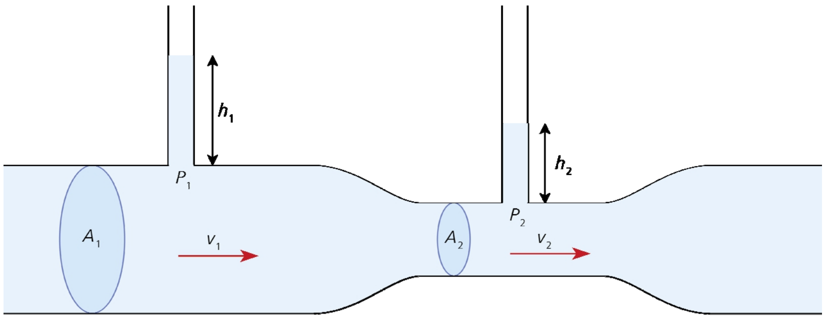
Steady flow – where the flowlines are the same as the streamlines.

Continuity equation – $A_1v_1 = A_2v_2$ ('what comes in must go out')

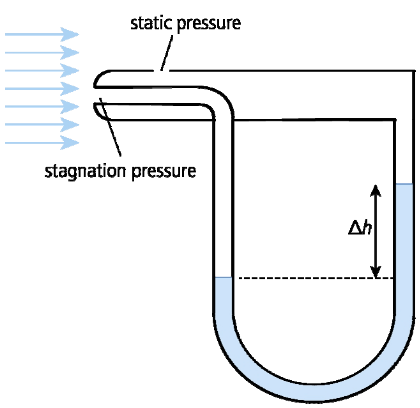
Bernoulli equation – $P_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2$ ('conservation of energy')



Venturi meter $-g\Delta h = \frac{1}{2}v_1^2\left[\left(\frac{A1}{A2}\right)^2 - 1\right]$



Pitot static tube – $P_2 - P_1 = \frac{1}{2}\rho v^2$



Viscosity (η) – force between two plates moving parallel to each other through a fluid $F=\eta A\frac{v}{L}$.

Stokes law $-F=6\pi\eta rv$

Reynolds number $-Re=\frac{vrp}{\eta}$ flow in a pipe becomes turbulent if $R_e > 1000$.

Damped harmonic motion

= when a force opposes the motion of a body executing SHM.

Light damping – amplitude reduces exponentially. $x=ae\left(\frac{-b}{m}\right)^t\cos\omega t$

Critical damping – body returns to equilibrium position as quickly as possible without crossing it.

Over damping – displacement from equilibrium is exponentially related to time (doesn't oscillate but returns to equilibrium position slowly).

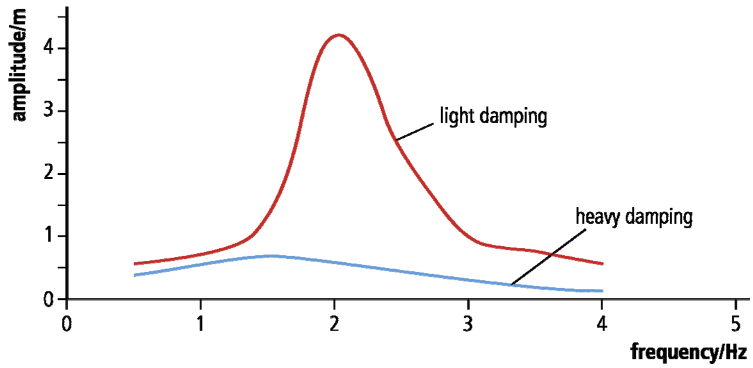
Q factor – $Q=\frac{\text{energy stores}}{\text{energy lost per cycle}}$.

Forced vibrations and resonance

When a system is forced to oscillate by a sinusoidally varying force.

Resonance – when the driving frequency = natural frequency.

Resonance curve



$$Q \text{ value } -Q=2\pi\times\text{resonant } f\times\frac{\text{energy stored}}{\text{energy loss}}$$

Phase difference

Driver lower f than natural – π phase difference.

Driver same f as natural – $\frac{\pi}{2}$ phase difference.

Driver higher f than natural – 0 phase difference.

Summary facts 11

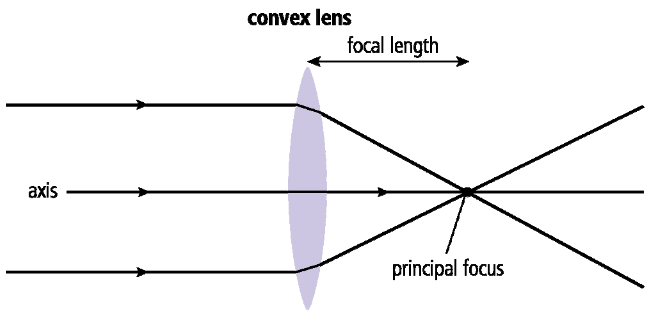
Imaging

Lenses

Convex – converging (+focal length)

$$Power = \frac{1}{f}$$

Concave – diverging (– focal length)



