



MARKSCHEME

November 1999

MATHEMATICAL STUDIES

Standard Level

Paper 2

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Paper 2 Markscheme
Instructions to Examiners

1 All marking must be done using a **red** pen.

2 **Abbreviations**

The markscheme may make use of the following abbreviations:

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

C Marks awarded for **Correct** statements

R Marks awarded for clear **Reasoning**

AG **Answer Given** in the question and consequently marks are **not** awarded

3 **Follow Through (ft) Marks**

Questions in this paper were constructed to enable a candidate to:

- show, step by step, what he or she knows and is able to do;
- use an answer obtained in one part of a question to obtain answers in the later parts of a question.

Thus errors made at any step of the solution can affect all working that follows. Furthermore, errors made early in the solution can affect more steps or parts of the solution than similar errors made later.

To limit the severity of the penalty for errors made at any step of a solution, **follow through (ft)** marks should be awarded. The procedures for awarding these marks require that all examiners:

- (i) penalise an error when it **first occurs**;
- (ii) **accept the incorrect answer** as the appropriate value or quantity to be used in all subsequent parts of the question;
- (iii) award **M** marks for a correct method, and **A(ft)** marks if the subsequent working contains no further errors.

Follow through procedures may be applied repeatedly throughout the same problem.

The errors made by a candidate may be: arithmetical errors; errors in algebraic manipulation; errors in geometrical representation; use of an incorrect formula; errors in conceptual understanding.

The following illustrates a use of the **follow through** procedure:

Markscheme		Candidate's Script	Marking	
\$ 600 × 1.02	M1	Amount earned = \$ 600 × 1.02	✓	M1
= \$ 612	A1	= \$602	×	A0
\$ (306 × 1.02) + (306 × 1.04)	M1	Amount = 301 × 1.02 + 301 × 1.04	✓	M1
= \$ 630.36	A1	= \$ 620.06	✓	A1(ft)

Note that the candidate made an arithmetical error at line 2; the candidate used a correct method at lines 3, 4; the candidate's working at lines 3, 4 is correct.

However, if a question is transformed by an error into a **different, much simpler question** then:

- (i) **fewer** marks should be awarded at the discretion of the Examiner;
- (ii) marks awarded should be followed by '(d)' (to indicate that these marks have been awarded at the **discretion** of the Examiner);
- (iii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

4 Using the Markscheme

- (a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Thus, the working out must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme.

In this case:

- (i) a mark should be awarded followed by '(d)' (to indicate that these marks have been awarded at the **discretion** of the Examiner);
 - (ii) a brief **note** should be written on the script explaining **how** these marks have been awarded.
- (b) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin \theta}{\cos \theta}$ for $\tan \theta$.
 - (c) As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7, 1·7, 1,7; different forms of vector notation such as \vec{u} , \overline{u} , \underline{u} ; $\tan^{-1} x$ for $\arctan x$.

5 Accuracy of Answers

- (a) In the case when the accuracy of answers is **specified in the question** (for example: “all answers should be given to four significant figures”) **A** marks are awarded **only if** the correct answers are given to the accuracy required.
- (b) When the accuracy is **not** specified in the question, then the general rule applies: .

Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.

In this case, the candidate is **penalised once only IN EACH QUESTION** for giving a correct answer to the wrong degree of accuracy. Hence, on the **first** occasion in a question when a correct answer is given to the wrong degree of accuracy **A** marks are **not** awarded. But on **all subsequent occasions** in the same question when correct answers are given to the wrong degree of accuracy then **A** marks **are** awarded.

NOVEMBER 1999

Additional instructions for Assistant Examiners

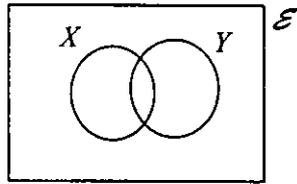
1. SAMPLES

All examiners are reminded that samples should be sent to the Team Leader by the fastest means possible. IBCA will reimburse examiners for any costs incurred.

2. PAPER 2 EXAMINERS – PART MARKS

Assistant examiners are asked to indicate on candidates' scripts the part marks that they have allocated for each part. To help identify these marks, they have now been incorporated into the markschemes. Where the markscheme has part marks *e.g. [3 marks]* after a part solution, assistant examiners should note the candidates' marks for that part of the question alongside the solution. They should also write down the total for the question at the end of each question.

