



MATHEMATICAL METHODS

Standard Level

Thursday 4 November 1999 (afternoon)

Paper 1

1 hour 30 minutes

A

Candidate name:	Candidate category & number:								
	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 12.5%; height: 20px;"></td> <td style="width: 12.5%;"></td> </tr> </table>								
<p>This examination paper consists of 20 questions. The maximum mark for each question is 4. The maximum mark for this paper is 80.</p> <p style="text-align: center;">INSTRUCTIONS TO CANDIDATES</p> <p>Write your candidate name and number in the boxes above.</p> <p>Do NOT open this examination paper until instructed to do so.</p> <p>Answer ALL questions in the spaces provided.</p> <p>Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.</p>									

B

QUESTIONS ANSWERED
ALL

C

EXAMINER	TEAM LEADER
TOTAL /80	TOTAL /80

D

IBCA
TOTAL /80

EXAMINATION MATERIALS

Required:
IB Statistical Tables
Calculator
Ruler and Compasses

Allowed:
A simple translating dictionary for candidates not working in their own language
Millimetre square graph paper

FORMULAE

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Arithmetic series: $S_n = \frac{n}{2} \{2a + (n-1)d\}$

Geometric series: $S_n = \frac{a(r^n - 1)}{r - 1}$, $r \neq 1$

Arc length of a circle: $s = r\theta$

Area of a sector of a circle: $A = \frac{1}{2} r^2 \theta$

Area of a triangle: $A = \frac{1}{2} ab \sin C$

Statistics: If (x_1, x_2, \dots, x_n) occur with frequencies (f_1, f_2, \dots, f_n) then the mean m and standard deviation s are given by

$$m = \frac{\sum f_i x_i}{\sum f_i} \quad s = \sqrt{\frac{\sum f_i (x_i - m)^2}{\sum f_i}}, \quad i = 1, 2, \dots, n$$

Newton-Raphson formula: (For finding a root of $f(x) = 0$)

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Integration by parts: (Analytical Geometry and Further Calculus Option only)

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

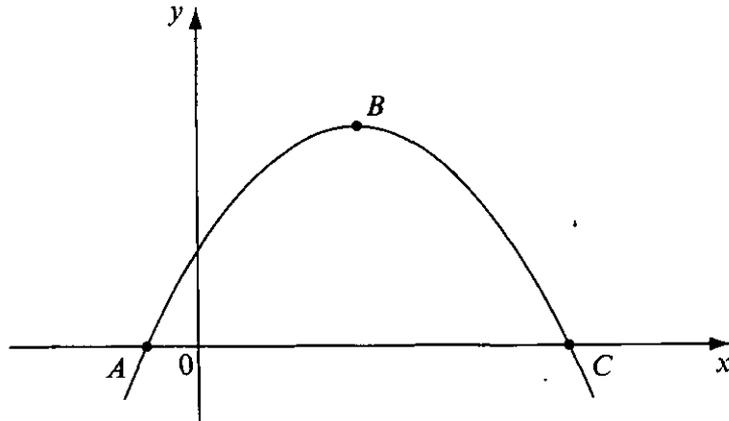
Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary, or on extra sheets of paper provided these are securely fastened to this examination paper.

1. An arithmetic series has five terms. The first term is 2 and the last term is 32 . Find the sum of the series.

Working:

Answer:

2. The diagram shows the parabola $y = (7 - x)(1 + x)$. The points A and C are the x -intercepts and the point B is the maximum point.



Find the coordinates of A , B and C .

Working:

Answers:

3. For the events A and B , $p(A) = 0.6$, $p(B) = 0.8$ and $p(A \cup B) = 1$.

Find

(a) $p(A \cap B)$

(b) $p(\complement A \cup \complement B)$.