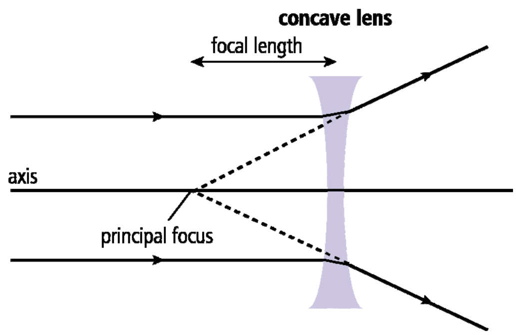
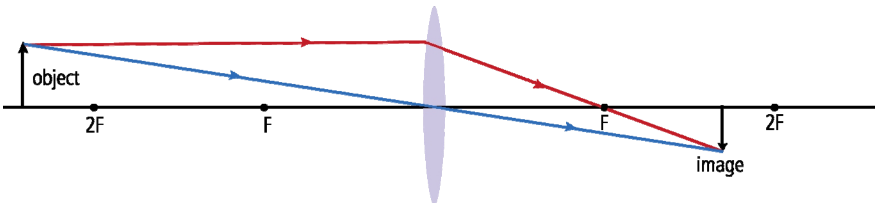


Image formation convex

further than *f* – smaller, closer, inverted, real.



Closer than *f* – larger, further, upright, virtual.

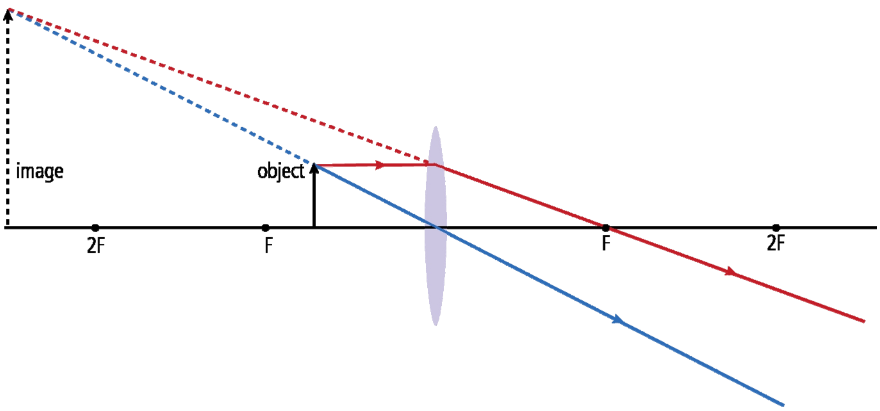
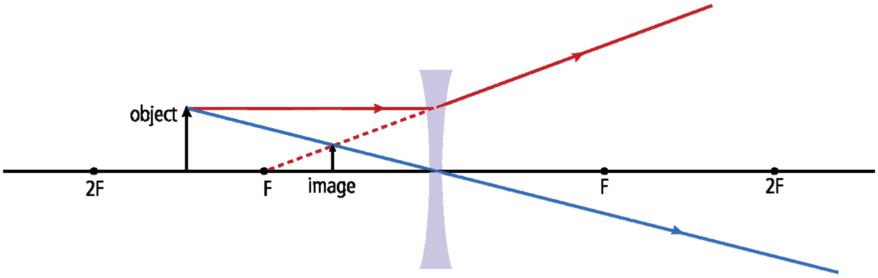


Image formation concave

Smaller, closer, upright, virtual.



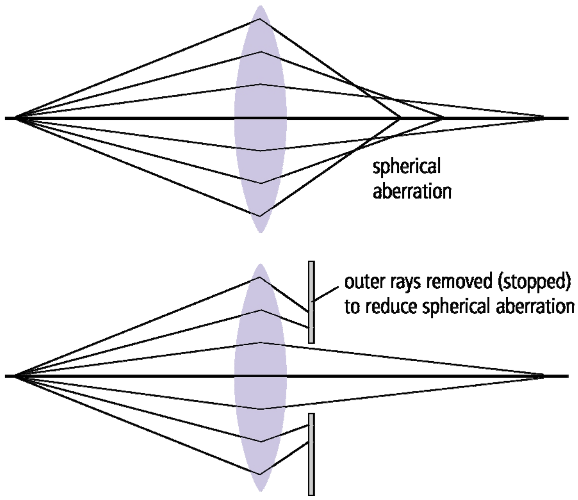
lens formula $-\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$ 'real is positive'

linear magnification m = $\frac{\text{height of image}}{\text{height of object}}=\frac{v}{u}$ (+ve upright, –ve inverted)