1. (i) (a)



(A1)

Note: Award (A1) for a diagram correctly labelled with X , Y and \mathcal{E}

[1 mark]

(b) (i) $(X \cap Y) = \{6, 12\}$

(A1)

(ii) $X \cap \bar{Y} = \{2, 4, 8, 10, 14\}$

(A2)

(c) $(X \cup Y)' = \bar{(X \cup Y)} = \{1, 5, 7, 11, 13\}$
 $n(X \cup Y)' = 5$

[3 marks]

(A1)

(A1)

[2 marks]

(ii) (a) (i) "The food supply is adequate and the visitors are hungry but the oven is not working," (or equivalent statement).

(A2)

(ii) "Either the oven is working and the food supply is adequate or the visitors are not hungry," (or equivalent statement).

(A2)

[4 marks]

(b) $(p \wedge q) \Rightarrow (p \vee q)$

(A2)

Notes: Award (A1) for $(p \wedge q)$ and $(p \vee q)$, (A1) for \Rightarrow .

[2 marks]

(c)

p	q	$(p \wedge q)$	$(p \vee q)$	$(p \wedge q) \Rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

(A2)

Therefore, $(p \wedge q) \Rightarrow (p \vee q)$ is a tautology

(R1)

[3 marks]

Notes: Follow through from part (ii) (b).
 Award [$\frac{1}{2}$ mark] for each correct bold column and round up.
 Award (R1) for a correct conclusion based on truth values in column headed $(p \wedge q) \Rightarrow (p \vee q)$.

[Total: 15 marks]

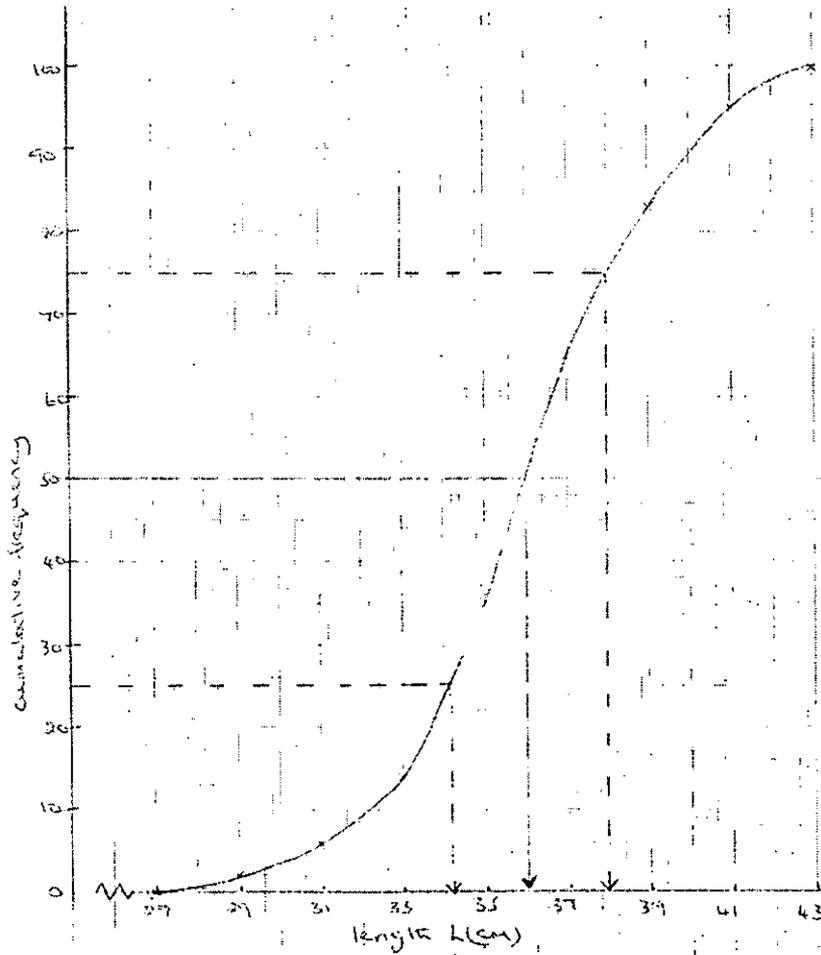
2. (a)

L (cm)	f	$\sum f$
≤ 29	2	2
≤ 31	4	6
≤ 33	8	14
≤ 35	21	35
≤ 37	30	65
≤ 39	18	83
≤ 41	12	95
≤ 43	5	100

Notes: Award (A1) for four correct entries in the column headed $\sum f$.
Award (A2) for all 8 correct.

(A2)
[2 marks]

(b)



(A3)
[3 marks]

Notes: Award (A1) for both axes and correct scale.
Award [$\frac{1}{2}$ mark] for each correctly plotted point and round up to a maximum of [2 marks].

Question 2 continued

- | | | | | |
|-----|------|---|-------------------------------|--------------|
| (c) | (i) | Median length of mackerel = $36 \text{ cm} \pm 0.2 \text{ cm}$
= 36 cm | } | (M1)
(A1) |
| | (ii) | Interquartile range of mackerel length = $3.8 \pm 0.2 \text{ cm}$
= 4 cm | } | (M1)
(A1) |
| | | | (read from candidate's curve) | |

[4 marks]

(d) (i) $p(L > 40) = \Phi\left(Z > \frac{40-35}{5}\right)$ (M1)

= $\Phi(Z > 1) = 1 - \Phi(Z < 1)$

= $1 - 0.8413$

= $0.1587 = 0.159$ (3 s.f.) (A1)

(ii) $p(L < 28) = \Phi\left(Z < \frac{28-35}{5}\right)$

= $\Phi(Z < -1.4) = \Phi(Z > 1.4)$ (M1)

= $1 - 0.9192 = 0.0808$ (A1)

(iii) $p(28 < L < 40) = \Phi\left(\frac{28-35}{5} < Z < \frac{40-35}{5}\right)$ (M1)

= $\Phi(-1.4 < Z < 1)$

= $(1 - 0.1587 - 0.0808)$

= $0.7605 = 0.761$ (3 s.f.) (A1)

[6 marks]

[Total: 15 marks]

3. (a) (i) $\text{Cost of loan} = 4000(1.01)^{12} - 4000$ (M1)
 $= \$ 507.30$ (A1)

Note: Accept \$ 4507.30 and then \$ 4450 for part (b)(i)

(ii) $\text{Equivalent S.I. Rate} = \frac{507.30(100)}{4000}$ (M1)
 $= 12.662503\dots$
 $= 12.7 \% (3 \text{ s.f.})$ (A1)

[4 marks]

(b) (i) $\text{Deposit} = 25 \% \text{ of } \$ 4000$
 $= \$ 1000$ (A1)

(ii) $\text{Cost of the loan} = 287.50(12) - 3000 \text{ (or equivalent)}$ (M1)
 $= \$ 450.00$ (A1)

[3 marks]

(c) (i) Option A. Because she doesn't need a deposit (or equivalent appropriate explanation). (R1)

(ii) Option B. Because it is cheaper by $\$(507.30 - 450.00) = \$ 57.30$ (or equivalent appropriate explanation). (R1)

[2 marks]

(d) $x + 19y = 1540$ (M1)
 $x + 29y = 2140$ (M1)
 $\Rightarrow x = 400$ (A1)
 $y = 60$ (A1)

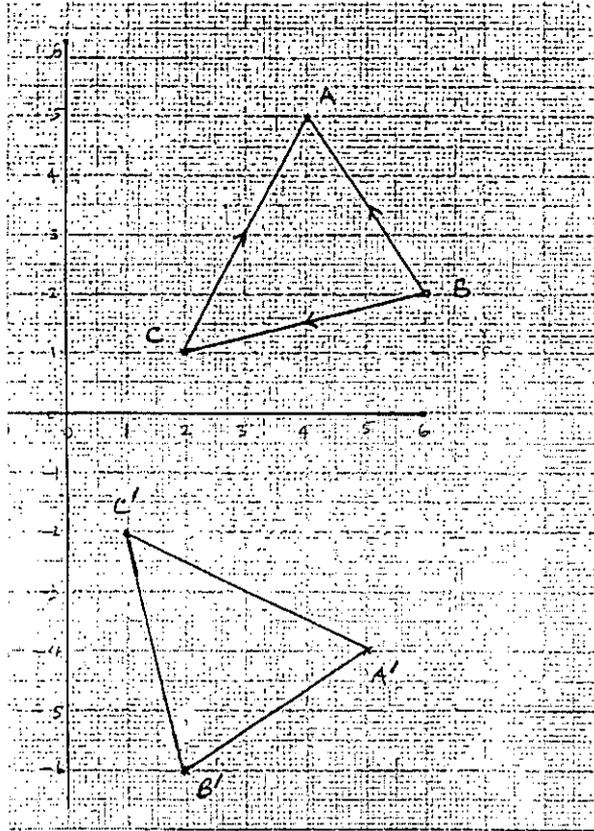
[4 marks]

(e) $400 + (n - 1)60 = 4000$ (M1)
 $\Rightarrow n = 61$
 Therefore, 61 months. (A1)

[2 marks]

[Total: 15 marks]

4. (a) (i)



(M2)

Note: Award [$\frac{1}{2}$ mark] for each point correctly plotted, and round up to maximum of [2 marks].

(ii) See same graph as for part (a) (i).

(M2)

Note: Award [$\frac{1}{2}$ mark] for each correct line segment properly labelled, and round up to maximum of [2 marks].

[4 marks]

(b) (i)
$$\vec{CA} = \begin{pmatrix} 4-2 \\ 5-1 \end{pmatrix}$$

(M1)

$$= \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

(A1)

(ii)
$$\vec{BC} = -\vec{CB} = -\begin{pmatrix} 6-2 \\ 2-1 \end{pmatrix}$$

(M1)

$$= \begin{pmatrix} -4 \\ -1 \end{pmatrix}$$

(A1)

(iii)
$$\vec{BA} = \vec{CA} + \vec{BC} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} -4 \\ -1 \end{pmatrix}$$

(M1)

$$= \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

(A1)

[6 marks]
continued...

Question 4 continued

(c) (i) See same graph as for part (a).

(M2)

Notes: Award [*½ mark*] for each correct vertex of triangle $A'B'C'$, and round up to maximum of [*2 marks*].

(ii) $\vec{A'B'} = -\vec{B'A'}$

$$= -\begin{bmatrix} 5-2 \\ -4+6 \end{bmatrix}$$

(M1)

$$= \begin{bmatrix} -3 \\ -2 \end{bmatrix}$$

(A1)

[4 marks]

(d) $T = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(A1)

[1 mark]

[Total: 15 marks]

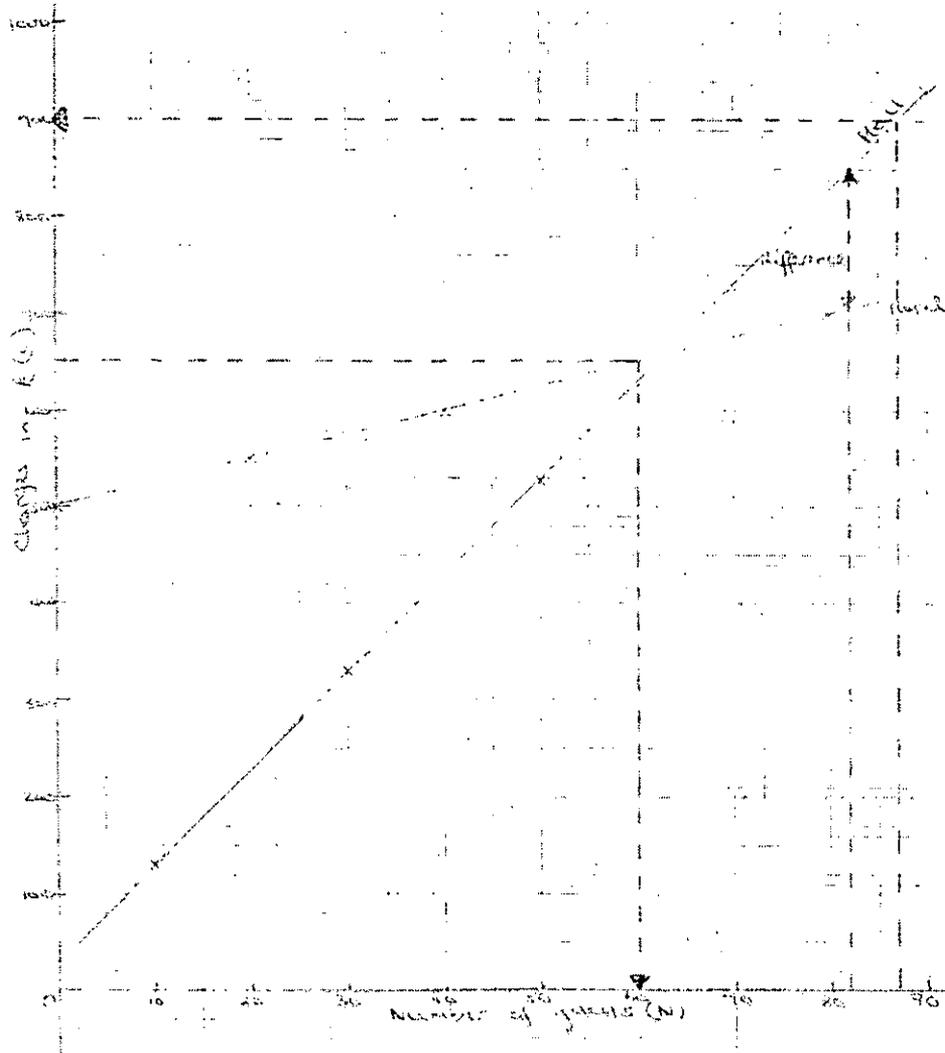
5. (a) (i)