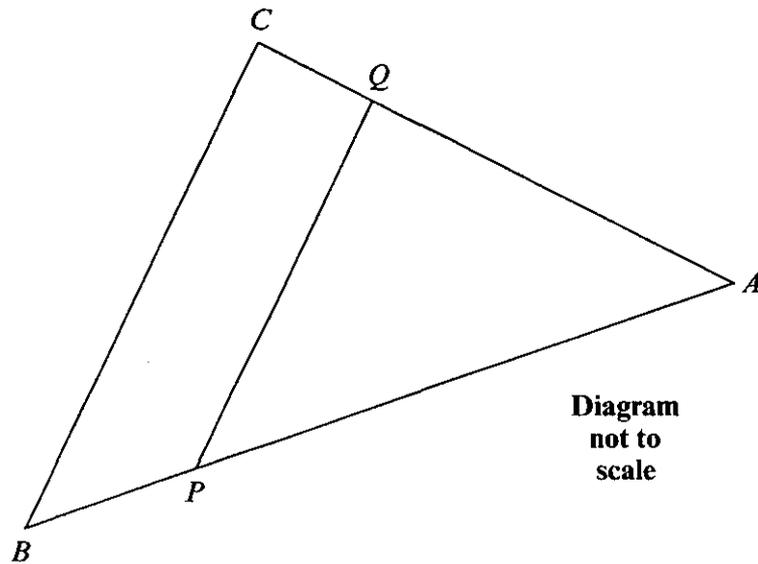
Working:

Answers:

(a) _____

(b) _____

4. In the diagram, triangle ABC is an enlargement of triangle APQ . $PQ = 15$ cm and $BC = 20$ cm and the area of triangle APQ is 36 cm².



- (a) Find the scale factor of the enlargement.
(b) Find the area of triangle ABC .

Working:

Answers:

(a) _____

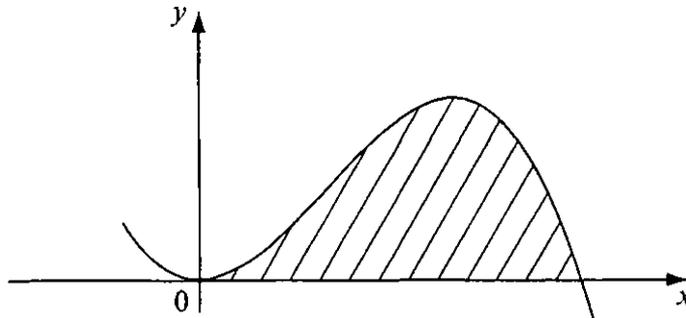
(b) _____

5. At a conference of 100 mathematicians there are 72 men and 28 women. The men have a mean height of 1.79 m and the women have a mean height of 1.62 m . Find the mean height of the 100 mathematicians.

Working:

Answer:

6. The diagram shows part of the graph of $y = 12x^2(1 - x)$.



- (a) Write down an integral which represents the area of the shaded region.
- (b) Find the area of the shaded region.

Working:

Answers:

(a) _____

(b) _____

7. Differentiate with respect to x :

(a) $(x^2 + 1)^2$

(b) $\ln(3x - 1)$.

Working:

Answers:

(a) _____

(b) _____

8. The mean of the population x_1, x_2, \dots, x_{25} is m . Given that $\sum_{i=1}^{25} x_i = 300$ and

$$\sum_{i=1}^{25} (x_i - m)^2 = 625, \text{ find}$$

(a) the value of m ;

(b) the standard deviation of the population.

Working:

Answers:

(a) _____

(b) _____

9. Solve the equation $3 \sin^2 x = \cos^2 x$, for $0^\circ \leq x \leq 180^\circ$.

Working:

Answers:

10. A and B are 2×2 matrices, where $A = \begin{bmatrix} 5 & 2 \\ 2 & 0 \end{bmatrix}$ and $BA = \begin{bmatrix} 11 & 2 \\ 44 & 8 \end{bmatrix}$. Find B .

Working:

Answer:

11. The quadrilateral $OABC$ has vertices with coordinates $O(0, 0)$ $A(5, 1)$ $B(10, 5)$ and $C(2, 7)$.

(a) Find the vectors \vec{OB} and \vec{AC} .

(b) Find the angle between the diagonals of the quadrilateral $OABC$.