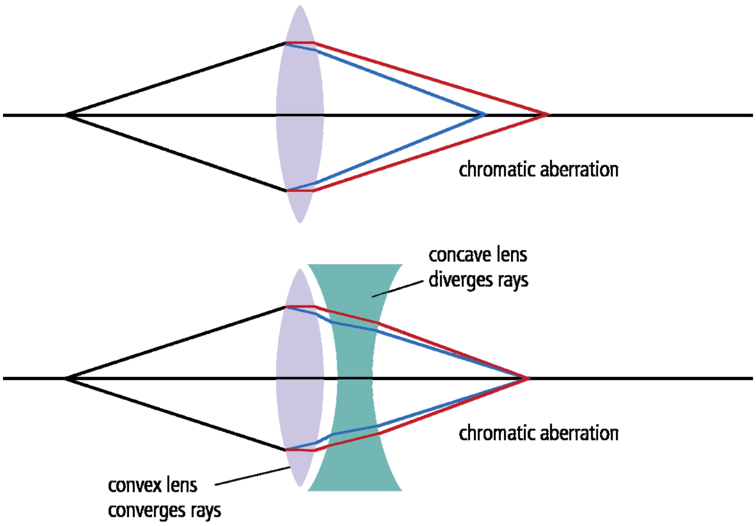
angular magnification M = $\frac{\text{angle subtended by image}}{\text{angle subtended by object at unaided eye}}$

For magnifying glass M = $\frac{f}{25}$ (image at infinity) M= 1 + $\frac{f}{25}$ (image at near point).

Spherical aberration – fixed by stopping lens.



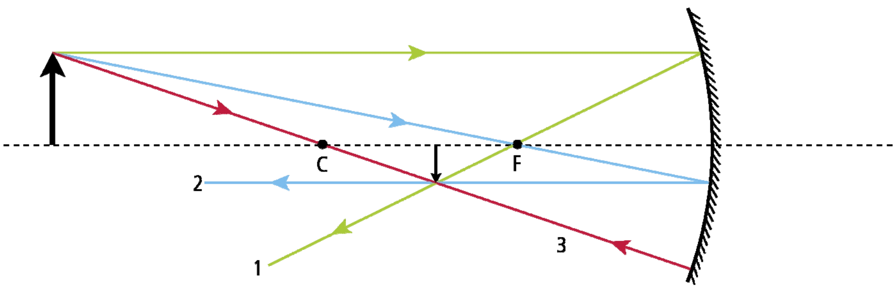
Chromatic aberration – fixed by achromatic doublet. Light most converged by convex but most dispersed by concave causing colours to focus at same place.

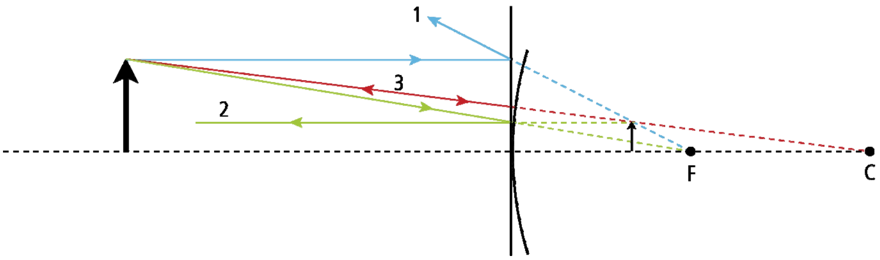


Mirrors

Convex – diverging

Concave – converging



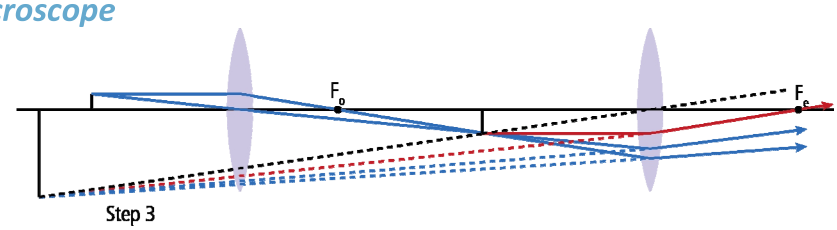


mrror formula – $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$ 'real is positive'

linear magnification m = $\frac{\text{height of image}}{\text{height of object}}=-\frac{v}{u}$ (+ve upright, –ve inverted)

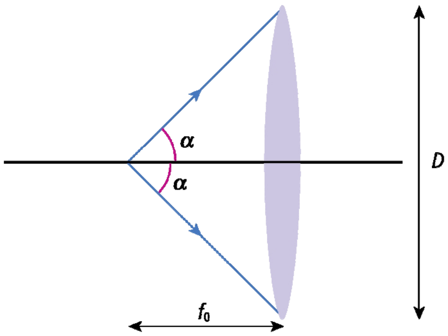
Imaging instrumentation

Compound microscope

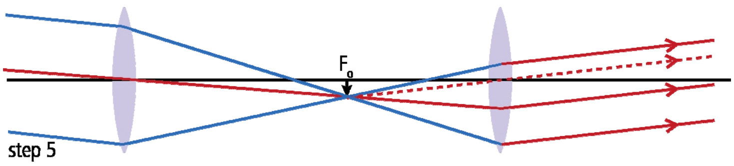


angular magnification = $m_e \times m_o$

Resolution – distance between resolvable points $d = \dfrac{0.61\lambda}{\sin\alpha}$

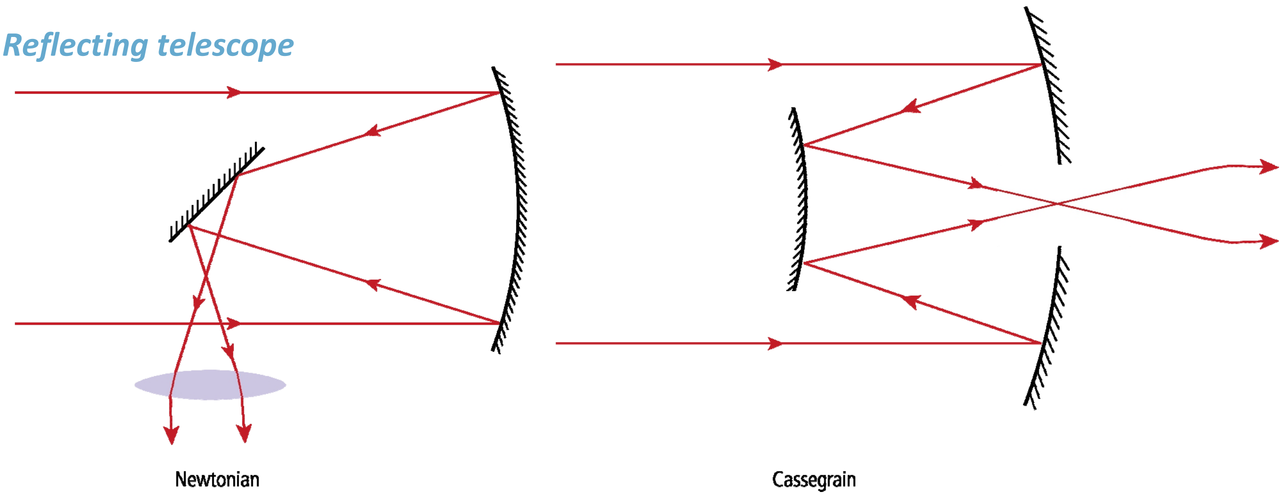


Astronomical telescope



Angular magnification $M = \dfrac{f_0}{f_e}$

Reflecting telescope

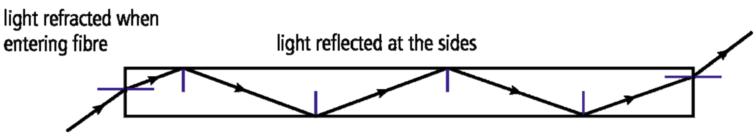


Single dish radio telescope – large wavelength so large dish required for high resolution. Image built up by scanning object.

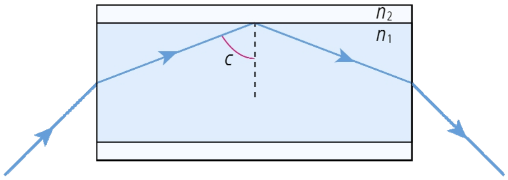
Radio interferometer – two dishes point at same object, phase difference between signals can be used to build image.

Fibre optics

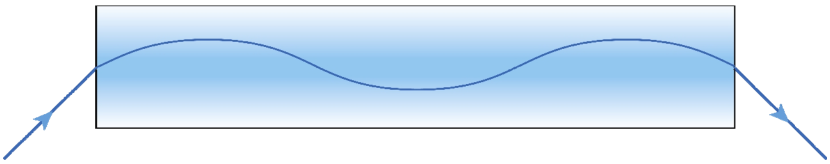
Light totally internally reflects along fibre if angle of incidence at side is greater than $c = \sin^{-1}\left(\frac{1}{n}\right)$



Step indexed – have a low *n* cladding.



Graded index – *n* gets gradually less towards the edge of the fibre– less waveguide dispersion.



Monomode fibre – very thin so waveguide dispersion is minimal.

$$\text{attenuation} = 10\log_{10}\left(\frac{P}{P_0}\right)$$

Medical imaging

X rays

Interaction of X rays with matter – Compton scattering related to *Z* of absorber. Photoelectric absorption related to energy of electrons and *Z*. So if absorber has high *Z* it absorbs X-rays more effectively.

$$\text{attenuation} = 10\log_{10}\left(\frac{I}{I_0}\right)$$

Linear attenuation coefficient (μ) – fractional decrease in intensity per cm.

$$I = I_0 e^{-\mu x}$$

Half-value thickness – the thickness that would reduce intensity by half.

$$X_{\frac{1}{2}} = \frac{0.693}{\mu}$$

Improving image

Collimating the beam – gives a clearer image.

Reducing beam width – removes X-rays scattered from the edges.

Using a grid – removes scattered X-rays.

Barium meal – increases contrast.

Image intensifier – increases intensity.

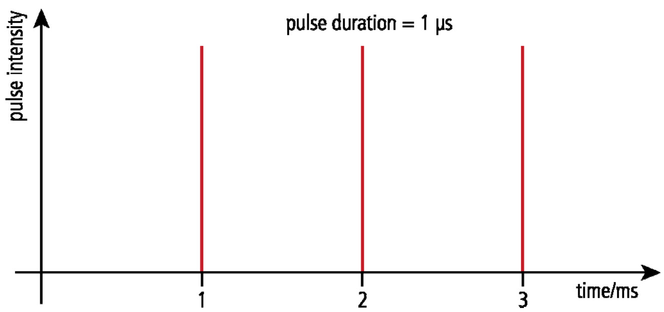
Ultrasound

High frequency sound >20 kHz

Produced and detected by the piezoelectric effect. When alternating pd applied, crystal vibrates and when vibrated produces an alternating pd.

Ultrasound waves reflect off the boundary between different tissues.

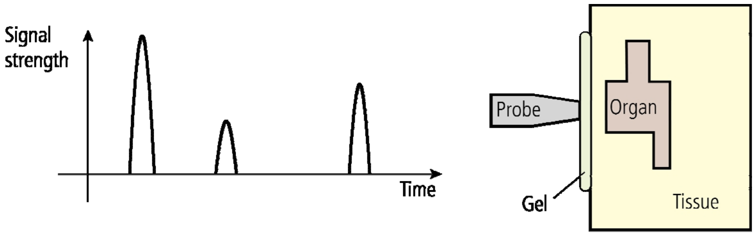
Signal is a series of pulses. *Pulse length* must be such that transmitted pulse is finished before first reflection is received. *Time between pulses* must be such that last reflection must have returned to detector before next pulse is sent.



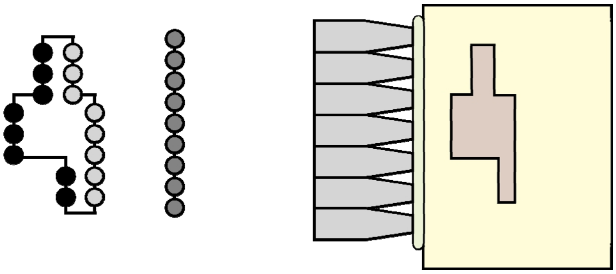
Acoustic impedance – $Z = \rho c$. More ultrasound is reflected if the tissues have very different acoustic impedances.

$$\frac{I_r}{I_0} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$$

A-scan – graph showing reflected pulses.

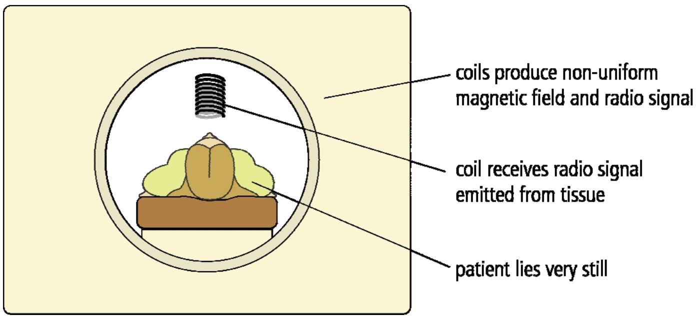


B-scan –image made of dots.



Nuclear magnetic resonance (NMR)

when placed in a magnetic field hydrogen nuclei can have spin up or spin down → spin down is at a higher energy than spin up → radio frequency (rf) EM waves can excite the nucleus from up to down → radio waves are emitted when the nucleus goes from spin down to spin up → the frequency of radio waves required to excite the nuclei is related to magnetic field → by placing the body in a non-uniform field it is possible to find out where the radio waves emitted come from → different tissue types have different relaxation times → by measuring the relaxation times and frequencies it is possible to build up an image of the different types of tissue



	Advantages	Disadvantages
X-ray	High quality image Quick and relatively cheap	X-rays are ionizing so over exposure by patient or radiologist is dangerous can't be used on pregnant women not so good for viewing soft tissue

	Advantages	Disadvantages
Ultrasound	not ionizing so not dangerous, can be used to view unborn babies cheap	not all organs can be viewed not very high resolution
MRI	not ionizing so not dangerous to patient or staff high-quality image bones don't get in the way of radio waves so particularly good for viewing the brain good contrast between different types of soft tissue	expensive each scan takes a long time (about 45 minutes)

Summary facts 12

Astrophysics

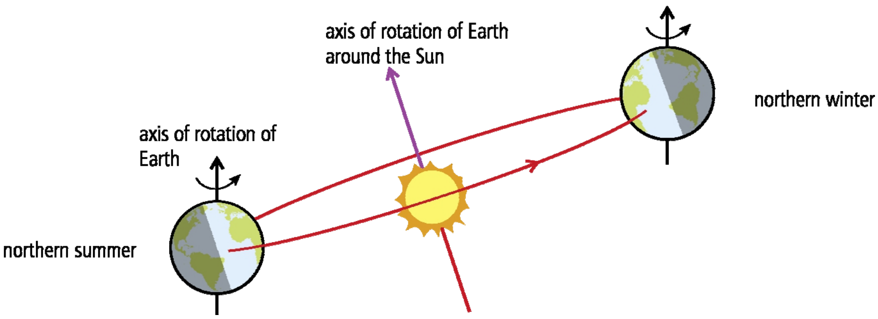
Objects in the Universe

The Sun = our closest star

Orbited by the Earth and other planets.

time period of Earth's orbit = 365.35 days

Tilt of axis causes seasons.



The Moon – orbits the Earth, time period 27.3 days.

Planet – a celestial body in orbit around the Sun that dominates its neighbourhood.

The planets closest to the Sun have the shortest time periods (Mercury is closest, Neptune is furthest).

Asteroids – lumps of rock that orbit the Sun between Mars and Jupiter.

Comet – a lump of rock and ice with an elongated elliptical orbit. Becomes visible when close to the Sun as the solar wind blows ice off in the form of a tail.

Stars – closest (not counting the Sun) Alpha Centauri (4 ly).

Position relative to each other is almost fixed, they move a bit due to parallax.

Binary star – two stars that orbit each other.

Cepheid variable – a star with variable brightness.

Red giant – a large red star (obviously).

White dwarf – (guess)

Constellation – a pattern made out of stars.

Stellar cluster – a group of stars that are near to each other.

Galaxy – a very large group of stars (hundreds of billions) 10^5 ly across separated by 10^6 ly.

Galaxy cluster – a group of galaxies.

The observable Universe – everything that you can see. This has a finite size 10^{11} ly but the whole Universe could be infinite.

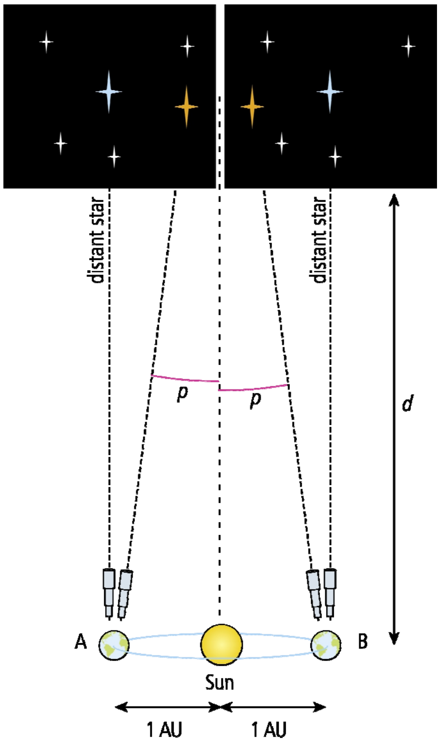
Astronomical distances

Astronomical unit – the average distance between the Sun and the Earth (1 AU = 1.5×10^{11} m).

Light year – the distance travelled by light in 1 year (1 ly = 9.46×10^{15} m).

Parsec – the distance to two points that subtend an angle of 1 arcsec (1pc = 3.26 ly).

Stellar parallax – a way of calculating the distance to a star by measuring its angular shift relative to distant stars as the Earth orbits the Sun. Only used up to 100 pc.



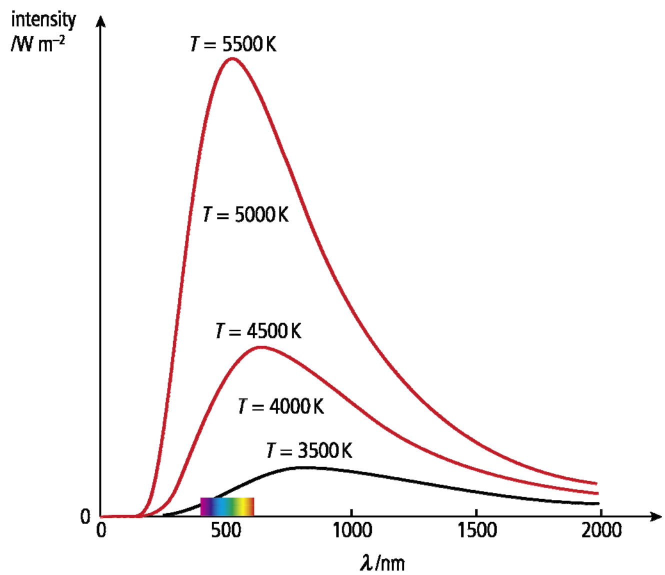
$$d(\text{pc}) = \frac{1}{p}(\text{arcsec})$$

Light from the stars

Luminosity (L) – the total power radiated from a star. The Sun has a luminosity $L_\odot = 3.84 \times 10^{26}$ W. The luminosity of other stars is often quoted as a multiple of this value $L_\odot$.