<i>N</i>	10	30	50	70	90
<i>C</i>	130	330	530	730	930

(A2)

Notes: Award [*½ mark*] for each correct bold entry and round down to a maximum of [*2 marks*].

(ii)



(M3)

Notes: Award (*M1*) for both axes correctly labelled and with suitable scales.
Award [*½ mark*] for each correct point and round up to a maximum of [*2 marks*].

(iii) $C = 10N + 30$

(A1)

[6 marks]

continued...

Question 5 continued

- (b) (i) "The local hotel will charge £ 500 plus half of five (or two and a half) times the number of guests" (or equivalent statement). (A2)

Note: Award (A1)(A0) if partly correct.

- (ii)

<i>N</i>	0	20	40	80
<i>C</i>	500	550	600	700

(A2)

Note: Award [*½ mark*] for each correct bold entry, and round up to a maximum of [*2 marks*].

- (iii) On same graph as for part (a) (ii). (M2)

Note: Award [*½ mark*] for each point correctly plotted, and round up to a maximum of [*2 marks*]. [6 marks]

- (c) Explanations abound: Award marks only for any correct explanation arising from candidate's own graph in parts (a) (ii) and (b) (iii). (R2) [2 marks]

Notes: Award (R1) for each correct point given. Maximum [*2 marks*].

- (d) (i) £ 900 (M1)(A1)
 (ii) 60 guests (M1)(A1)
 (iii) £ 145.00 (M1)(A1) [6 marks]

Notes: For parts (d) (i) to (d) (iii), follow through with candidate's own graphs. Answers given here are obtained by calculation and should serve only as a guide.

[Total: 20 marks]

6. (a) (i) "The number of Kombi and Danfo minibuses used each day cannot exceed 7 because the maximum number of drivers provided by the college is 7 and no driver is allowed to repeat the journey." (A2)

Note: Award (A2) if and only if the correct reason or explanation (*italics*) is also given, otherwise award (A1) or (A0) as appropriate.

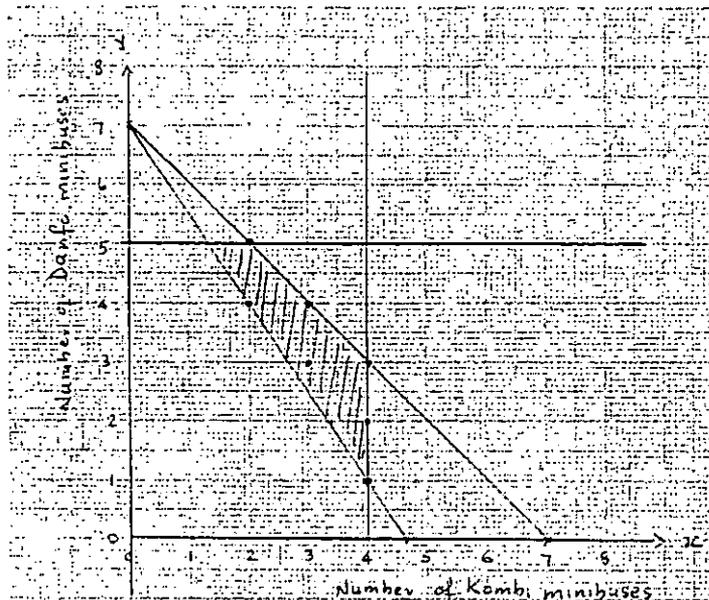
(ii) $15x + 10y \geq 70$ (M1)
 $\Leftrightarrow 3x + 2y \geq 14$ (A1)

- (iii) Additional information in table is that: maximum number of Kombi (x) and Danfo (y) types of minibuses are 4 and 5 respectively. (A2)

Note: Examiners to use their discretion in allocating this (A2) mark.

