



**MATHEMATICAL STUDIES**

**Standard Level**

Wednesday 5 May 1999 (morning)

Paper 2

1 hour 30 minutes

This examination paper consists of two sections, Section A and Section B.  
Section A consists of 4 questions.  
Section B consists of 3 questions.  
The maximum mark for Section A is 60.  
The maximum mark for each question in Section B is 20.  
The maximum mark for this paper is 80.

**INSTRUCTIONS TO CANDIDATES**

Do NOT open this examination paper until instructed to do so.

Answer all FOUR questions from Section A and ONE question from Section B.

Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.

**EXAMINATION MATERIALS**

Required:

IB Statistical Tables

Millimetre square graph paper

Calculator

Ruler and compasses

Allowed:

A simple translating dictionary for candidates not working in their own language

**FORMULAE**

**Sine Rule:**  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

**Cosine rule:**  $a^2 = b^2 + c^2 - 2bc \cos A$        $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

**Arithmetic sequences:**  $u_n = a + (n - 1)d$        $S_n = \frac{n}{2}(a + l) = \frac{n}{2}\{2a + (n - 1)d\}$

**Geometric sequences:**  $u_n = ar^{n-1}$        $S_n = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1$

**Simple interest:**  $I = \frac{Crm}{100}$

**Compound interest:**  $I = C \left(1 + \frac{r}{100}\right)^n - C$

**Statistics:** If  $(x_1, x_2, \dots, x_n)$  occur with frequencies  $(f_1, f_2, \dots, f_n)$  then the mean  $m$  and standard deviation  $s$  are given by

$$m = \frac{\sum f_i x_i}{\sum f_i} \quad s = \sqrt{\frac{\sum f_i (x_i - m)^2}{\sum f_i}}, \quad i = 1, 2, \dots, n$$

A correct answer with **no** indication of the method used will normally receive **no** marks. You are therefore advised to show your working.

**SECTION A**

Answer all **FOUR** questions from this section.

1. [Maximum mark: 15]

The profit ( $P$ ) in Swiss Francs made by three students selling homemade lemonade is modelled by the function

$$P = -\frac{1}{20}x^2 + 5x - 30$$

where  $x$  is the number of glasses of lemonade sold.

(a) **Copy** and complete the table below

$x$	0	10	20	30	40	50	60	70	80	90
$P$		15			90			75	50	

[3 marks]

(b) On graph paper draw axes for  $x$  and  $P$ , placing  $x$  on the horizontal axis and  $P$  on the vertical axis. Use suitable scales. Draw the graph of  $P$  against  $x$  by plotting the points. Label your graph.

[5 marks]

(c) Use your graph to find

(i) the maximum possible profit;

[1 mark]

(ii) the number of glasses that need to be sold to make the maximum profit;

[1 mark]

(iii) the number of glasses that need to be sold to make a profit of 80 Swiss Francs;

[2 marks]

(iv) the amount of money initially invested by the three students.

[1 mark]

(d) The three students Baljeet, Jane and Fiona share the profits in the ratio of 1 : 2 : 3 respectively. If they sold 40 glasses of lemonade, calculate Fiona's share of the profits.

[2 marks]

2. [Maximum mark: 15]

Let  $\mathcal{E} = \{x : 1 \leq x < 17, x \in \mathbb{N}\}$ .

$P$ ,  $Q$  and  $R$  are the subsets of  $\mathcal{E}$  such that

$P = \{\text{multiples of four}\}$  ;

$Q = \{\text{factors of 36}\}$  ;

$R = \{\text{square numbers}\}$  .

(a) List the elements of

(i)  $\mathcal{E}$  ;

(ii)  $P \cap Q \cap R$  .

[2 marks]

(b) Describe in words the set  $P \cup Q$  .

[1 mark]

(c) (i) Draw a Venn diagram to show the relationship between sets  $P$ ,  $Q$  and  $R$  .

[2 marks]

(ii) Write the elements of  $\mathcal{E}$  in the appropriate places on the Venn diagram.

[3 marks]

(d) Let  $p$ ,  $q$  and  $r$  be the statements

$p$  :  $x$  is a multiple of four;

$q$  :  $x$  is a factor of 36 ;

$r$  :  $x$  is a square number.

(i) Write a sentence, in words, for the statement;

$$(p \vee r) \wedge \neg q$$

[2 marks]

(ii) Shade the region on your Venn diagram in part (c)(i) that represents  $(p \vee r) \wedge \neg q$  .

[1 mark]

(iii) (a) Use a truth table to determine the values of  $(p \vee r) \wedge \neg q$  . Write the first three columns of your truth table in the following format.

$p$	$q$	$r$
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

[3 marks]

(b) Write down one possible value of  $x$  for which  $(p \vee r) \wedge \neg q$  is true.

[1 mark]

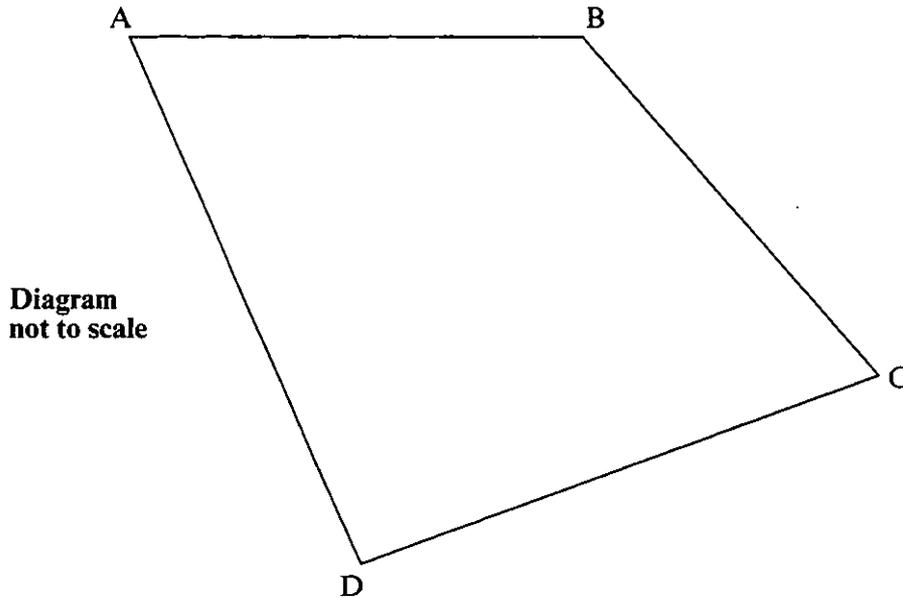
3. [Maximum mark: 15]

A machine dispenses liquid into containers in such a way that the volume of liquid dispensed is normally distributed with mean 672 ml and standard deviation 50 ml .

- (a) What is the probability that a container selected at random contains
- (i) less than 700 ml ? *[2 marks]*
  - (ii) between 640 ml and 700 ml ? *[5 marks]*
- (b) The volume of liquid content printed on the containers is such that 33% of all containers filled by the machine contains less than the printed volume.
- (i) On a normal curve represent this situation. *[1 mark]*
  - (ii) What  $z$  value corresponds to 33% on a standard normal distribution? *[2 marks]*
  - (iii) What volume of liquid is printed on the container? *[2 marks]*
- (c) Two containers are chosen at random. Calculate the probability that exactly one of the containers contains more than the printed volume. *[3 marks]*

4. [Maximum mark: 15]

A farmer marks the corners of a field with four posts which are labelled A, B, C and D as shown in the diagram.



The following information is known

- B is 260 metres due east of A ;
- C is on a bearing of  $150^\circ$  from B ;
- C is 260 metres from B ;
- D is 200 metres from C ;
- D is 315 metres from A .

- (a) Copy the diagram to show A, B, C and D and clearly mark the given bearings and distances. [3 marks]
- (b) Calculate
  - (i) the size of angle ABC ; [2 marks]
  - (ii) the length of AC ; [2 marks]
  - (iii) the size of angle ADC . [2 marks]
- (c) Calculate the area of the field ABCD . [3 marks]
- (d) Calculate the bearing of A from C . [3 marks]

**SECTION B**

Answer *ONE* question from this section.

5. [Maximum mark: 20]

*Note: For this question, it is important that you show your working and explain your method clearly.*

(i) A box contains 10 coloured light bulbs, 5 green, 3 red and 2 yellow. One light bulb is selected at random and put into the light fitting of room A .

(a) What is the probability that the light bulb selected is

(i) green?

[1 mark]

(ii) not green?

[1 mark]

A second light bulb is selected at random and put into the light fitting in room B .

(b) What is the probability that

(i) the second light bulb is green given the first light bulb was green?

[1 mark]

(ii) both light bulbs were not green?

[2 marks]

(iii) one room had a green light bulb and the other room does not have a green light bulb?

[3 marks]

A third light bulb is selected at random and put in the light fitting of room C .

(c) What is the probability that

(i) all three rooms have green light bulbs?

[2 marks]

(ii) only one room has a green light bulb?

[3 marks]

(iii) at least one room has a green light bulb?

[2 marks]

*(This question continues on the following page)*

*(Question 5 continued)*

- (ii) It is known that 5% of all AA batteries made by Power Manufacturers are defective. AA batteries are sold in packs of 4.

Find the probability that a pack of 4 has

- (a) exactly two defective batteries;

*[3 marks]*

- (b) at least one defective battery.

*[2 marks]*

6. [Maximum mark: 20]

(i) Triangle ABC has vertices at A(2, 1), B(2, 4) and C(5, 5).