Apparent brightness (b) – power per m^2 $\left(b = \frac{L}{4\pi d^2}\right)$

Stellar spectra – as from a black body.



Stefan–Boltzmann law – $L = A\sigma T^4$

Wein's displacement law – $\lambda = \dfrac{0.0029}{T}$

Absorption lines – as light passes through the outer layers of the star some wavelengths are absorbed giving information about the chemical composition of the star (assume since star material is all mixed up then the outer part is made of the same stuff as the inside).

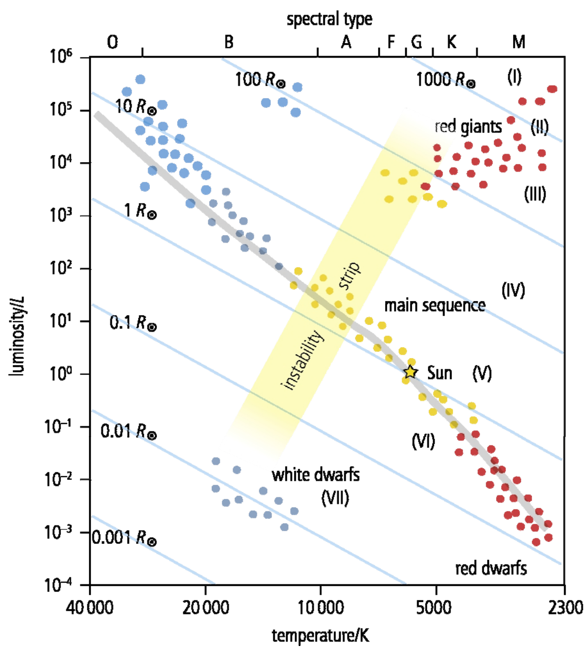
> 72% hydrogen, 25% helium

The temperature of a star can be found from the relative brightness of the spectral lines.

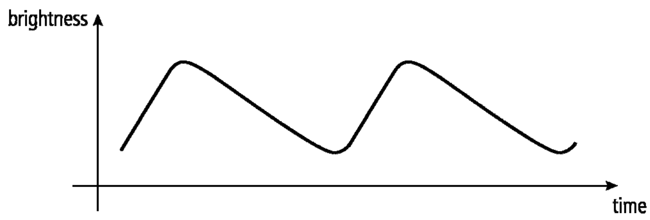
HR diagram

Can plot temperature *vs* luminosity and see size and type of star.

Mass–luminosity relationship – for main sequence stars $L \propto M^{3.5}$



Cepheid variables – have a regularly changing brightness whose time period is related to luminosity. Knowing luminosity and brightness, we can calculate distance. This technique is called a standard candle.



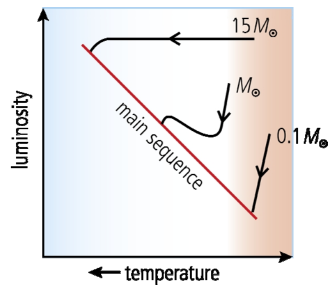
Stellar evolution

Giant molecular cloud GMC – a swirling cloud of gas and dust.

Jeans criterion – the minimum mass of gas needed to collapse into a star $M_J = \frac{3kTR}{2Gm}$

Protostar – the end of GMC collapse and the beginning of a star.

Proton–proton cycle – $4\,^1_1\text{H} \rightarrow \,^4_2\text{He} + 2\text{e}^+ + 2\nu_e + 2\gamma$



Main sequence – once on main sequence, star is stable; time on main sequence depends on mass.

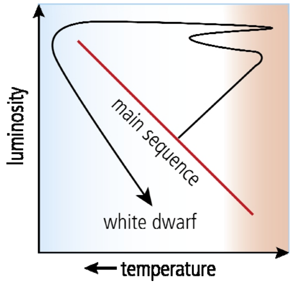
$$\frac{\Delta t}{\Delta t_{\odot}} = \left(\frac{M}{M_{\odot}}\right)^{-2.5}$$

After main sequence – evolution path depends on the mass of the core.

hydrogen fusion continues until hydrogen used up → fusion slows → pressure less → core collapses → temp. increases → outer layers expand

Electron degeneracy – Pauli exclusion principle says two electrons cannot occupy the same quantum mechanical state.

Chandrasekhar limit – the maximum mass of the core of a white dwarf 1.4 M_{Sun}

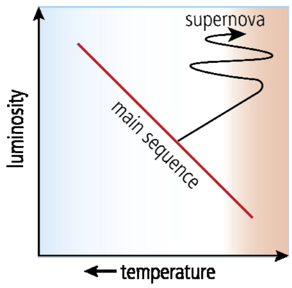


Type Ia supernova – when a white dwarf takes in more mass from a neighbouring star and exceeds the Chandrasekhar limit leading to a violent collapse and explosion. Luminosity is always 10^{10} L_{Sun} so can be used as standard candles.