[6 marks]

- (b) (i)



(M5)

Notes: Award (M1) for both axes correctly labelled, and (M1) for correct scales.
 Award (M1) for both lines $x = 4$, $y = 5$
 Award (M1) for each of the lines: $x + y = 7$ and $3x + 2y = 14$

- (ii) For shading correct region. (See graph for correct region). (A2)

Note: Follow through with candidate's inequality obtained in part (a) (ii).

- (iii) (2, 4), (4, 3), (2, 5), (4, 1), (3, 4), (3, 3), (4, 2) (A2)

Notes: Follow through from (b) (ii), i.e. candidate's region and award (A1) for any three correct combinations. Maximum [2 marks].

[9 marks]

Question 6 continued

(c) (i) $C = 23x + 15y$ (A1)

(ii)
$$\left. \begin{array}{l} 2(23) + 4(15) = 106 \\ 4(23) + 3(15) = 137 \\ 2(23) + 5(15) = 121 \\ 4(23) + 1(15) = 107 \\ 3(23) + 3(15) = 114 \\ 3(23) + 4(15) = 129 \\ 4(23) + 2(15) = 126 \end{array} \right\} \text{ (M2)}$$

Therefore, minimum daily cost = £ 106. (A1)

Note: Accept other correct methods. Follow through from parts (b) (iii) and (c) (i).

(iii) From part (c) (ii) $2(23) + 4(15) = 106$ is the cheapest alternative.
Hence 2 Kombi and 4 Danfo or (2, 4). (A1)

Note: Correct answer (2, 4) only.

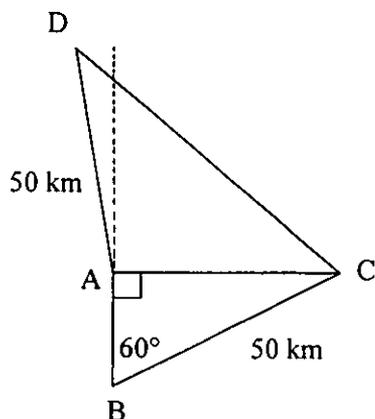
[5 marks]

[Total: 20 marks]

7.

Note: Throughout Question 7, watch out for and accept alternative methods.

(i) (a) (i)



(A2)

Notes: Award (A1) for $\hat{BAC} = 90^\circ$.
Award (A1) for $\hat{ABC} = 60^\circ$ and $BC = 50$ km.

(ii) $AC = 50 \sin 60^\circ$

(M1)

$= 43.3$ km (3 s.f.)

(A1)

[4 marks]

(b) See diagram above.

(M2)

Notes: Award (M1) for correctly positioning point D.
Award (M1) for labelling $AD = 50$ km.

[2 marks]

(c) **Notes:** For parts (c) (i) and (ii) award (M1) for choosing the correct formulae.
Award (M1) for substituting correctly in any formulae chosen.
Award (A1) for the correct answer.

(i) $DC^2 = (50)^2 + AC^2 - 2(50)(AC)\cos 120^\circ$
 $= 6540.063508\dots$

(M2)

Therefore, $DC = 80.9$ km (3 s.f.)

(A1)

(ii) $\hat{ACD} = \arcsin \left[\frac{50 \sin 120^\circ}{DC} \right]$

(M2)

$= \arcsin 0.5354385\dots$

$= 32.4^\circ$ (3 s.f.)

(A1)

[6 marks]

continued...

Question 7 (i) continued

(d) Bearing of D from C $\approx (270 + 32.4^\circ) \approx 302.4^\circ$ (M1)

Note: Award (M1) for any correct method, towards obtaining the bearing of D from C, e.g. $(270 + 32.4)^\circ$.

$\approx 302^\circ$ (3 s.f.) (A1)

(ii) (a) Coordinates of P: (2, 2, 3) [2 marks] (A1)

(b) (i) $AC^2 = 2^2 + 4^2 = 20$ [1 mark] (M1)

$EC^2 = 3^2 + AC^2 = 3^2 + 20 = 29$ (M1)

Therefore, $EC = \sqrt{29}$
 ≈ 5.39 (3 s.f.) (A1)

(ii) $\hat{A}EC = \arctan\left(\frac{AC}{AE}\right) = \arctan\left(\frac{\sqrt{20}}{3}\right)$ (M1)

$\approx 56.1^\circ$ (3 s.f.) (A1)

[5 marks]

[Total: 20 marks]