(a) (i) Draw a set of axes for  $-5 \leq x \leq 5$  and  $-5 \leq y \leq 5$ . [1 mark]

(ii) Plot and label the points A, B and C. Join the points A, B and C. [2 marks]

Triangle ABC is reflected in the  $y$ -axis to give triangle A'B'C'.

(b) (i) On the same axes used in part (a) draw and label triangle A'B'C'. [2 marks]

(ii) Write down the coordinates of the vertices of triangle A'B'C'. [1 mark]

Triangle ABC is transformed by the matrix  $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$  to give triangle A''B''C''.

(c) (i) Calculate the coordinates of the vertices of triangle A''B''C''. [3 marks]

(ii) On the same axes as part (a) draw the triangle A''B''C''. [1 mark]

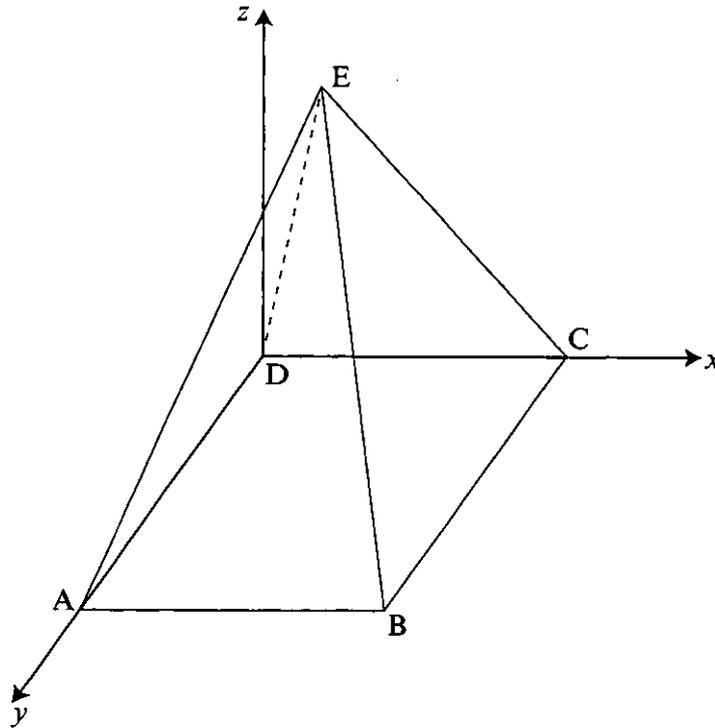
(iii) Describe the transformation  $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ . [1 mark]

(iv) What single transformation will map triangle A'B'C' onto A''B''C''? [2 marks]

(This question continues on the following page)

(Question 6 continued)

- (ii) ABCDE is a square based right pyramid. The coordinates of A and D are  $(0, 6, 0)$  and  $(0, 0, 0)$ , respectively.



- (a) Write down the coordinates of B. [1 mark]
- M is the mid-point of [DB].
- (b) Write down the coordinates of M. [2 marks]
- The height of the pyramid is 7.
- (c) Write down the coordinates of E. [1 mark]
- (d) Calculate the length of BE. [3 marks]

7. [Maximum mark: 20]

Yasmin and Ismael wish to cultivate tulips and carnations. The following information is known.

They have 16 hectares of land.

It costs £20 per hectare for tulip bulbs and £12 per hectare for carnation seeds.

They have £240 to spend on bulbs and seeds.

At least 6 hectares of carnations need to be cultivated.

- (a) Let  $x$  be the number of hectares of tulips and  $y$  the number of hectares of carnations to be cultivated.
- (i) Explain why  $20x + 12y \leq 240$ . [2 marks]
  - (ii) Simplify the inequality in part (a)(i). [1 mark]
  - (iii) Explain why  $x \geq 0$  and  $y \geq 6$  are valid statements. [2 marks]
  - (iv) Write down an inequality which shows that the maximum amount of land for cultivation is 16 hectares. [1 mark]
- (b) (i) On graph paper draw axes for the number of hectares of tulips ( $x$ ) and the number of hectares of carnations ( $y$ ). Place  $x$  on the horizontal axis and  $y$  on the vertical axis. [2 marks]
- Draw the lines which define the inequalities in part (a). [4 marks]
- (ii) Indicate by shading the region defined by the four inequalities. [1 mark]
  - (iii) Write down the coordinates of the vertices of the defined region. [2 marks]

It is known that 7000 tulips per hectare can be cultivated and 10 000 carnations per hectare can be cultivated. The profit per tulip is £0.85 and the profit per carnation is £0.45.

- (c) (i) What is the profit per hectare for
- (a) tulips?
  - (b) carnations? [2 marks]
- (ii) Write an expression for the total profit  $P$  in terms of  $x$  and  $y$ . [1 mark]
- (iii) Showing your method clearly, determine the maximum profit Yasmin and Ismael can make. [2 marks]