Neutron star – if mass of core is greater than 1.4 M_{Sun} electron degeneracy cannot prevent further collapse. Protons combine with electrons to give neutrons.

Neutron degeneracy – prevents the collapse of a neutron star.

Type II supernova – formed when the neutron degeneracy stops the collapse of the core, outer layers bounce off the core causing an explosion.

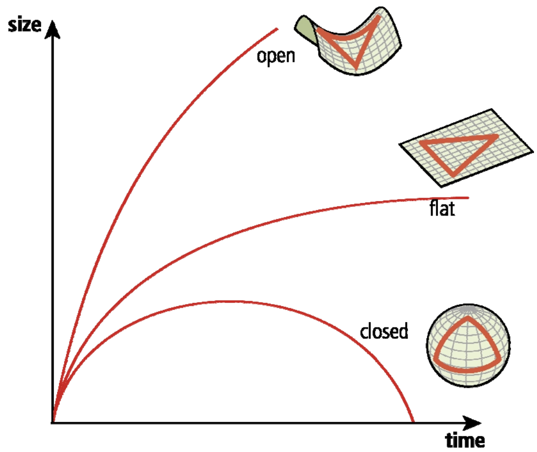


Openheimer–Volkhoff limit – the minimum size of core needed to form a black hole $3M_{Sun}$.

Cosmology

Cosmological principle – the Universe is homogeneous and isotropic. The Earth is not in a special place so observations from here apply everywhere.

General relativity – Einstein's theory of gravity. Mass curves space–time. Curvature of Universe determines its fate.



Critical density (ρ_c) – the density that would give a flat Universe. Can be found by applying Newton's gravity.

$$\rho_c = \frac{3v^2}{8\pi GR^2}$$

Hubble's law – the recessional velocity of a distant galaxy is directly proportional to its distance.

Hubble constant (H_0) – $H_0 = \frac{v}{d}$ (about 72 km s^{-1} Mpc^{-1})

$\frac{1}{H_0}$ = the age of the Universe (careful with units). In terms of H_0 critical density $\rho_c = \frac{3v^2}{8\pi GR^2}$

z parameter – the fractional increase in wavelength of a galaxy $z = \frac{\Delta\lambda}{\lambda_{em}} = \frac{v}{c}$

Expanding Universe – Hubble's law → implies that space is expanding.

Scale factor (R) – a measure of the size of the Universe relative to today $z + 1 = \frac{1}{R_{(tem)}}$

Big Bang model – if the Universe is expanding there must have been a time when it was much smaller. It seems like the Universe was created by some big explosion. However, just because space is expanding does not mean it had to have been created from a point. If the Universe is infinite now, then it has always been infinite, just that the distance between points in the past was less.

Development of the Universe

before 10^{-43} s no physical model (Planck time) → sea of quarks and leptons → annihilation of particle–antiparticles to photons which create particle pairs → space expands, photons no longer create pairs, photons dominate → nuclei form, Universe opaque – black body radiation → space expands, photons no longer have energy to ionize – atoms form → Universe becomes transparent (3000 K)

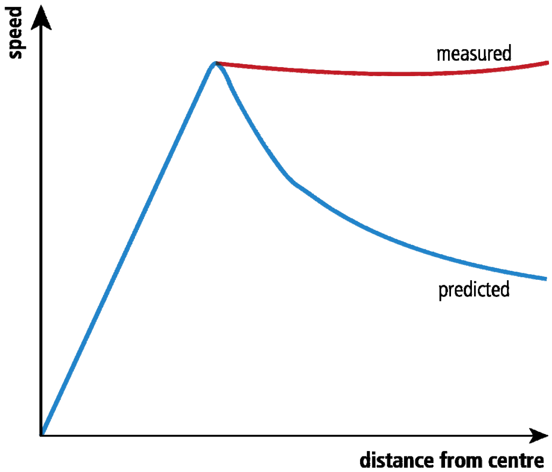
Cosmic microwave background (CMB) – Blackbody radiation left over from the time when the Universe was opaque. Homogeneous and isotropic (on large scale).

Black body peak – temperature = 2.73 K, consistent with original black body radiation from 3000 K source.

Dark matter – the rotation of galaxies shows that they contain more mass than we can see.

MACHO – massive compact halo object (e.g. black hole and brown dwarf).

WIMP – weakly interacting massive particle (e.g. neutrino-like particles).



Dark energy – the Universe is expanding at an ever-increasing rate so seems to be pushed out by some sort of negative energy.

Wilkinson microwave anisotropy probe (WMAP) – found that CMB is not isotropic but on a small scale has some variation. This can be explained by quantum fluctuations in the early Universe. Modelling the motion of the early Universe leads to the conclusion that to get what we see today the Universe must be flat.