Working:

Answers:

(a) _____

(b) _____

12. Find the coefficient of a^3b^4 in the expansion of $(5a + b)^7$.

Working:

Answer:

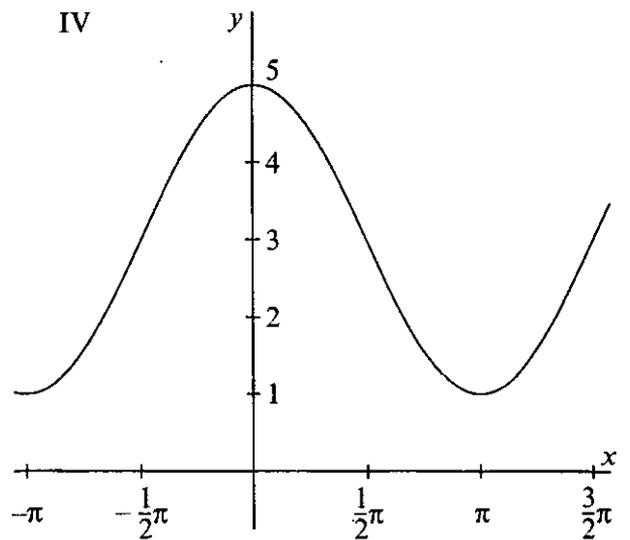
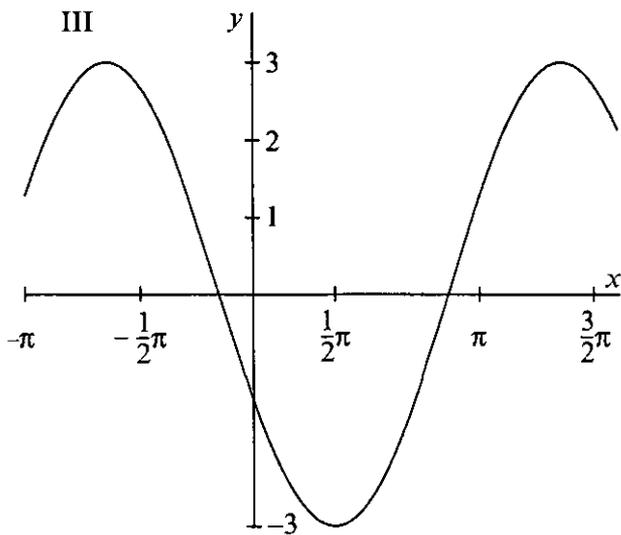
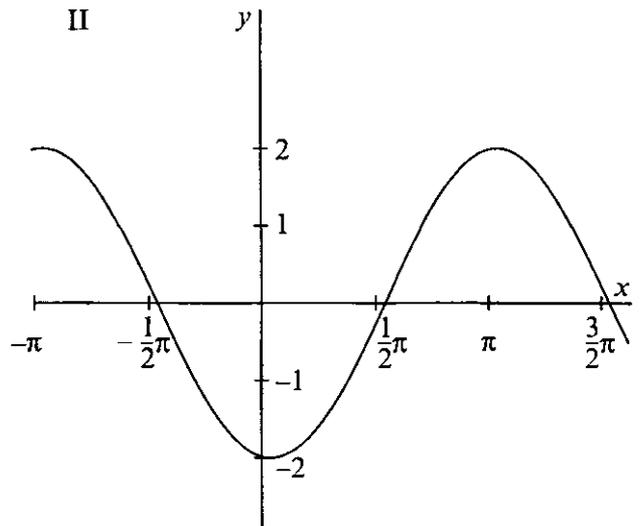
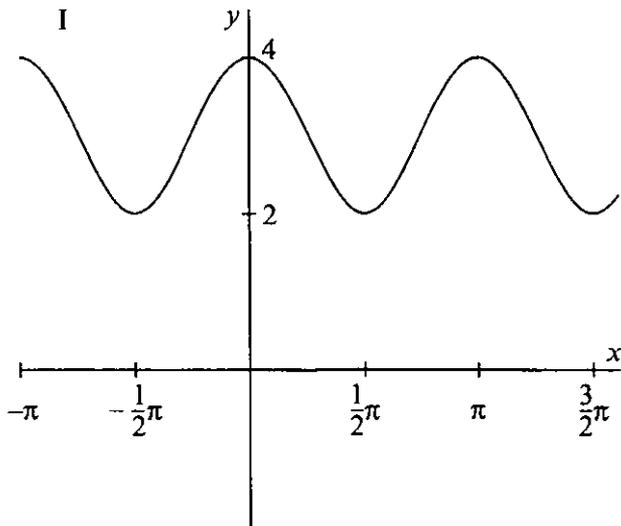
13. Three of the following diagrams I , II , III , IV represent the graphs of

(a) $y = 3 + \cos 2x$

(b) $y = 3 \cos(x + 2)$

(c) $y = 2 \cos x + 3$.

