



**MATHEMATICAL STUDIES**

**Standard Level**

Tuesday 4 May 1999 (afternoon)

Paper 1

1 hour 30 minutes

**A**

Candidate name:	Candidate category & number:								
	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 12.5%; height: 20px;"></td> <td style="width: 12.5%;"></td> </tr> </table>								
<p>This examination paper consists of 20 questions. The maximum mark for each question is 4. The maximum mark for this paper is 80.</p> <p style="text-align: center;"><b>INSTRUCTIONS TO CANDIDATES</b></p> <p>Write your candidate name and number in the boxes above.</p> <p>Do <b>NOT</b> open this examination paper until instructed to do so.</p> <p>Answer <b>ALL</b> of the questions in the spaces provided.</p> <p>Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.</p>									

**B**

QUESTIONS ANSWERED
ALL

**C**

EXAMINER	TEAM LEADER
TOTAL /80	TOTAL /80

**D**

IBCA
TOTAL /80

**EXAMINATION MATERIALS**

- Required:  
 IB Statistical Table  
 Calculator  
 Ruler and compasses

- Allowed:  
 A simple translating dictionary for candidates not working in their own language

## FORMULAE

**Sine Rule:**  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

**Cosine rule:**  $a^2 = b^2 + c^2 - 2bc \cos A$        $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

**Arithmetic sequences:**  $u_n = a + (n - 1)d$        $S_n = \frac{n}{2}(a + l) = \frac{n}{2}\{2a + (n - 1)d\}$

**Geometric sequences:**  $u_n = ar^{n-1}$        $S_n = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1$

**Simple interest:**  $I = \frac{Crn}{100}$

**Compound interest:**  $I = C\left(1 + \frac{r}{100}\right)^n - C$

**Statistics:** If  $(x_1, x_2, \dots, x_n)$  occur with frequencies  $(f_1, f_2, \dots, f_n)$  then the mean  $m$  and standard deviation  $s$  are given by

$$m = \frac{\sum f_i x_i}{\sum f_i} \quad s = \sqrt{\frac{\sum f_i (x_i - m)^2}{\sum f_i}}, \quad i = 1, 2, \dots, n$$

*Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary, or on extra sheets of paper provided these are securely fastened to this examination paper.*

1. Calculate  $3.7 \times 16.2^2 - 500$ , writing your answer

(a) correct to two decimal places;

(b) (i) correct to three significant figures;

(ii) in the form  $a \times 10^k$ , where  $1 \leq a < 10$ ,  $k \in \mathbb{Z}$ .

*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) (i) \_\_\_\_\_

(ii) \_\_\_\_\_

2. At Jumbo's Burger Bar, Jumbo burgers cost £ $J$  each and regular cokes cost £ $C$  each. Two Jumbo burgers and three regular cokes cost £5.95.

(a) Write an equation to show this.

(b) If one Jumbo Burger costs £2.15, what is the cost, in pence, of one regular coke?

<i>Working:</i>	<i>Answers:</i> (a) _____ (b) _____
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3. The mean of the ten numbers listed below is 5.5.

4, 3,  $a$ , 8, 7, 3, 9, 5, 8, 3

(a) Find the value of  $a$ .

(b) Find the median of these numbers.

<i>Working:</i>	<i>Answers:</i> (a) _____ (b) _____
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4.  $A$  is the matrix  $\begin{pmatrix} 2 & 1 \\ -3 & 2 \end{pmatrix}$  and  $B$  is the matrix  $\begin{pmatrix} -3 & 4 \\ 3 & 0 \end{pmatrix}$ .

Calculate:

(a)  $2A$

(b)  $A + B$

(c)  $AB$

*Working:*

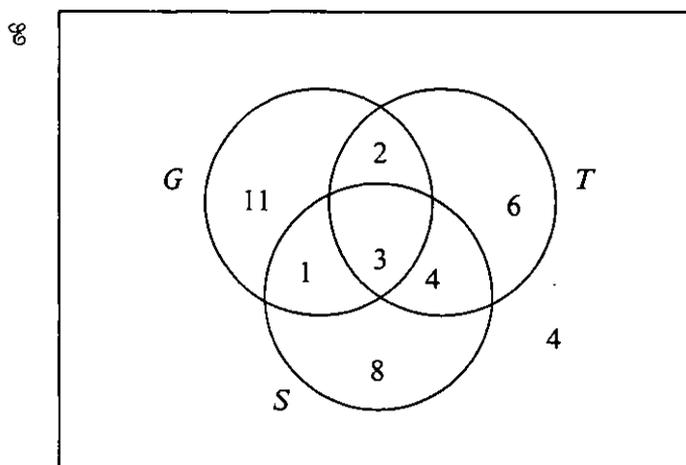
*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_

(c) \_\_\_\_\_

5. The sports offered at a retirement village are Golf ( $G$ ), Tennis ( $T$ ) and Swimming ( $S$ ). The Venn diagram shows the numbers of people involved in each activity.



- (a) How many people
- (i) only play golf?
  - (ii) play both tennis and golf?
  - (iii) do not play golf?
- (b) Shade the part of the Venn diagram that represents the set  $\complement(G \cap S)$ .

*Working:*

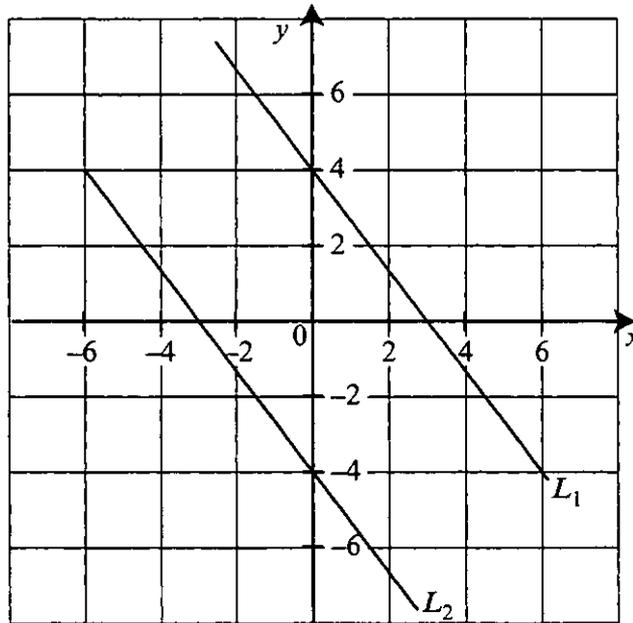
*Answers:*

(a) (i) \_\_\_\_\_

(ii) \_\_\_\_\_

(iii) \_\_\_\_\_

6. In the diagram, the lines  $L_1$  and  $L_2$  are parallel.



- (a) What is the gradient of  $L_1$ ?
- (b) Write down the equation of  $L_1$ .
- (c) Write down the equation of  $L_2$  in the form  $ax + by + c = 0$ .

*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_
- (c) \_\_\_\_\_

7. A woman deposits \$100 into her son's savings account on his first birthday. On his second birthday she deposits \$125, \$150 on his third birthday, and so on.
- (a) How much money would she deposit into her son's account on his 17th birthday?
  - (b) How much in total would she have deposited after her son's 17th birthday?

*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_

8. The height of a vertical cliff is 450 m. The angle of elevation from a ship to the top of the cliff is  $23^\circ$ . The ship is  $x$  metres from the bottom of the cliff.

(a) Draw a diagram to show this information.

Diagram:

(b) Calculate the value of  $x$ .

*Working:*

*Answer:*

(b) \_\_\_\_\_

9. The table below shows the relative frequencies of the ages of the students at *Ingham High School*.

Age (in years)	Relative frequency
13	0.11
14	0.30
15	0.23
16	0.21
17	0.15
Total	1

- (a) If a student is randomly selected from this school, find the probability that
- (i) the student is 15 years old;
  - (ii) the student is 16 years of age or older.

There are 1200 students at *Ingham High School*.

- (b) Calculate the number of 15 year old students.

*Working:*

*Answers:*

(a) (i) \_\_\_\_\_

(ii) \_\_\_\_\_

(b) \_\_\_\_\_

10. The functions  $f$  and  $g$  are defined as

$$\left. \begin{array}{l} f: x \mapsto 2x + 3 \\ g: x \mapsto x - 5 \end{array} \right\} x \in \mathbb{R}$$

(a) Find  $g^{-1}(x)$ .

(b) Evaluate:

(i)  $f(-3)$

(ii)  $(f \circ g)(6)$

*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) (i) \_\_\_\_\_

(ii) \_\_\_\_\_

11. A piece of ground in the shape of a rectangle is measured to the nearest metre. Its length is 17 m and its width is 8 m.

(a) Write the range of possible values for the length of the rectangle in the form

$$a \leq \text{length} < b.$$

(b) Calculate the minimum possible area of the rectangle.

*Working:*

*Answers:*

(a) \_\_\_\_\_  $\leq \text{length} <$  \_\_\_\_\_

(b) \_\_\_\_\_

12. In an experiment researchers found that a specific culture of bacteria increases in number according to the formula

$$N = 150 \times 2^t,$$

where  $N$  is the number of bacteria present and  $t$  is the number of hours since the experiment began.

Use this formula to calculate

- (a) the number of bacteria present at the start of the experiment;
- (b) the number of bacteria present after 3 hours;
- (c) the number of hours it would take for the number of bacteria to reach 19 200.

*Working:*

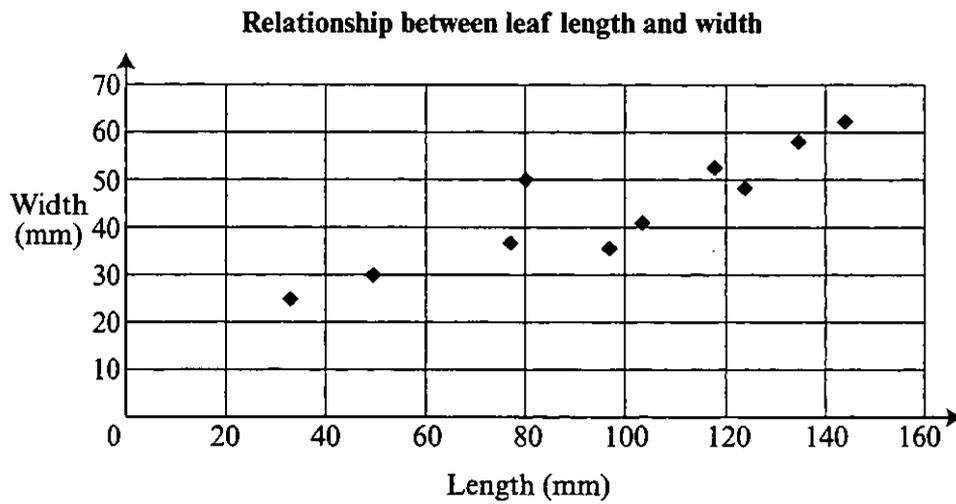
*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_

(c) \_\_\_\_\_

13. The length and width of 10 leaves are shown on the scatter diagram below.



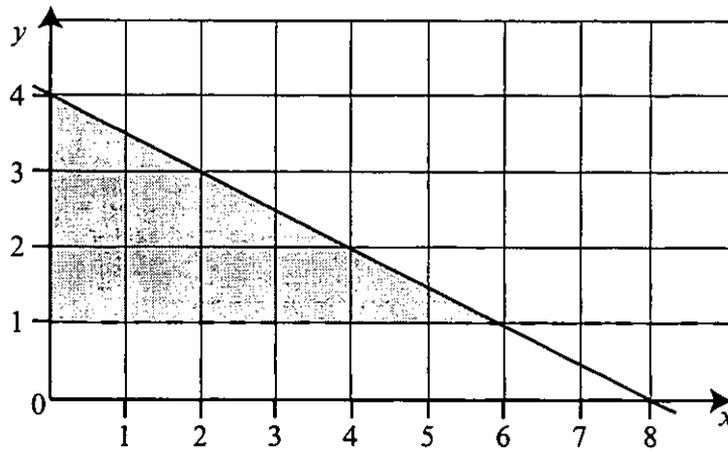
- (a) Plot the point  $M(97, 43)$  which represents the mean length and the mean width.
- (b) Draw a suitable line of best fit.
- (c) Write a sentence describing the relationship between leaf length and leaf width for this sample.

*Working:*

*Answer:*

(c) \_\_\_\_\_  
\_\_\_\_\_

14. Write down the three inequalities that uniquely represent the shaded region shown on the diagram.



*Working:*

*Answers:*

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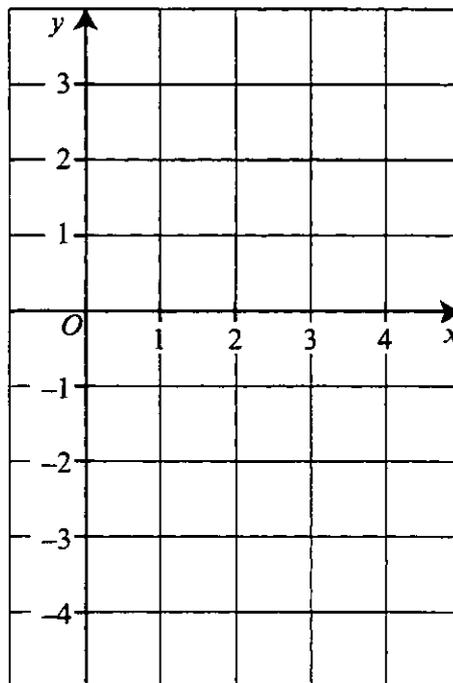
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15. On the diagram below  $O$  is the origin.

(a) Draw and label directed line segments to represent the vectors:

(i)  $\vec{OA} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

(ii)  $\vec{AB} = \begin{pmatrix} 0 \\ -6 \end{pmatrix}$



(b) Calculate the length of  $\vec{OB}$ .

*Working:*

*Answer:*

(b) \_\_\_\_\_

16. At what interest rate, compounded annually, would you need to invest \$100 in order to have \$125 in 2 years?

*Working:*

*Answer:*

17. The table below shows the number of left and right handed tennis players in a sample of 50 males and females.

	Left handed	Right handed	Total
Male	3	29	32
Female	2	16	18
Total	5	45	50

If a tennis player was selected at random from the group, find the probability that the player is

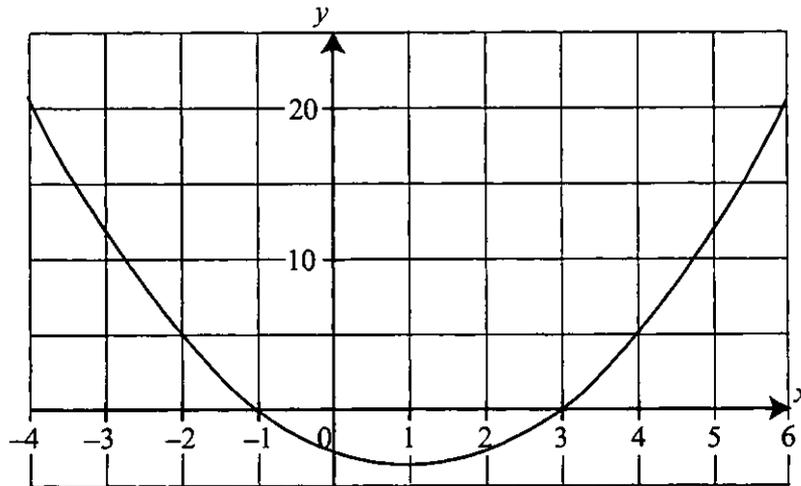
- (a) male and left handed;
- (b) right handed;
- (c) right handed, given that the player selected is female.

*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_
- (c) \_\_\_\_\_

18. The graph of  $y = x^2 - 2x - 3$  is shown on the axes below.



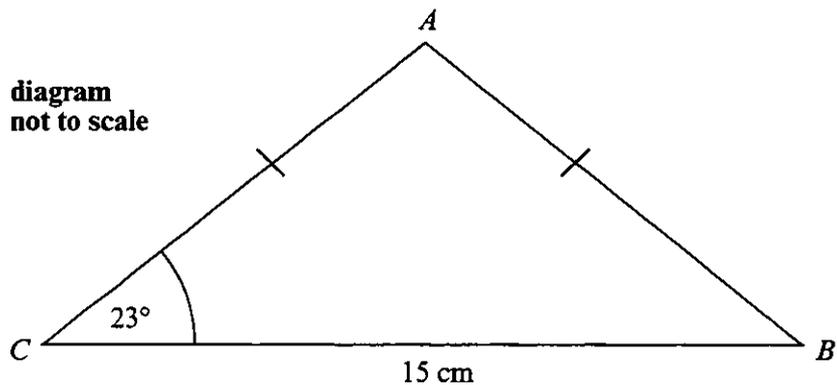
- (a) Draw the graph of  $y = 5$  on the same axes.
- (b) Use your graph to find:
  - (i) the values of  $x$  when  $x^2 - 2x - 3 = 5$
  - (ii) the value of  $x$  that gives the minimum value of  $x^2 - 2x - 3$

*Working:*

*Answers:*

- (b) (i) \_\_\_\_\_
- (ii) \_\_\_\_\_

19. In the diagram, triangle  $ABC$  is isosceles.  $AB = AC$ ,  $CB = 15$  cm and angle  $ACB$  is  $23^\circ$ .



Find

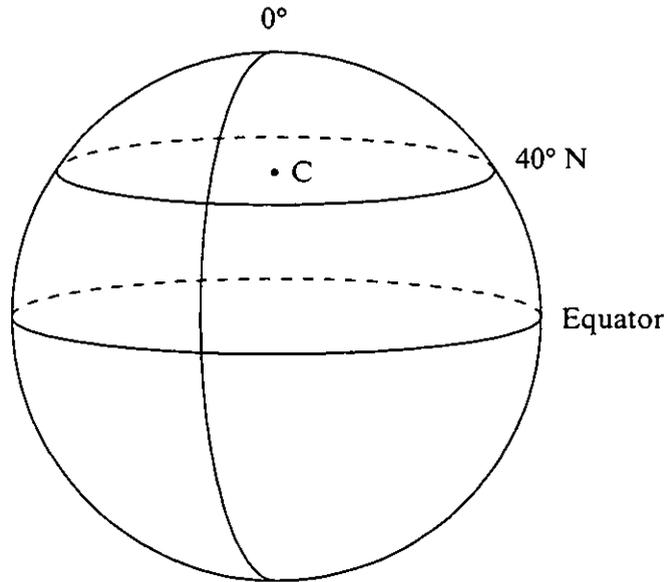
- (a) the size of angle  $CAB$ ;
- (b) the length of  $AB$ .

*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_

20. The circumference of the minor circle made by the line of latitude  $40^\circ \text{ N}$  is approximately 30 800 km. C is the centre of this circle.



City A is located at  $(40^\circ \text{ N}, 125^\circ \text{ E})$  and city B is located at  $(40^\circ \text{ N}, 20^\circ \text{ W})$ .

- (a) On the diagram above indicate the approximate locations of city A and city B and the size of the angle ACB.
- (b) Calculate the distance between city A and city B.

*Working:*

*Answer:*

(b) \_\_\_\_\_