Identify which diagram represents which graph.



(This question continues on the following page)

(Question 13 continued)

Working:

Answers:

(a) _____

(b) _____

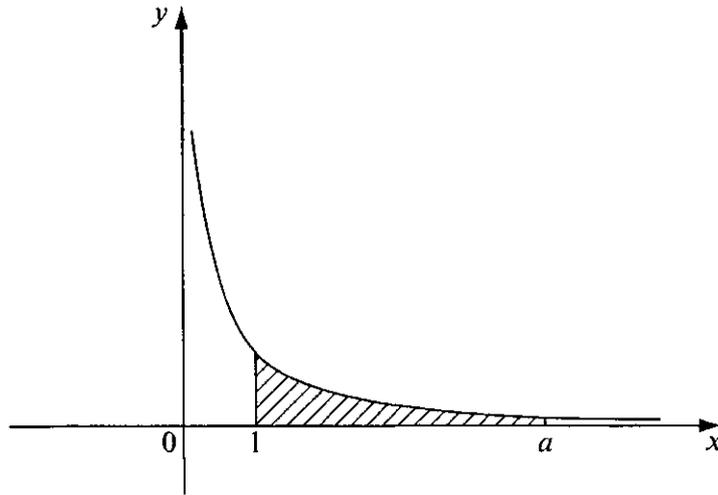
(c) _____

14. Solve the equation $4^{3x-1} = 1.5625 \times 10^{-2}$.

Working:

Answer:

15. The diagram shows part of the graph of $y = \frac{1}{x}$. The area of the shaded region is 2 units.



Find the exact value of a .

Working:

Answer:

16. The function f is given by $f(x) = \sqrt{\ln(x - 2)}$. Find the domain of the function.

Working:

Answer:

17. The two lines $2x + y = 0$ and $ax + by = 5$ are perpendicular and intersect at the point $(1, -2)$. Find the values of a and b .

Working:

Answers:

18. A population of bacteria is growing at the rate of 2.3 % per minute. How long will it take for the size of the population to double? Give your answer to the nearest minute.

Working:

Answer:

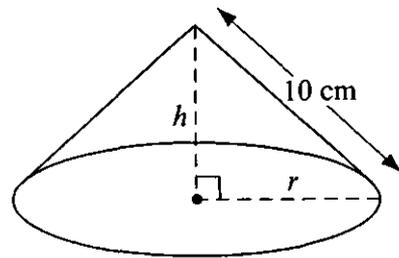
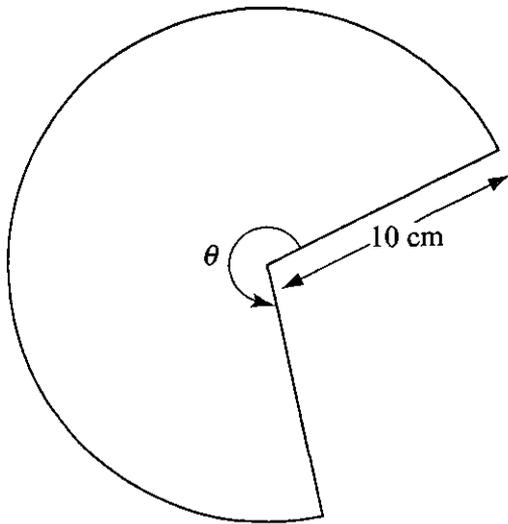
19. Let $f(x) = \sqrt{x}$, and $g(x) = 2^x$. Solve the equation

$$(f^{-1} \circ g)(x) = 0.25.$$

Working:

Answer:

20. The diagrams show a circular sector of radius 10 cm and angle θ radians which is formed into a cone of slant height 10 cm. The vertical height h of the cone is equal to the radius r of its base. Find the angle θ radians.



Working:

Answer: